Abstract - The standard measure of the 75–year Social Security imbalance is 1.87 percent of payroll, but raising taxes this much would not make finances sustainable. We consider exact and approximate infinite horizon measures based on different strategies for extending the projections past 75 years. These indicate imbalances of 3.1 to 5.7 percent of payroll. We also suggest a simpler common sense measure: the tax increase necessary to make the Trust Fund Ratio equal in the 74th and 75th years. Under the Trustees’ mortality projection, it indicates 3.1 percent imbalance, with a more rapid mortality decline of 4.2 percent. All sustainability measures indicate greater imbalance than is currently thought.

INTRODUCTION

Despite its current surplus and growing Trust Fund, the U.S. Social Security system is in long–term imbalance, primarily due to future population aging ushered in by the retirement of the baby boom generations. According to the most recent Trustees report at the time of this study (Board of Trustees, 2002), the Office of the Actuary of the Social Security Administration (SSA) projects that even with legislated increases in the normal retirement age in the coming decades, the Trust Fund will be exhausted in 2038. The assessment in the 2002 Trustees Report differs only slightly from those of 2000 and 2001. The projected date of exhaustion is 2041 instead of 2037 or 2038, and the 75–Year Summary Actuary Balance is –1.87 percent of payroll instead of –1.89 percent or –1.86 percent.

Imbalance is measured as the present value of future taxes minus future benefits, plus initial assets, divided by GDP in 1994. Calculations are presented for real discount rates of 3, 5, and 7 percent, and rates of productivity growth of 1, 1.5, and 2 percent. Comparisons reported in the text were for a discount rate of 3 percent and productivity growth rate of 1 percent, corresponding to the assumptions of the Trustees Report (2000). The reported imbalance for the U.S. is 62 percent of GDP. For Belgium, Norway, Portugal and Sweden it is over 200 percent, and for New Zealand and Denmark it is over 300 percent.
nonetheless a prominent problem that attracts a great deal of attention from politicians, policy analysts, academics, and the general public.

Indeed, in the U.S., the long-run financial problems of Social Security are once again in the news and on the political agenda. Why can’t Social Security be fixed and stay fixed? One reason is the way we commonly measure long-term financial balance in the system. The most prominent measure of long-run solvency of the Trust Fund, and the most common test of proposed policies, is the 75-Year Actuarial Balance. Unfortunately, it has serious limitations for this purpose because it assesses solvency only over a 75-year horizon. Fixing the system according to this criterion, while it would ensure its balance for the initial 75 years, would leave it in serious imbalance thereafter. Furthermore, since one year from now, the end date of the 75-year horizon will have moved a year farther into the future period of deficit spending, the newly calculated 75-Year Actuarial Balance will find even a “fixed” system to be newly out of balance. This measurement problem can be solved by constructing measures that require sustainability beyond the 75-year horizon, perhaps even over an infinite horizon. Such measures can also be used to evaluate competing policy proposals for restoring solvency.

Infinitely many policies are consistent with sustainable Social Security. Some of these would raise taxes immediately and build up a large trust fund so as to avoid additional tax increases in the future. Others might raise taxes slowly, ending up with higher taxes for future generations than under the immediate tax increase policies. Still others might do nothing for now, leading to a large trust fund debt. This also could be a sustainable policy, provided that it included a plan to raise taxes in the future, in order to stabilize the trust fund debt. These various policies would imply very different patterns of intergenerational transfers. We do not argue here for one or another of these. Our point is only that the choice of policy should be made with eyes open to the longer-run future. Any proposed policy should provide explicitly for the indefinite continuation of the system beyond the initial 75-year horizon.

Some reform proposals involve privatization and/or investment in equities. We do not consider such proposals here. Instead, we focus on proposals that keep the current structure of Social Security: a partially funded tax and transfer program, with the fund invested in U.S. Treasury bonds. Such proposals might change taxes (raising the rate, removing the cap) or benefits (reducing the COLA, raising normal retirement age, indexing benefits to life expectancy, making benefits need-based). For simplicity sake, we will discuss only changes in taxes, but the extension to changes in benefits is simple and straightforward.

MEASURING THE IMBALANCE: LIMITS OF THE 75-YEAR ACTUARIAL BALANCE MEASURE

How big is the imbalance in the long-term finances of the Social Security system (OASDI)? The most commonly used measure is the 75-Year Actuarial Balance. Virtually all the plans that have been proposed by politicians and analysts are based on this measure, and discussions in the media revolve around it as well. The 75-Year Actuarial Balance equals the current amount in the trust fund, plus the present value of projected taxes minus expenditures over the next 75 years, minus the present value of one year’s projected costs in 75 years, all expressed as a percentage of the present value of projected taxable payroll over the next 75 years. According to the year 2000 estimate by the Social Security Office of the Actuary, the 75-year Actuarial Balance is 1.89 percent. That is, the payroll tax rate would
need to be raised by 1.89 percentage points, from its current level of 12.4 percent to 14.29 percent, in order to achieve long–term actuarial balance (Board of Trustees, 2000). If the payroll tax rate were immediately and permanently raised by this amount, then there would be sufficient funds to make all necessary expenditures over the next 75 years, and to have a trust fund equal to one year’s costs at the end of this period.

This measure indicates whether the system will remain solvent over the remaining lifetime of almost all participants currently contributing to the system. In 75 years, surviving contributors currently age 20 would be age 95. The difficulty is that even with the payroll increase of 1.89 percent indicated by the 75–year Actuarial Balance, after 79 years the Trust Fund would be exhausted and taxes would cover only 78 percent of projected costs. Then the payroll tax rate would have to be raised by a further 4.3 percentage points to 18.6 percent in order to cover costs, or benefits would have to be cut correspondingly (Board of Trustees, 2000). After 75 years, the system would be in a far more precarious situation than it is now. For this reason, some analysts have called for additional measures of the long–term soundness of the system (The 1999 Technical Panel).

Figure 1 illustrates this problem. It plots the projected Trust Fund level under the current tax rate, and after a 1.89 percent increase, over the next 80 years. Since it would take about 4.5 years to exhaust the final balance equal to one year’s costs \( \frac{1}{1 - .78} = 4.5 \), the trust fund would hit 0 between 2079 and 2080, and would drop increasingly rapidly thereafter.

Here is a different way to look at the same point: Each year, the Trustees issue a new Report that updates the 75–year Actuarial Balance. Each new Balance includes one more year of deficit far in the future, when expenditures exceed tax revenues, and the previous year’s actuarial balance grows by the interest rate. For this reason, referred to as “change in the valuation period,” each year the new Balance tends to be worse than the previous. The year 2001 report indicates that the “change in the valuation period” since the previous year’s report resulted in a worsening of the 75–year Actuarial Balance by .07 percent (Board of Trustees, 2001). Under the Trustees Report’s assumptions, this source of change is a certain and completely predictable result of the passage of an additional year, and is a consequence of choosing a finite horizon. Although the size of the annual change is small, it is repeated systematically year after year, and so it cumulates. Based on the last twelve Trustees Reports, the cumulated change since 1989 due to valuation period is .74. The size of the changes has increased steadily over that period, and will presumably be larger in the coming decade. Since this deterioration in the 75–year Actuarial Balance is predictable,

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3 According to the 2001 report, the 75-year Summary Actuarial Balance is 1.86 percent. This slight improvement (+0.03 percent) over 2000 is primarily due to the use of updated data on birth rate and immigration rate in demographic assumptions. See section IV.B.7 “Reasons for Change in Actuarial Balance from Last Report” and Table IV.B9 (Board of Trustees 2001, pages 64-66) for detailed discussion on the changes in assumptions between two reports. Throughout the analysis, we have used the information from the 2000 Trustees report, instead of the 2001 report. This is because the elaborate program we use to generate deterministic and stochastic Social Security projections has built into it the assumptions of the 2000 Trustees Report.

4 The payroll tax rate is currently set at 12.4 percent. However, taxes on benefits generate additional revenues, so that the current tax income is 12.65 percent of payroll rather than 12.4 percent. By 2075, this is projected to rise to 13.34 percent (all numbers based on Board of Trustees, 2000 Table III.A.2). The calculations reported in this paragraph reflect this increased revenue from taxes on benefits.

5 Since there are other changes, which also alter the AB75, the net result from all causes may be either improvement or deterioration.
it should be possible to construct a measure which reflects it in advance, and therefore does not change over time as the horizon is shifted farther into the future.

The most recent Technical Advisory Panel for Social Security endorsed this kind of concern with “sustainability” over the longer term. Stating that “emphasis on the 75–year actuarial balance is misleading,” the panel noted that “many designers of reform try to reach balance simply by targeting their plans only at the 75–year actuarial deficit . . . [and] . . . usually end up in a situation where their reforms only last a year before being shown out of 75–year balance again” (1999). However, the panel also recognized that “there is no way to demonstrate long-term patterns of sustainability . . .” (1999). This paper attempts to fill this gap.

Figure 1. Social Security Fund Projections Under Current Law and after a 1.89% Payroll Tax Increase, 2000–2080

![Diagram showing Social Security Fund Projections](image)

Note: Trust Fund Projection: For 2000–75, authors’ calculation based on the information from the 2000 Trustees’ Report (see footnote 4 in the text or detail). For 2076–80, the figures are based on linear extrapolation of those between 2070 and 2075. Income and Cost Rates are under current law, and are taken from Table III.A.2 :Table III.A.2, 2000 Trustees Report.

A SIMPLE CRITERION FOR SOCIAL SECURITY SUSTAINABILITY: THE FLAT FUND RATIO

As a first step to address the sustainability of Social Security, we suggest a simple common sense criterion: The Trust Fund Ratio should be constant or flat at the end of the 75–year projection horizon (that is, it should be equal in the 74th and 75th years of the projection), or equivalently that the Trust Fund should be growing at the same rate as Costs. We call this the “Flat Fund Ratio” criterion, and the Flat Fund Ratio tax is the constant payroll tax rate that would be necessary to achieve a Flat Fund Ratio for the last two years at the end of the projection horizon. The difference between the current tax rate and the Flat Fund Ratio tax rate is a measure of the imbalance in the system,
expressed as a proportion of the present value of payroll. It is directly comparable to the current 75-year Actuarial Balance measure. It appears that this criterion, or one like it, was used to assess the proposals of the 1994–96 Advisory Council (Goss, 1999). To maintain the Trust Fund Ratio constant at the end of the forecast period, the Fund must be just large enough so that the interest earned on the Fund is large enough to make up the difference between the costs and the tax revenues, plus enough to make the Fund grow at exactly the rate that costs grow, so that it will be able to do the same in the next year as well.

The Flat Fund Ratio Criterion requires that $F(t)/C(t)$ be constant over time in the neighborhood of some designated year $T$, which we will generally take to be 75 years after the projection baseline, consistent with the horizon used by the Trustees. Note that if it is constant between, say, 2074 and 2075, that does not imply that it will be constant thereafter; in general it will not. The condition implies that $F(T)$ must grow at the same rate as $C(T)$, say $\beta$, or that $F(T + 1) = (1 + \beta)F(T)$. We can also express $F(T + 1)$ in terms of the flows into and out of the system between $T$ and $T + 1$, yielding: $F(T + 1) = rF(T) + (\tau + \theta)W(T) - C(T)$, where $W(T)$ is the taxable payroll in year $T$, $\tau$ is the current payroll tax rate, $\theta$ is some unknown increase in the payroll tax rate, which is assumed to be implemented in year $T$, and $r$ is the rate of interest earned on the trust fund. Equating the two expressions for $F(T + 1)$, we find that to keep the fund/expenditure ratio constant between time $T$ and $T + 1$ requires:

$$[1] \quad (\tau + \theta)W(T) - C(T) = F(T)(\beta - r).$$

Taking $F(T)$ as given, we can solve this for the tax increase, $\theta$, which would be necessary to keep the fund ratio constant around time $T$ if it were instituted starting in year $T$. If the fund is projected to be zero at $T$, then $\theta$ will be the tax increase necessary to make tax revenues equal costs in year $T$. Otherwise, interest on the fund, whether positive or negative, must be taken into account.

To illustrate the use of equation [1], suppose that current policy was maintained through 2075. To achieve sustainability thereafter, the necessary payroll tax rate starting in 2075 would be 23 percent, a 10.5 percent increase. In this case, the payroll tax increase must be large enough to pay interest on the large Trust Fund deficit that would have been accumulated if current policy were maintained. Alternatively, suppose that the payroll tax were immediately and permanently increased by 1.89 percent. Then to achieve sustainability after 2075, the payroll tax rate would have to be raised by 6 percentage points to 18.4 percent, requiring an additional tax rate increase of 4.1 percent, on top of the original 1.89 percent. In this case, the Fund would be projected to be near zero in 2075, so the tax increase brings tax revenues up to meet costs.

More to the point, however, we would like to calculate the tax increase necessary to achieve a flat fund ratio at $T$ if it were instituted immediately, rather than in 2075. In this case, a current change in taxes will also change the fund balance at time $T$, so that $F(T)$ should be treated as a function of the tax rate: $F(T) = F(T, \tau)$, where the tax rate is understood to be constant up to time $T$. We can now consider a permanent increase in the tax rate that is instituted today, call it $\delta$, as opposed to $\theta$ which is a tax imposed starting at time $T$. The difficulty in applying the condition above is that we do not know $F(T, \tau + \delta)$, which will depend on the present value of payroll up to time $T$, $PV(W, T)$. The Appendix derives the equation for $\delta$ in terms of the fund balance projected under the current tax regime, $\tau$, and the present value of payroll over the forecast horizon, $PV(W, T)$, as follows:
ESTIMATE OF FLAT FUND RATIO TAX BASED ON TRUSTEES REPORT

All the items on the right side of equation [2] can be found in the Trustees Report for any recent year, or calculated from data in it, using either nominal or constant dollar values. \( \beta \), the growth rate of costs, can be calculated as \( \ln \left( \frac{C(T)}{C(T-1)} \right) \). The present value of taxable payroll is calculated using the projected value of payroll for each year through \( T \).

Using the year 2000 Trustees Report, we find \( \delta \) is 3.1 percent. This means that in order to achieve a projected Trust Fund Ratio in 2075 equal to the Ratio in 2074, it would be necessary to raise the payroll tax rate immediately from 12.4 percent to 15.5 percent. This is in contrast to the 1.89 percent that would be required to achieve long–run balance according to the conventional 75–year Summary Actuarial Balance measure. Note that both criteria use projections over the same 75–year horizon, and neither makes any implicit or explicit assumption about what happens after 2075.

Figure 2 plots projected Trust Fund Ratios through 2100, based on the year 2000 Trustees Report assumptions, which are here assumed to continue to hold from 2075 to 2100. As expected, we see that the Ratio falls to zero in 2037 under current policy, and that it falls to zero in 2078 if the payroll tax rate is immediately raised by 1.89 percent. We also see that if the payroll tax rate is immediately raised by 3.1 percent, then the Ratio rises until 2074, and remains constant at this peak value in 2075, meeting the Flat Fund Ratio criterion. However, thereafter it begins to decline. We see that the Flat Fund Ratio

\[
F(T, \tau) + \frac{\tau W(T) - C(T)}{r - \beta} e^{\gamma PV(W, T) + \frac{W(T)}{r - \beta}}
\]

Note: Fund/Cost ratio, or trust fund ratio, is the ratio of the assets at the beginning of the year to the expenditures during that year (Trustees' Report 2000, page 222). See text for definition of the tax rate regimes. The ratios are based on the authors' calculation using Stochastic Social Security Simulator (MVR, 2001) to generate the SSA's Intermediate projections over a 100–year horizon and apply appropriate tax rate increases to calculate increased tax contributions to the trust fund.

Figure 2. Trust Fund/Cost Ratios for Different Tax Rate Regimes, Based on SSA Intermediate Projections, 2000–2099

Note: Fund/Cost ratio, or trust fund ratio, is the ratio of the assets at the beginning of the year to the expenditures during that year (Trustees' Report 2000, page 222). See text for definition of the tax rate regimes. The ratios are based on the authors' calculation using Stochastic Social Security Simulator (MVR, 2001) to generate the SSA's Intermediate projections over a 100–year horizon and apply appropriate tax rate increases to calculate increased tax contributions to the trust fund.
criterion does not insure long–run sustainability, if the Trustees Report assumptions are continued into the future. It is a weak definition of sustainability, yielding a low estimate of the tax increase needed for sustainability. This observation suggests that we should explore more rigorously the conditions necessary for a truly sustainable system and the relation of the Flat Fund Criterion to such conditions.

INFINITE HORIZON MEASURES OF SUSTAINABILITY

Although the Flat Fund Ratio criterion has a common sense appeal, it is ad hoc and the choice of a 75–year horizon is arbitrary. Extension of the Actuary’s calculations of Summary Actuarial Balance for horizons longer than 75 years shows that the imbalance continues to rise as the horizon lengthens, still growing slowly after 300 years. In this section we will consider three strategies for carrying out infinite horizon calculations of balance, and show that the Flat Fund Ratio criterion is equivalent to one of these strategies.

Prior to 1965 the Actuaries assessed solvency over an infinite horizon or “in perpetuity” (Myers, 1959). After 1965, the infinite horizon was replaced by a 75–year horizon on the recommendation of the Advisory Council (Goss, 1999). According to Goss this had a relatively small effect on the long–run cost projections at that time, because costs were projected to remain flat in any case, rather than rising exponentially as they do now. Starting in 1973, the projections began to assume a changing time path for earnings and benefits, since new legislation linked benefits to past earnings. Currently, a variety of measures are employed. While the projected date of fund exhaustion and the 75–year actuarial balance are best known and most influential, many other figures for the short, medium, and long run are also calculated and presented. However, no current measure is similar to the former infinite–horizon measure.

There is good reason to require balance over an infinite horizon. Social Security is structured in a very particular way. Financing current benefits largely out of taxes on current workers automatically creates new obligations to pay benefits in the future. The young generation that pays benefits to the old generation must itself be supported in its old age by the next young generation. Social Security is built on the concept of linked benefits and contributions across generations over an infinite time horizon. The net obligations at any moment to those who have paid in but not yet received benefits are the system’s implicit debt and the program could not be shut down without either canceling the implicit debt—a policy that no one recommends—or providing for its repayment. Thus, as long as one considers policies to continue Social Security in its current form, assessment and balance over an infinite horizon are not only appropriate but also necessary. In this regard, our approach is quite similar to Generational Accounting.7

The most direct approach to the infinite horizon assessment is to continue the 75–year projection farther into the future, say for 300 or 500 years, when the power of discounting would render further exten-

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6 Some policy alternatives, such as privatizing Social Security and replacing it with a system of funded private retirement accounts, do not require an infinite time horizon since they generally include a plan for repayment of this implicit debt.

7 Like Generational Accounting (Auerbach et al., 1991; Auerbach et al., 1999) we start with the premise that the budget must be balanced over an infinite horizon, and consider the kinds of policy changes that would be required to achieve this balance. Our approach also differs in a number of ways. Generational Accounting is based on generations, while we focus on periods. Generational Accounting is applied in a comprehensive way to government budgets, whereas we only examine Social Security finances. These differences arise from the practicalities of implementation rather than from important conceptual differences.
sion irrelevant. This extended projection can be achieved by simply continuing the long–run rates of covered wage growth and real interest rates assumed for the 75–year horizon. For the demography, the values of fertility, mortality, and immigration assumed at the end of the 75–year horizon can likewise be continued. This is the approach used in Generational Accounting, which also requires very long forecast horizons (see Auerbach et al., 1999). Goss (1999) reports an infinite horizon actuarial balance of −4.7 percent based on the assumptions of the 1995 Trustees Report (1999). The Infinite Horizon imbalance in the system would then be 4.7 percent of the present value of pay–roll, in contrast to the 2.19 percent of pay–roll indicated by the 75–year Actuarial Balance calculation in the 1995 report.

Although this approach is straightforward, many people already believe that the 75–year horizon is too distant to be meaningful, and that detailed projections over longer horizons suggest a false precision. A simpler projection assumption is that after 75 years (or some other interval, T), the system will have settled into a steady state in which rates of growth of costs and tax revenues are thereafter constant, although not necessarily equal. That is, after T years, the tax income in year t, I(t) = τW(t), and the costs in year t, C(T), grow at constant exponential rates α and β, respectively. With this assumption, we can work from the situation at the end of the detailed projection (for example, after T = 75 years), using the information published in the Trustees Report, without extending the detailed projection beyond T. We could seek a specific one–time, immediate and permanent payroll tax rate increase δ∞ that would achieve actuarial balance over the infinite horizon under this steady–state condition. δ∞ would then measure the Social Security imbalance as a proportion of the present value of pay–roll over the infinite horizon, in a manner exactly analogous to the currently used 75–year Actuarial Balance measure.

The Appendix derives the following expression for δ∞:

$$\delta_\infty = -\frac{F(T) + \frac{I(T)}{r - \alpha} - \frac{C(T)}{r - \beta}}{e^{rT} PV(W, T) + \frac{W(T)}{r - \alpha}}.$$  \[3\]

First consider the general case, in which the growth rates of costs and tax revenues differ (α ≠ β). Calculating these from the last two years of the Trustees 75–year projection period we find that tax revenues grow at α = 1.16 percent per year, while costs grow more rapidly at β = 1.47 percent per year. One cause of this discrepancy is the projected continuation of mortality decline, which leads to an ever–rising old–age dependency ratio, as we will discuss below. When these growth rates are not equal, we will call the resulting calculation of imbalance the Unstable Infinite Horizon imbalance.

In the other case, growth rates of pay–roll and costs are equal (α = β). The Trustees’ projections indicate that under current policy, tax revenues will cover only two–thirds of costs in 2075. Under this assumption, the proportional gap would remain constant thereafter under current policy. We will call the resulting calculation of imbalance the Stable Infinite Horizon imbalance. Under this condition, equation [3] is slightly simplified, and becomes:

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8 Although details are not given, it appears that Goss calculated the 500–year Actuarial Balance from extended projections of this sort. We have replicated this result by extending our population projection out for 300 years based on the 1995 Trustees Report assumptions, and then after year 75, assuming that the tax income per person 20–64 continues to grow at the same exponential rate as observed at 75 years, and that the benefit per person 65+ likewise continues to grow at the same exponential rate thereafter.

9 While the mathematical expression below is not transparent, the intuition is very simple. According to Trustees’ projections, a tax increase in 2000 of 1.89 percent would result in a Trust Fund near zero in 2075, at which...
Comparison shows that this is identical to Equation [2] for the Flat Fund Ratio $\delta$. The Appendix shows that any system that is balanced over an infinite horizon and is stable from time $T$ must have a constant Trust Fund Ratio from time $T$, thus satisfying the Flat Fund Ratio criterion. We can conclude that the commonsensical Flat Fund Ratio criterion, which requires no projection or equivalent assumptions past the Trustees’ projection horizon, would be numerically equivalent to the infinite horizon measure under the assumption that the growth rates of payroll and costs were constant and equal after the Trustees’ projection horizon. This observation clarifies the meaning of the Flat Fund Ratio criterion. It also suggests why the Trust Fund Ratio declines after 2075 in Figure 2, even after the tax increase indicated by the Flat Fund Ratio criterion: $\alpha$ does not equal $\beta$ so the stable assumption is not met. While the indicated tax increase does make equal the Fund Ratio in 2074 and 2075, thereafter the ratio declines since costs grow more rapidly than tax revenues.

The second and fourth columns of Table 1 present measures of imbalance, based on the Trustees’ projection assumptions. The Trustees’ main measure of imbalance is the 75–year Summary Actuarial Balance, which in the year 2000 report was 1.89 percent. As already discussed, the Flat Fund Ratio measure is 3.1 percent, which equals the Stable Infinite Horizon measure. For the more realistic Unstable Infinite Horizon measure, which acknowledges that after 2075, costs grow more rapidly than payroll, we find 3.7 percent. Finally, if we simply continue a more detailed projection over a 300–year horizon, we find 3.5 percent. All these alternative measures of imbalance are substantially greater than the conventional measure.

These measures are expressed relative to the present value of payroll, but we can also express them in current dollars. The 75–year Summary Actuarial Balance measure is multiplied times the present value of payroll through 2075, yielding 3.08 trillion dollars. A cash infusion of this

$$F(T) + \frac{I(T) - C(T)}{r - \beta} \text{ if } \alpha = \beta.$$
amount today would keep the system afloat until 2075, according to the Actuaries’ projections. The Flat Fund Ratio measure also has a 75–year horizon, since projections are not continued past this point. Over this horizon, the imbalance is 5.05 trillion dollars, or 64 percent greater. The Stable Infinite Horizon measure is identical over the 75–year horizon, but over the infinite horizon, the present value of imbalance is 6.90 trillion dollars. The Unstable Infinite Horizon measure is higher still, at 8.37 trillion. The 300–year horizon calculation has a slightly lower imbalance at 7.76 trillion, because the long–run demography implies a slowing of the rate of increase of costs, as we discuss in the next section.

DEMOGRAPHY IN THE VERY LONG RUN

Consider the behavior of very long–run demographic projections in relation to the stable and unstable infinite horizon assumptions. Most long–term demographic projections assume that fertility will remain constant at some level after a short initial period of transition, and assume that the annual flow of net immigrants will be a constant number. Mortality, however, is often projected to continue to decline indefinitely, in the sense that it is still declining at the end of the forecast horizon.

In the long run, survival to age 65 will approach 1.0; it is already projected by the Actuaries to be .87 for the generation born in 2000 (sexes combined, cohort projection). The U.S. Total Fertility Rate is projected by the Actuaries to approach 1.95, which would lead eventually to population decline if there were no net immigration. Net immigration is assumed to remain constant at roughly one million per year (Board of Trustees, 2001). As time passes, the population up to age 65 will increase or decrease until the net immigration flow is just sufficient to offset the natural decrease due to low fertility. At that point, the population below age 65 will reach a steady state age distribution, with constant size.

Now consider the population above age 65, Pop(65+). Each year, it will be augmented by an inflow equal to the number of 64–year olds in the previous year, a number that eventually will be constant from year to year, call it Pop(65). Assume that all net immigrants arrive or depart before the age of 65. Then the size of Pop(65+) will approximately equal this inflow times the life expectancy at age 65, $e_{65}$. Since $e_{65}$ will continue to increase, so will Pop(65+), and so will the old–age dependency ratio. Vaupel (1986) has shown that if age–specific death rates continue to decline at a constant proportional rate over the long term, then $e_{65}$ will eventually rise at a constant linear rate. In particular, if mortality at all ages over 65 declines at 1 percent per year, then $e_{65}$ will rise at approximately one year per decade. A constant linear increase in $e_{65}$ will imply a declining proportional rate of increase in $e_{65}$, eventually approaching zero as the centuries pass.\[10\]

We can conclude that for a long time, the growth rate of the elderly population, and therefore of costs, will exceed the growth rate of the working–age population and therefore of covered wages and tax revenues. Therefore, the stable assumption is initially wrong, and will un-

\[10\] By arguments given in Vaupel (1986), we know that $e_{65}(t)$ will be increasing by roughly .1 years for each one year increase in $t$. After 300 years, $e_{65}$ will be roughly 30 years greater. Since $e_{65}(2000)$ is about 18 years, an increment of .1 years per year initially contributes a growth rate of .1/18 = .55 percent. In 2075, $e_{65}$ may be about 25 years, and the growth rate will have fallen to .40 percent. After 300 years, $e_{65}$ may be 48 years, and the growth rate will then be only .1/48 = .2 percent. Thus the elderly population will continue to grow after the working–age population has ceased to grow, and its growth rate will initially (in 2075) exceed that of the working–age population by about .4 percent, which will be halved after another 225 years.
derestimate the pressures on the system. In the very long run, however, the assumption of equal rates of change of taxes and expenditures is appropriate, but this convergence occurs at a slower rate than observed initially (say in 2075), and during the transition to the long run, a larger gap between taxes and expenditures opens up due to the slowing of population growth, increasing $e_{65}$ and consequent population aging. These points are confirmed by a demographic projection over a 300–year horizon.\textsuperscript{11}

We will present results calculated according to each of these strategies. However, our general view is that there is little value in spinning out over several centuries the implications of detailed demographic assumptions that are so fragile. A slight change in the assumptions for fertility or immigration would have a large effect on the outcome. The appeal of the stable and unstable assumptions is that they are rough and simple, and can be implemented from existing projections. We expect that the stable assumption will lead to an underestimate of the imbalance, while the unstable assumption will lead to an overestimate, for the reasons just discussed.

HOW FAST WILL MORTALITY DECLINE?

Many analysts believe that the Intermediate mortality assumption of the Trustees Report represents an unrealistically slow rate of mortality decline (Stoto and Durch, 1993; Technical Panel Report, 1999: 3, 22, 64–7; Lee and Miller, 2001). Lee and Carter (1992) developed a new method for forecasting mortality, which projected life expectancy gains at almost double the rate projected by the Social Security Actuaries at that time. These Lee–Carter forecasts are incorporated in new stochastic and deterministic population forecasts by Lee and Tuljapurkar (1994 and 2000). Although the Actuaries have raised their life expectancy forecast slightly in response to the recommendation of the Technical Advisory Panel (Board of Trustees, 2000), their forecast of gains is still substantially below the recommendation and the Lee–Carter forecast. An updated application of the Lee–Carter method forecasts life expectancy in 2075 of 85.9 years, for sexes combined, in contrast to just under 83 in the 2000 Trustees Report. If mortality follows the Lee–Carter path, then the system imbalance will be greater in 2075, with a larger deficit and a greater Trust Fund debt. In addition, the elderly population will be growing more rapidly in 2075 than under the Actuaries’ projections, at .74 percent versus .49 percent, with other demographic assumptions unchanged.

Table 1 also presents estimates of imbalance that are based on the Lee–Carter mortality forecast but otherwise with the same projection assumptions as the first column, identical to the year 2000 Trustees Report. These estimates were made using the MVR–Berkeley Social Security simulation program in deterministic mode (unpublished runs; see Lee and Tuljapurkar, 2000.\textsuperscript{12} The third column shows how much the imbalance increases with more rapid mortality decline. The 75–year Actuarial Balance measure is .5 percent higher, and the gap is larger for the sustainability measures, rising to 1.1 percent, then 2.0 percent. The sustainability criteria attach more weight to events far in the future compared to the 75–year measure, although they discount the future at the same rate.

\textsuperscript{11} The projection shows the rate of growth of the 65+ population declining from .69 percent per year in 2075, to .14 percent per year in the year 2300. Over this same period, the growth rate of the working–age population declines from .14 percent per year to .05 percent per year. Neither remains constant, obviously, and the difference between the two decreases from .55 percent initially to .09 percent in 2300.

\textsuperscript{12} The program can be run on the web at http://simsoc.demog.berkeley.edu).
FURTHER ASPECTS OF SUSTAINABILITY POLICIES

We have proposed measures of long run imbalance, expressed as the immediate, constant and perpetual tax increase that would be needed to achieve balance. However, these are just measures, and balance could be restored by an infinite variety of trajectories of altered tax or benefit adjustments. With minor modification, Equations [1], [3], and [4] can be used to calculate a sustainable tax rate starting at any future date, given a projected fund balance for that date. More specifically, for any proposed tax and benefit trajectories up to 2075, the implied Fund balance in 2075 could be calculated, and the tax increase needed for sustainability could then be found. Each trajectory of taxes and benefits has a different implication for intergenerational transfers. One simple policy would be to have tax rates increase linearly from now until 2075, ending in a tax rate in that year that would be sustainable thereafter under the stable infinite horizon assumption. The equations can be used to find that the terminal tax rate in 2075 and thereafter would be 17.7 percent of payroll under the Trustees Report assumptions.

Figure 3 plots the Trust Fund Ratio under various tax rate regimes from 2000 to 2099, using the Lee–Tuljapurkar projections with more rapid mortality decline. Under the current legislation scenario, the Fund Ratio peaks in 2013 and declines to zero in 2034, a little sooner than under the Actuaries’ assumptions. With a tax increase of 2.4 percent, which is the 75-year Actuarial Balance under the Lee–Tuljapurkar projections, the Ratio would peak around 2020 and decline to zero in 2078. With a tax increase of 4.2 percent to meet the Flat Fund Ratio criterion of balance, the ratio would peak in 2074–2075. By construction, it is flat between these two dates. If the Stable assumption were true, then the fund ratio would remain constant thereafter, but Figure 3 shows that it is projected to decline after 2075, as in Figure 2. If the tax is raised above 4.5 percent, as under the 300-year Actuarial Balance measure (4.6 percent increase) and the Unstable Infinite Horizon measure (5.7 percent increase), then the fund ratio continues to rise throughout the 21st century. The rapid and continuing increase in the Trust Fund Ratio under the Unstable Infinite Horizon tax is unappealing and politically unrealistic. It continues to rise in order to provide for a long-run future in which the old-age dependency ratio is expected to continue to increase.

Figure 3 also shows that under the linear increase policy, the fund ratio would remain remarkably flat throughout the century, particularly after 2045, tracking the increase in costs quite closely, at a ratio of about 3.5. At this ratio, interest from the fund covers about 10 percent of annual costs.

As one might imagine, the sizes of future Trust Fund implied by these sustainable tax scenarios are impressively high. Under current unsustainable policy, Social Security would accumulate a debt of 29 trillion dollars (2000) by 2075 if it had the authority to borrow; an amount just equal to projected GDP. With the more rapid mortality decline, that would rise to 36 trillion. If taxes were immediately raised by 1.89 percent, then by construction a Trust Fund equal to one year’s cost would be accumulated, or 1.45 trillion (slightly more if rapid mortality decline is anticipated). The Flat Fund Ratio tax (or Stable Infinite Horizon tax) would generate a Fund of 21 trillion in 2075 under the Trustees’ assumptions, which would be 71 percent of projected GDP in 2075. It would generate a Fund of 29 trillion if more rapid mortality decline were anticipated. For the Unstable Infinite Horizon, the Fund in 2075 would be about twice these last sizes, and growing rapidly. Finally, with a policy of gradual linear increase to a sustainable
tax rate in 2075, a more modest Fund of six or seven trillion would be accumulated under the two mortality projections.

All these projections assume that Social Security operations do not affect labor productivity, GDP growth, or interest rates. However, either a debt of a Fund in 2075 equal to GDP, or a substantially increased payroll tax rate, would be inconsistent with the this assumed independence.

CONCLUSION

The 75–year Summary Actuarial Balance, on which policy discussions are currently based, measures solvency over the lifetimes of current workers, but not over the lifetimes of the workers in coming decades who will pay for the retirement of current workers. We must assess system solvency for these workers as well. The sustainability and balance measures developed in this paper fill this need. Unlike the standard 75–year Actuarial Balance, they should require revision only when new information becomes available, and not because of the predictable change in valuation period as time passes. In particular, we propose the Flat Fund Ratio as the basic measure of long–run solvency as a more suitable basis for policy discussions and public debate. Under the 2000 Trustees Report assumptions, the Flat Fund Ratio measure indicates that the system is 3.1 percent out of balance; and with more rapid mortality decline (which we believe more likely), the imbalance rises to 4.2 percent. These figures are substantially higher than the 75–Year Actuarial Balance measure of 1.89 percent.

There are a number of reasons why the Flat Fund Ratio measure is an appealing lower bound estimate of the imbalance. Its rationale is intuitive and consistent with common sense. It is a natural exten-
percent increase in the payroll tax rate would correspond to only 1.7 percent of GDP on a continuing basis. That is, if taxes were immediately and permanently raised by 1.7 percent of GDP today, the system would be put on a sustainable path. Recall, however, that the median imbalance in OECD countries is two and a half times as great as in the U.S., and that some countries have imbalances six times the size of that in the U.S. We have not been able to estimate the Flat Fund Ratio Tax for these other countries, but there is every reason to expect that as for the U.S., it would indicate greater problems than are apparent from the 75–year Actuarial Balance measure reported in Roseveare et al. (1996). For these countries, with their more generous pensions, longer life, lower fertility and earlier retirement, the fiscal problems will be far more painful to confront and resolve.

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We define the following variables, all measured in real terms, net of inflation:

- $F(t)$: the level of the Social Security fund in year $t$
- $W(t)$: the taxable payroll in year $t$
- $C(t)$: the cost in year $t$ for benefits and administration
- $I(t)$: the tax income in year $t$ (payroll tax plus tax on benefits, but not including interest)
- $r$: the interest rate, which is taken equal to the rate earned on the Trust Fund.
- $\tau$: the current tax rate relative to payroll
- $\delta$: the size of a one time immediate and permanent increase in the payroll tax rate needed to achieve a stated goal, such as 75–year Actuarial Balance ($\delta_{75}^{AB}$), Flat Fund Ratio ($\delta_{75}^{FF}$), Stable Infinite Horizon ($\delta_{\infty}^{S}$) or Unstable Infinite Horizon ($\delta_{\infty}^{U}$).
- $T$: the time horizon for an evaluation, which can be finite or infinite

Additional notation will be introduced as necessary.

**Actuarial Balance**

Define the present values of income from taxes and of costs over a time horizon of $T$ years. This presupposes a policy which specifies the tax rate and the costs of benefits over the horizon $T$.

1. $PV(I, T) = \int_0^T e^{-rt}I(s)ds$
2. $PV(C, T) = \int_0^T e^{-rt}C(s)ds$

The actuarial balance over horizon $T$, relative to this policy, is defined as:

$$AB(T) = PV(I, T) + F(0) - PV(C, T + 1)$$

This is usually expressed as a proportion of the present value of taxable payroll over the same horizon, $PV(W, T)$. We will call this proportion $-\delta_{T}(T)$, where $\delta$ can be interpreted as the size of the immediate and permanent increase in the payroll tax rate (relative to the tax rate of the policy) which would achieve balance, or set $AB(T) = 0$, leaving a fund at time $T$ equal to the next year’s costs. In these expressions, $T$ can take any positive value, including infinity. In the calculations of the Actuaries, $T = 75$.

**Actuarial Balance with an Infinite Time Horizon**

It will be useful to divide each infinite integral into two parts: the first over the time period for which a detailed projection is available, which is 75 years in the case of the Social Security system, and the second from this point to infinity, for which detailed projections are not available and a simplifying assumption must be made. The equation for the infinite horizon actuarial balance, given above with $T = \infty$, can be rewritten as:

$$AB^\infty = F(0) + \int_0^T e^{-rt}[I(t) - C(t)]dt + \int_T^\infty e^{-rt}[I(t) - C(t)]dt.$$

Inspection of the first two terms on the right indicates that this is the fund balance at time $T$, discounted to time 0, or $e^{-rT}F(t)$. (Note that this is almost identical to $AB(T)$, differing only by the exclusion of the present value of one year’s costs at the end of the period, $e^{-rT}C(T + 1)$). This can be evaluated using data contained in the Trustees Report.\(^{14}\) The infinite integral requires additional assumptions, as discussed in the text.

\(^{14}\) There are complexities related to tax income derived from taxation of benefits rather than taxation of payroll. We assume that income from the taxation of benefits in year $s > T$, $\theta(s)$, represented as a fraction of expenditures, remains constant. When growth rates of payroll and expenditures differ, $\theta(s)$, expressed in terms of an additional tax rate levied to payroll, will also grow at a constant rate of $\beta - \alpha$. While incorporated in actual calculations, the representation of $\theta(s)$ was not included in this appendix to simplify the mathematical presentation.
1. **Unstable Assumption**: Assume that from \( T \) on, \( I(s) \) and \( C(s) \) grow at constant exponential rates \( \alpha \) and \( \beta \), with both less than \( r \). Substituting and simplifying, we find:

\[
[A.5] \quad AB^\infty = e^{-rT}F(T) + e^{\alpha - r} \int_T e^{\alpha - \beta} dt \\
- e^{\beta - r} C(T) \int_T e^{\beta - r} dt
\]

Relative to the present value of payroll, this is:

\[
[A.6] \quad AB^\infty = e^{-rT} \left[ F(T) + \frac{I(T)}{r - \alpha} - \frac{C(T)}{r - \beta} \right].
\]

Note that the \( AB \) and \( \delta \) are defined relative to a baseline policy, here taken to be continuation of current policy, and \( F, I, C \) and \( W \) all likewise refer to this baseline policy.

2. **Stable Assumption**: Both \( I(S) \) and \( C(S) \) grow at the constant exponential rate \( \beta \) less than \( r \). In this case, the expressions derived above become:

\[
[A.8] \quad AB^\infty = e^{-rT} \left[ F(T) + \frac{I(T) - C(T)}{r - \beta} \right].
\]

\[
[A.9] \quad \delta^{AB^\infty} = - \frac{F(T) + \frac{I(T) - C(T)}{r - \beta}}{e^{rT} PV(W, T) + \frac{W(T)}{r - \beta}}.
\]

The Flat Fund Ratio Criterion

Consider the stable infinite horizon case. Assuming balance, so that \( AB^\infty = 0 \), we have:

\[
[A.10] \quad F(T) = \frac{I(T) - C(T)}{\beta - r}.
\]

The trust fund evolves according to

\[
[A.11] \quad \frac{dF}{dt} = rF + I(t) - C(t).
\]

Combining these we get:

\[
[A.12] \quad \frac{dF}{dt} = BF(T).
\]

Since, by assumption, \( C \) is also growing at the exponential rate \( \beta \), the Fund Ratio will be constant or flat after \( T \), including the case where \( F(T)=0 \). This establishes that the Flat Fund Ratio after \( T \) is a necessary condition for balance over an infinite horizon, provided that the system is stable after \( T \).

So a sustainable policy that achieves steady state will have the fund growing at the same rate as costs, and therefore the Flat Fund Ratio criterion will be satisfied after \( T \).