Abstract - This note investigates the impact of profit tax evasion on firms' output decisions in a Cournot oligopoly setting in which the market structure is determined endogenously. It is shown that tax evasion intensifies market entry and raises aggregate output, while production of each incumbent firm decreases. Therefore, tax evasion choices affect activity decisions and an evadable profit tax distorts the market outcome.

INTRODUCTION

A firm’s decision to evade taxes may be separable from its activity choice. This separability feature has important implications since, for example, the neutrality of a profit tax is preserved in the presence of tax evasion opportunities. Synthesizing previous analyses, Yaniv (1995) establishes the conditions that guarantee that a firm’s input or output choices are independent of its evasion decision. There are a number of settings, however, in which the separability feature may not hold. First, this will be the case if the firm cannot attain its preferred extent of tax evasion and its choice variable is the fraction of the tax base declared, instead of the absolute amount (Kreutzer and Lee, 1986; Wang and Conant, 1988; Yaniv, 1995, 1996). Activity and evasion choices may not be separable either if, second, the extent of tax evasion exerts a direct negative effect on gross profits (Kreutzer and Lee, 1988; Virmani, 1989) or, third, the detection probability and/or the penalty rate are determined endogenously by, e.g., the firm’s reported revenues or costs (Marelli, 1984; Marelli and Martina, 1988; Virmani, 1989; Lee, 1998). Finally, separability will not hold if the uncertainty caused by tax evasion is complemented by a second source of uncertainty regarding either the outcome of the firm’s activity (Yaniv, 1995) or the investment decision (Panteghini, 2000).

All of the above contributions take the market structure as given. However, tax evasion occurs because firms are better off when misreporting than when telling the truth to tax authorities. Hence, tax evasion affects the firms’ payoffs and the gains from entering the market. The innovation of our analysis is to endogenize the market structure. We rule out all causes for a non–separability of output and evasion choices established thus far, and develop a Cournot oligopoly model.
with profit tax evasion and an endogenous number of firms. Prior to the decisions about the activity level and the amount of profit tax evasion, firms choose whether to enter the market at positive costs. We can show that tax evasion influences the production level of each firm and alters aggregate output. The reason is that, under a given market structure, tax evasion enhances the expected payoff of each firm. This increase will attract new firms. As a consequence, output of the incumbent firms falls. The additional output of the new firms outweighs the decline in the production of the firms already in the market so that aggregate output increases. In sum, we identify the endogeneity of the market structure as a further reason why profit tax evasion affects individual and aggregate production. Hence, under oligopoly with endogenous market structure, profit taxes will not be neutral and we shall argue below that tax evasion will tend to render the market structure less efficient.

MODEL AND RESULTS

Consider an industry with a continuum of potential firms that will produce a homogeneous good once they have entered the market. To ensure that non–marginal firms have positive gross profits in equilibrium and an incentive to evade a tax on profits, firms are assumed to differ in production costs. A firm of type \( \theta \) has costs \( C(x, \theta) \), given a production quantity of \( x \) units, with \( C_x > 0 \) and \( C_{xx} \geq 0 \). Moreover, we assume \( C(0, \theta_1) \geq C(0, \theta_2) \) for \( \theta_1 > \theta_2 \) and \( C_{x\theta} \geq 0 \), with strict inequality in at least one of these two conditions. Hence, a firm with a large \( \theta \) has higher fixed and/or higher marginal production costs than a firm with a small \( \theta \). Note that these conditions imply \( C_{\theta} > 0 \). The cost parameter \( \theta \) is continuously distributed over the interval \([\theta, \bar{\theta}]\) with \( \theta < \bar{\theta} \), according to the distribution function \( F(\theta) \) and the density function \( f(\theta) = dF(\theta)/d\theta \). Without loss of generality, the mass of firms is normalized to unity.

A firm makes three decisions at most. First, it decides whether to enter the market at costs \( k \). If it enters, it will subsequently select output and the extent of tax evasion. To ensure a subgame perfect solution, we start with the latter decisions and take market entry as given, so that entry costs \( k \) are sunk. Consider a firm of type \( \theta \) and assume that this firm produces an output of \( x \) in the second stage. Let \( \bar{x} \) be the output of all other firms operating in the market. Aggregate output is \( y := x + \bar{x} \), and \( P(y) \) with \( P' < 0 \) defines the inverse demand function. We focus on the most relevant case of strategic substitutes defined in Bulow, Geanakoplos and Klemperer (1985). Marginal profits of any one firm will then decrease with the output of the firm’s rivals. Formally we have \( P' + xP'' < 0 \). This implies \( 2P' + xP'' - C_x < 0 \), which is a well–known necessary condition for the stability of the equilibrium in oligopoly models (Dixit, 1986).

Each firm has to pay a profit tax at rate \( \tau \). The tax base of firm \( \theta \) is

\[
\Pi = xP(x + \bar{x}) - C(x, \theta) - k.
\]

The firm can evade taxes by understating profits. Let \( s \) be the absolute amount of understatement. The tax authority audits firms with an exogenous probability \( q \), and if a firm is audited, tax evasion will be detected with certainty. In this case, the firm has to pay the full amount of taxes due and a penalty that is proportional to the amount of taxes evaded. The penalty rate is exogenous and denoted by \( \delta > 0 \). After–tax profits of the firm will equal

\[^{1}\text{Virmani (1989) also considers a model with (free) entry of firms. However, since the output market is always perfectly competitive, the market structure is effectively exogenous.}\]
if tax evasion remains undetected, and

\[\Pi^d = (1 - \tau)\Pi - \delta s\]

if the tax authority catches the firm evading taxes. In line with most of the literature referred to in the introduction, a firm is assumed to maximize the expected utility of profits. It has an increasing and strictly concave utility function \(U\). Without loss of generality, we normalize the reservation utility, i.e., the utility resulting from not entering the market and producing a quantity of zero, to \(U(0) = 0\). Expected utility of the firm, taken entry as given, reads

\[EU = (1 - q)U(\Pi^\tau) + qU(\Pi^d).\]

For a given market structure, the firm maximizes [4] with respect to output and evasion, taking into account [1]–[3]. Supposing an interior solution, which implies \(q < 1/(1 + \delta)\), and denoting optimal values by an asterisk, the first–order conditions can be written as

\[1 - q)U'[(1 - \tau)[x^* P(y) - C(x^*,\theta) - k] + \tau s^*] - \delta qU'[(1 - \tau)[x^* P(y) - C(x^*,\theta) - k] - \delta s^*] = 0,\]

\[P(y) + x^* P'(y) - C_s(x^*,\theta) = 0.\]

Equation [5] states that expected marginal benefits from tax evasion equal expected marginal costs in terms of the utility loss caused by the penalty. According to [6], marginal revenues just offset marginal production costs. This condition for the firm’s optimal output is the same as the respective requirement in the absence of tax evasion. More specifically, output does not depend on the evasion variable \(s\) and the tax enforcement parameters \(q\) and \(\delta\). For a given market structure, therefore, the separability property derived in earlier contributions holds. However, this property does not necessarily imply that the firms’ output is not affected by tax evasion. As we will now show, endogenizing the market structure creates a link between output and the extent of tax evasion.

From equations [5] and [6], the optimal amount of tax evasion \(s^*\) can be written as a function of the cost parameter \(\theta\), aggregate output \(y\) and the detection probability \(q\), i.e., \(s^* = S(\theta, y, q)\), while, due to the separability property, the optimal output level \(x^*\) depends solely on the cost parameter \(\theta\) and aggregate output \(y\), i.e., \(x^* = X(\theta, y)\).3

Equation [7] states that expected marginal benefits from tax evasion equal expected marginal costs in terms of the utility loss caused by the penalty. According to [7], low–costs firms have a higher output than high–costs firms and an increase in aggregate production reduces the firm’s output. The latter property is due to our assumption of strategic substitutes. Inserting \(x^* = X(\theta, y)\) and \(s^* = S(\theta, y, q)\) into [4] defines the maximum expected utility \(EU^*(\theta, y, q)\) of the firm, provided it enters the market. Differentiating \(EU^*(\theta, y, q)\) with respect to the cost parameter \(\theta\) for a given aggregate output level \(y\), and taking into account [5]–[7] yields

\[\frac{C_{sy}}{P' - C_{sx}} < 0,\]

\[X_y = -\frac{P' + xP''}{P' - C_{sx}} < 0.\]

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\[\frac{C_{sy}}{P' - C_{sx}} < 0,\]

\[X_y = -\frac{P' + xP''}{P' - C_{sx}} < 0.\]
Hence, the maximum expected utility is a decreasing function of \( \theta \) so that, for any given aggregate output \( y \), high-costs firms have lower expected utility than low-costs firms.

This property allows us to formalize the market entry decision in the first stage and, thus, to determine the overall equilibrium of the oligopoly game. A firm of type \( \theta \) will enter if the maximum expected utility resulting from entry, \( EU^*(\theta, y, q) \), is not less than the reservation utility \( U(0) = 0 \) of abstaining from such an action. Let \( \theta^o \) be the cost parameter of the marginal firm, i.e., of the firm that obtains an expected utility equal to the reservation utility and that, accordingly, is just indifferent between entering and not entering. Suppose, further, that all firms with a cost parameter \( \theta \in [\theta^o, \theta] \) enter the market, while the remaining firms do not.

Expected utility of the marginal firm and aggregate output can then be written as

\[
EU^*(\theta^*, y^*, q) = 0, \tag{9}
\]

\[
y^* = \frac{1}{1 + \delta} X(\theta^*, y^*) \tag{10}
\]

Equations [9] and [10] determine the equilibrium of our oligopoly game. To see this, note that, according to [8] and [9], all firms \( \theta \in [\theta^o, \theta] \) have a non-negative payoff from entering the market, while the expected payoff of all firms \( \theta \in [\theta^o, \theta] \) is negative. Hence, none of the firms in the market has an incentive to exit and none of the firms outside the market could increase its payoff by entering. The entry stage attains an equilibrium.

This equilibrium is unique. To demonstrate the claim, suppose we have an equilibrium with aggregate output \( y^* \). Due to [8], we know that \( EU^*(\theta^o, y^*, q) > 0 \) implies \( EU^*(\theta, y^*, q) > 0 \) for all \( \theta \in [\theta^o, \theta] \). This means that if, in the equilibrium, firm \( \theta_i \) is in the market, all firms with a lower cost parameter have to be in the market as well. Similar, \( EU^*(\theta^o, y^*, q) < 0 \) implies \( EU^*(\theta, y^*, q) < 0 \) for all \( \theta \in [\theta^o, \theta] \), i.e., if firm \( \theta^o \) is not in the market, then no firm with a higher cost parameter will be in the market.

From these two properties it follows that, in every equilibrium, there exists a \( \theta^o \in [\theta^o, \theta] \), which divides the whole set of firms into a subset of firms that are in the market, i.e., all \( \theta \in [\theta^o, \theta] \), and a subset of firms not in the market, i.e., all \( \theta \in [\theta^o, \theta] \).

To complete the proof of uniqueness, we finally show that firm \( \theta^o \) is the one with zero expected utility. This is proven by contradiction. Suppose \( EU^*(\theta^o, y^*, q) > 0 \). Then there is a small \( \varepsilon > 0 \) such that \( EU^*(\theta^o + \varepsilon, y^*, q) \geq 0 \), i.e., firm \( \theta^o + \varepsilon \) is also in the market. But this contradicts the property that, in equilibrium, all firms \( \theta \in [\theta^o, \theta] \) cannot be in the market. An analogous argument excludes \( EU^*(\theta^o, y^*, q) < 0 \). Hence, we unambiguously have \( \theta^o = \theta^* \) and \( y^* = y^* \) as determined by equations [9] and [10].

Having established a unique equilibrium of the oligopoly game, a comparative static analysis clarifies the impact of tax evasion on aggregate output \( y^* \) and output per firm \( x^* = X(\theta^*, y^*) \). Note that [9] and [10] define the marginal firm \( \theta^o \) and aggregate output \( y^* \) as functions of the detection probability \( q \), and that our model includes a world without evasion as a special case for \( q \geq 1/(1 + \delta) \). Thus, a decrease in \( q \), starting from a value of \( q = 1/(1 + \delta) \), simulates the transition from a world without tax evasion to a world in which evasion becomes profitable. Totally differentiating [9] and [10] yields

\[
\frac{d\theta^o}{dq} = \frac{1}{\Delta} \left[ 1 - \int_{\theta^o}^{\theta} X(\theta^*, y^*)dF(\theta) \right], \quad EU^*_q < 0, \tag{11}
\]

\[
\frac{dy^*}{dq} = \frac{1}{\Delta} X(\theta^*, y^*) f(\theta^*) EU^*_q < 0, \tag{12}
\]
since $\Pi' < \Pi^n$, $2P' + xP'' - C_{xx} < 0$ and [8] implies $1 - X_y = (2P' + xP'' - C_{xx})/(P' - C_{xx}) > 0$, and

$$1 - \int_{\theta} o X_y(\theta, y^*)dF(\theta) = 1 - F(\theta^*)$$

$$+ \int_{\theta} o [1 - X_y(\theta, y^*)]dF(\theta) > 0,$$

$EU_\theta^* = U(\Pi') - U(\Pi^n) < 0,$

$EU_y^* = (1 - \tau)x^* P'(1 - X_y^*):$

$$\cdot [(1 - q)U'(\Pi^*) + qU'(\Pi')]< 0,$$

$\Delta = -X(\theta^*, y^*) f(\theta^*)EU_y^*$

$$- \left[1 - \int_{\theta} o X_y(\theta, y^*)dF(\theta)\right]EU_y^* > 0.$$

With respect to the output of a non–marginal firm $\theta \in [\theta^*, \theta^r]$, we use [7] and [12] and obtain

$$[13] \quad \frac{dx^*}{dq} = X_y^* \frac{dy^*}{dq} > 0.$$

Equations [11]–[13] contain our main results. They show that profit tax evasion will have an impact on output if the market structure is endogenous. According to equation [12], aggregate output is increased by tax evasion, while [13] implies that the output of each firm operating in the market will be reduced. The intuition is as follows: for a given market structure, tax evasion increases expected utility of all firms. As a consequence, the marginal firm in the absence of tax evasion will have a positive expected utility level if evasion becomes feasible. Moreover, there are firms whose expected utility from entry into the market has been negative without tax evasion but becomes at least zero in the presence of evasion activities. These firms enter the market. Formally, $\theta^r$ rises and the (new) marginal firm is characterized by a higher cost parameter according to [11]. Hence, tax evasion attracts new firms. The output of these new firms reduces output of the incumbents due to our assumption of strategic substitutes. But the additional production of the new firms more than compensates the reduction in the incumbents’ output so that aggregate output is enhanced by tax evasion.

These effects of tax evasion on the firms’ output are indirect. A change in the probability of being detected evading taxes does not affect the first–order condition [6] of the firm with respect to output. Instead, tax evasion changes the market structure and so induces the firms to adjust their output levels. This is the reason why, for a given market structure, we obtain the same separability result as the analyses referred to in the Introduction. But none of the previous contributions considers the case of an endogenous market structure. In contrast, we show that with an endogenous market structure, tax evasion influences output by changes in the number of firms and, in this sense, evasion and output decisions are no longer separable.

The non–separability result has important implications for the neutrality of profit taxes. One of the main motivations of previous studies to investigate the relation between tax evasion and activity choices was the question whether a profit tax can be used to induce a monopolist to increase its inefficiently low output. Those authors who obtain the separability property conclude that the conventional neutrality of a profit tax is true even in the presence of tax evasion, while authors disproving separability argue that under tax evasion a profit tax is no longer neutral. In our framework, we can raise a similar question. It is well known from the analysis by Mankiw and Whinston (1986) that unrestricted entry in a Cournot oligopoly with positive entry costs induces an inefficiently large number of firms and aggregate output, relatively to a second–best world in which the number of firms, but not the output per firm, can
be regulated. The reason is that the marginal entrant ignores that the incumbents’ output and profit levels are reduced by her entry. Thus, we may ask whether a profit tax is suitable to internalize this externality and which role tax evasion plays.

To answer this question, initially consider the case without tax evasion \((q \geq 1/(1 + \delta))\). The profit tax is then neutral because the market entry condition [9] reduces to \(x^*P(\cdot) - C(\cdot) - k = 0\). Hence, the profit tax alters neither the firms’ output decisions, as characterized by [6], nor the marginal firm. In the presence of tax evasion, by contrast, the non–separability result derived above invalidates the neutrality of the profit tax because the firms’ incentives to enter the market are changed. The number of firms and aggregate output are increased further since evasion raises profits and leads to additional entry. Hence, an evadable profit tax is likely to aggravate the inefficiency derived by Mankiw and Whinston (1986) instead of removing it.

CONCLUDING REMARKS

Extending previous studies to the case of an endogenous market structure, this paper proves that profit tax evasion may influence the activity levels of firms. In our setting, tax evasion increases the expected payoff from production so that more firms find it profitable to enter the market. Aggregate supply in the whole market rises since the increase in the number of firms more than compensates the reduction in output per firm. Finally, we would like to emphasize that the main rationale of our argument is also true in a more general setting comprising other types of imperfect competition and taxes. To see this, we can use a similar approach as Yaniv (1995). Analogous to his equation [1], define firm \(\theta\)'s legitimate net profits (i.e., net profits if this firm fully complies to the tax law) as \(G(x_1, x_2, \ldots, x_{\theta}, \ldots, x_n, \theta, k, \tau)\), where \(x_i\) is the activity level of firm \(i\) (for heuristic reasons we now assume a discrete firm space), \(\tau\) may be any kind of tax (not necessarily a profit tax) and all other variables have the same meaning as in our Cournot model. If the term \((1 - \tau)\Pi\) in [2] and [3] is now replaced by \(G\), firm \(\theta\)'s optimal activity level is determined by \(\partial G(x_1, x_2, \ldots, x_{\theta}, \ldots, x_n, \theta, k, \tau)/\partial x_{\theta} = 0\). This is the same condition as in the absence of tax evasion and, thus, for a given market structure, we would obtain the same separability result as Yaniv (1995). But with an endogenous market structure, tax evasion influences market entry and the dimension of the vector \((x_1, x_2, \ldots, x_{\theta}, \ldots, x_n)\). This, in turn, changes the individual firm’s activity level and invalidates the separability result. It will be an interesting task for future research to investigate the economic consequences of this non–separability for other taxes, such as value-added or emission taxes, and other types of competition such as Bertrand competition or product differentiation.

Acknowledgments

We thank an anonymous referee for very helpful and constructive comments. All errors remain ours.

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4 This inefficient entry result also holds in a framework with heterogeneous firms, as is the case in our model. A proof of this assertion is available from the authors upon request.

5 In the case of strategic complements, i.e., \(P + xP' > 0\), tax evasion exerts almost the same effects on the market equilibrium. The only exception is that the individual output of incumbent firms increases. Interestingly, however, evasion then tends to render the market structure more efficient as, without tax evasion, the number of firms is inefficiently small. Proofs of these assertions can be obtained from the authors upon request.
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