Abstract - Warm-glow utility mitigates concerns that public giving crowds-out private giving dollar-for-dollar. Warm glow also means that utility is decreasing in the giving of others, ceteris paribus, and the willingness to pay for altruism is smaller (at the margin) if altruistic households have a positive willingness to pay for warm glow. Consequently, a marginal redistribution of income that passes the Pareto test may fail the test if altruistic households receive warm glow. Numerical evaluation shows that passing the Pareto test is very sensitive to cross-price elasticities between charity and labor supply, the elasticity of charity with respect to warm glow, the tax rate of the rich, and the fraction of the population that is rich.

INTRODUCTION

Corneo and Gruner (2002) find that the demand for government redistribution is not motivated solely by consideration of a personal monetary gain. Such results bring renewed relevance to the work of Hochman and Rogers (1969, 1974) who argue that interdependent preferences imply that some redistributions of income make everyone better off. However, Warr (1982) shows that incremental fiscal redistribution is not necessarily Pareto improving because government redistribution crowds-out private charity dollar-for-dollar. If the rich view the consumption of the poor as a public good, public redistribution of income is only a Pareto improvement once private giving has been driven to zero. Andreoni (1989, 1990) shows that if households receive utility from the size of the public good and they receive a “warm glow” from the size of their own contribution, public redistribution does not crowd-out private charity dollar-for-dollar. Although Andreoni demonstrates that subsidies for private giving are preferred to direct government contributions, little is known about how warm glow alters the welfare effects of marginal redistributions of income, such as in Allgood and Snow (1998) and Dahlby (1998).1

In this paper, the efficiency of a marginal redistribution of income is considered, given that rich households are impure altruists.
altruists that make tax deductible gifts to private charities. Impure altruists receive utility from the amount consumed by the poor and from their own contribution to the consumption of the poor. One might expect that if households receive utility from giving to the poor, then marginal redistributions of income are more likely to pass the Pareto or cost–benefit test. However, there are two reasons why this may not be the case. First, government giving to the poor may crowd–out private giving, and this will generate utility losses because of lost warm glow. Of course, crowding–in of private charity has the opposite normative implications.

The second reason is best understood by thinking in terms of the marginal costs and benefits of giving to charity. A household gives to charity until the marginal benefit of contributing equals the marginal cost. The marginal cost of giving is one minus the marginal tax rate (MTR). For an impure altruist, the marginal benefit is the gain in utility from the increased consumption of the poor, the willingness to pay (WTP) for altruism, plus the gain in utility from an increase in one’s own contribution to the poor, the WTP for warm glow. The household does not receive warm glow if a dollar increase in the consumption of the poor is because of a government transfer (or giving by other households). In this case, the increase in utility equals the WTP for altruism, which is marginal cost minus the WTP for warm glow. On the other hand, if a rich household is a pure altruist, then the increase in utility due to a one dollar increase in the consumption of the poor is just one minus the MTR.²

Diamond (2006) acknowledges that warm glow may improve our description of how individuals and households behave. However, he argues for ignoring the effects of warm–glow utility on social welfare when considering optimal tax structures because warm glow amounts to preferences over how the public good is produced.³ Diamond compares the social welfare implication of models with and without warm glow by inserting a parameter that is set to zero when the social planner wants to ignore changes in utility arising from warm glow.

Diamond’s (2006) analysis highlights an important modeling decision that must be made for normative tax evaluation. Should households be treated as impure altruists, pure altruists, or even non–altruists? Even if households receive warm–glow utility, one may choose to not incorporate this element of preferences into the model. This paper identifies and quantifies the implications of these different assumptions about preferences for marginal redistributions of income.

The interpretation of the analytical model builds on the work of Andreoni (1989, 1990) and Diamond (2006), and the analytical model provides a general guide for how impure altruism alters the efficiency of marginal reforms. However, there is also a large literature that attempts to quantify the welfare costs of increasing the tax rate on labor income to fund a marginal redistribution of income. This literature mainly focuses on the marginal efficiency costs caused by reductions in labor supply.⁴ The general conclusion of this literature is that redistributing income does not pass the cost–benefit test because

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² The current paper assumes that households cannot see through the governments budget, as in Bernheim (1986), Boadway et al. (1989), and Andreoni and Bergstrom (1996).

³ In the words of Diamond (2006, p. 909), “With warm glow preferences, behavior is modeled as if it maximized utility that depends not just on the final allocation of resources but also on the process that results in that allocation.”

⁴ Browning and Johnson (1984), Ballard (1988), Triest (1994), Browning (1995), and Allgood and Snow (1998) are some of the works that measure the efficiency cost of marginal redistributions of income. These previous studies are uniform in their assumption that agents are linked only through the government and that the government is the only institution able to redistribute income.
such reforms generate a positive welfare cost as higher tax rates shrink the tax base. Warm glow and charitable giving introduce elasticities not used in past analysis, in addition to the welfare effects just discussed. The numerical evaluation of the model illustrates how sensitive calculations of marginal welfare cost are to the introduction of these elasticities and the importance of the WTP for altruism to the overall welfare effects of reforms.

The paper proceeds as follows. First, the model is developed and equations for evaluating marginal reforms to tax and spending policy are derived. Then a numerical example is provided. The final section provides concluding comments.

THE MODEL

Basics of the Model

Consider a model with two types of households, the rich (indexed by $r$) and the poor (indexed by $p$). For simplicity, there is only one poor household and the number of (identical) rich households is $n > 1$. Rich households are assumed to have greater labor market productivity $\delta_r > \delta_p$ and larger endowed income $M_r > M_p = 0$. If $W$ is the market wage rate, then an hour of labor earns a rich household $\delta_r W$ and a poor household earns $\delta_p W$. Households sell their labor supply $h$ in the market at these prices. Output for the economy is produced by a linear production function that is a function of a fixed supply of capital and aggregate effective labor supply $H = \delta_p h_p + n \delta_r h_r$.

Government collects taxes via a graduated tax structure on labor earnings with two MTRs. The two tax brackets are defined by the income level $y$. It is assumed that the labor income of the rich exceeds $y$, and the income of the poor does not. The tax rate for the rich, $m_r$, includes all taxes on labor income such as payroll taxes. Poor households pay payroll taxes, but it is assumed that the poor face a zero federal income rate on earned income (Rosen, 2005). As a result, the poor household does not explicitly pay taxes on other income sources, such as transfers received from private charities.

Government transfers to the poor often decrease as labor income increases. As Browning (1995) explains, the MTR on labor income for poor households may exceed that of richer households once implicit taxation is considered. The analysis that follows ignores this issue because the focus is on changes in transfers to the poor household, holding constant the MTRs that they face. The poor household has the tax burden

$$ R_p = m_p \delta_p W h_p,$$

where $m_p$ is the marginal tax rate faced by the poor household.

Charitable contributions $c_r$ by the rich are tax deductible, and the poor household does not make such contributions. All charitable contributions are given to the poor household as transfer $T_p^c = nc_r$. It is further assumed that the poor household receives the full charitable transfer $T_p^c$ as long as its income remains below some income threshold. Given the marginal nature of the analysis, the poor household remains poor and it remains the beneficiary of $T_p^c$, and any increases or decreases in this transfer.

The tax burden of a rich household is

$$ R_r = m_r (\delta_r W h_r - c_r) - N_r,$$

where $N_r = (m_r - m_p)y$. Aggregate tax revenue is $R^r = nR_r + R_p$, or

$$ R^r = n[m_r (\delta_r W h_r - c_r) - N_r] + m_p \delta_p W h_p,$$

Government spends this money on cash transfers to the poor $T_p^c$ and a publicly provided, public good consumed by all households and provided solely by the government. The government’s budget is always balanced, and the proportion of
aggregate tax revenue given to the poor is $β$, so that

$$βR^v = T_p^s.$$  

Between government and private charity, the total transfer to the poor household is $T_p = T_p^s + T_p^c$.

The poor household maximizes utility by the choice of leisure $ℓ_p$ and the numeraire, composite consumption good $x_p$, where utility is $u_p(x_p, ℓ_p)$. Utility $u_p$ is strictly quasi-concave, twice-differentiable, and increasing in both arguments. The poor household’s budget constraint is $x_p = w_pℓ_p + T_p$, where $w_p + (1 - m_p)δ_pW$. Consumption $x_p + w_pℓ_p$ must equal full income

$$I_p = w_pL + T_p^c$$

where $ℓ + h = L$ and $L$ is the time endowment. This yields the indirect utility function $V_p(w_p, I_p)$ and the labor supply function $h_p(w_p, I_p)$.

Rich households have utility $u_p(x_r, ℓ_r, c_r, x)$. Utility $u_r$ is quasi-concave, twice-differentiable, and increasing in all arguments. The first two arguments are the same as in the utility function of the poor. The third argument represents the “warm glow” a household receives from their private contribution to the poor. The last term shows that the utility of the rich is positively related to the consumption of the poor as in Hochman and Rogers (1969, 1974), Warr (1982), and Roberts (1984). Andreoni (1989, 1990) calls this preference type impure altruism because households care not only about the consumption of the poor, but they also care about their private contribution. Following Andreoni, pure altruism refers to having preferences $u_p(x_p, ℓ_p, c_p)$, and a non-altruist’s preferences are $u_p(x_p, ℓ_p)$.

Rich households choose $x_p$, $ℓ_p$, and $c_p$ to maximize utility given the budget set

$$x_p + γc_p = wh + M + N,$$

and the time constraint, where $γ = 1 - m_r$, and $w_r = γδW$. In addition, an interior solution is assumed ($h_r > 0$ and $c_r > 0$).

The model takes a simplified view of how government and charities assist the poor. For example, government and charitable assistance is often in-kind (non-cash). This distinction is not important unless the in-kind transfer is larger than the household’s utility maximizing quantity. Otherwise, an increase in an in-kind transfer is treated the same as an income effect (Rosen, 2005). Charitable organizations may provide assistance that the government does not provide. This would not drastically alter the current model if the different goods provided by government and charity are perfect substitutes or if the altruism of rich households is a function of the aggregate consumption of the poor. It may also be the case that charitable organizations provide assistance to those who are not helped by government. This could be modeled by introducing an additional group of poor households that receive charitable aid but not government aid (or less government aid).

Warm Glow and Private Provision of a Public Good

In this context, the consumption of the poor is a public good for rich households. Rich households are assumed to take the behavior of others as given, as is typical of models with private provision of public goods (Bergstrom, Blume, and Varian, 1986). For a single rich household, define the contributions of others to the consumption of the poor to be

$$x_{p,-r} = x_p - c_r.$$
Following Andreoni (1989, 1990), the Nash assumption is incorporated by substituting equation [7] into budget equation [6]. After adding \( w_r \) to both sides, a rich household’s full income budget constraint is

\[
x_r + \gamma x_p + w_r L + \gamma x_{p-r} + N_r + M_r = I_r = w_r L + \gamma x_{p-r} + N_r + M_r.
\]

Equation [8] shows that the value of what the household consumes must equal the sum of the value of the household’s endowments plus the value of the contributions of the rest of society to the consumption of the poor. Thus, this definition of income is similar to Becker’s (1974) notion of social income, which is also how Andreoni defines income.

A rich household’s maximization problem is now written

\[
\max_{x_r, \ell_r, x_p} u(x_r, \ell_r, x_p - x_{p-r}, x_p)
\]

subject to equation [8]. One implication of warm–glow utility is that, for a given \( x_p \), utility decreases as the giving by others increases, \( \partial u / \partial x_{p-r} < 0 \). Consider a dollar increase in the giving of others that is exactly offset by a dollar decrease in one’s own–giving. The altruism effect does not change because \( x_p \) is unchanged, but utility is lower from the third element of the utility function because \( x_p - x_{p-r} \) decreases. The indirect utility function is \( V(\gamma, w_r, I_r, x_{p-r}) \), and it follows that indirect utility is negatively related to its fourth argument, \( \partial V / \partial x_{p-r} \big|_{a_{p-r}=0} < 0 \). The third element of the indirect utility function is full income which includes the contributions of others, \( x_{p-r} \). Andreoni refers to the third term as the pure altruism effect and the fourth as the egoism effect. The first–order conditions for a rich household’s maximization problem imply that

\[
P_a + P_{wg} = \gamma
\]

where \( P_a \) is the willingness to pay for altruism, \( P_{wg} \) is the WTP for warm glow, and \( \gamma = 1 - m_r \).

The solution functions for this problem are labor supply \( h_r(\gamma, w_r, I_r) = L - \ell_r(\gamma, w_r, I_r) \), which is assumed to be independent of the egoism effect, and the rich household’s demand for the public good \( x_p \), given by \( f_r(\gamma, w_r, I_r, x_{p-r}) \). A rich household’s demand for the consumption of the poor \( f_r(\gamma, w_r, I_r, x_{p-r}) \) must satisfy

\[
c_r = f_r - x_{p-r}.
\]

Responses to fully specified marginal reforms can be evaluated by analyzing changes in \( f_r \) and changes in labor supply. Changes in charitable contributions could then be determined using equation [10]. However, little is known about the price and income effects of the demand for this type of public good, but there is a literature that estimates the responsiveness of charitable contributions to changes in prices and income. For this reason, changes in behavior are evaluated using the response of \( c_r \) to marginal changes. Consequently, the behavioral equations for rich households are assumed to be the demand for charitable contributions \( c_r(w_r, \gamma, I_r, x_{p-r}) \) and \( h_r \). Equation [10] can then be used to solve for the change in the consumption of the poor.

EVALUATING MARGINAL CHANGES

Normative Effects of Marginal Reforms

Marginal reforms are specified by changes in the proportion of revenue given to the poor household \( d\beta \), changes

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Marginal reforms are specified by changes in the proportion of revenue given to the poor household \( d\beta \), changes
in the definition of the income tax brackets $dy$, and changes in MTRs $dm_t$ and $dm_r$. For all reforms it is assumed that households are not bunched at the kink in the budget line so that no household changes tax brackets due to a reform. In general, households are assumed to respond to marginal reforms with marginal changes. For all reforms, $dN_r = 0$ and $dmp = 0$.

Changes in utility are evaluated using the money metric $NB_r = dV_r/V_r$, where $dV_r$ is the total differential of the indirect utility function and $V_r = \partial V_r/\partial I_r$ is the marginal utility of income. Marginal reforms are evaluated with two criteria. Assuming that redistribution improves the utility of the poor, a marginal reform passes the Pareto criterion if the utility of the rich does not fall, $NB_r \geq 0$. An advantage of the Pareto criterion is that it does not require inter-personal comparisons of utility. However, a reform that lowers the utility of even one household fails the Pareto criterion regardless of the potential gains. An alternative is to assume that the marginal utility of income is constant and equal across households. A reform then passes the cost–benefit test if $NB^a = nNB_r + NB_p > 0$.

The analysis below proceeds by first deriving an expression for the change in utility, or net benefit, for a rich household that is an impure altruist. When computing the marginal benefits and costs of a reform, the analyst begins with an initial set of data (hours worked, wages rates, non–labor income) and parameters (tax rates and elasticities). The researcher must decide if the initial position is generated by impurely altruistic households, purely altruistic households, or non–altruistic households. Much of the previous literature on measuring the welfare effects of marginal reforms models households as non–altruists (Browning and Johnson, 1984; Ballard, 1988; Triest, 1994). This approach assumes that there is no value to redistribution. Diamond (2006) acknowledges that warm glow may provide a better description of behavior, but he suggests that warm–glow utility should be ignored for normative tax analysis. The analysis to follow illustrates how the different assumptions about preferences alter both the benefits and the costs of marginal reforms.

The change in money metric utility for a rich household, or net benefit, given the maintained assumption that the pre–tax wage is constant ($d\delta_r = dW = 0$), is

$$[11] NB_{IA}^r = (\gamma - P_{wg})dx_p - (dR_r + WC_r^{IA}),$$

where

$$[12] WC_{IA}^r = -m\delta_r Wdh_r + (1 - P_{wg})dc,$$

and $dR_r$ is marginal tax revenue (the derivative of equation [2]). Equation [11] shows that the change in utility is decomposed into the marginal benefit from increasing the consumption of the poor and the marginal cost of the additional taxes. When discussing the first–order conditions for rich households, the WTP for warm glow was defined as the additional utility generated from a dollar increase in the household’s own charitable giving. An equivalent definition is to say that $P_{wg}$ equals (minus) the loss in utility due to a dollar increase in the giving of others, holding the consumption of the poor constant. The change in the implicit transfer $N_t$ is $dN_t = ydm_t + (m_t - m)d\gamma$ if $dmp = 0$. It is assumed that the government makes marginal changes in the tax bracket’s so that $N_t$ remains constant. Households do not change tax brackets’ given the assumption that they are not near the kink in the budget line. Allgood and Snow (1998) and Allgood (2003) discuss the role of these implicit transfers on the welfare effects of marginal reforms.

8 The change in the implicit transfer $N_t$ is $dN_t = ydm_t + (m_t - m)d\gamma$ if $dmp = 0$. It is assumed that the government makes marginal changes in the tax bracket’s so that $N_t$ remains constant. Households do not change tax brackets’ given the assumption that they are not near the kink in the budget line. Allgood and Snow (1998) and Allgood (2003) discuss the role of these implicit transfers on the welfare effects of marginal reforms.

9 See Appendix A for the derivation of equations (11) – (16).

10 In equation [9] the WTP for warm glow was defined as $P_{wg} = u/\lambda$. In deriving equations [11] and [12], the WTP for warm glow is $P_{wg} = -V_r^e/\gamma$ for $V_r^e = \partial V_r/\partial x_{r,-}$, $\gamma = -c$. The two definitions of the WTP for warm glow are equivalent, with one defined in terms of changes in $c$, and the other in terms of changes in $x_{r,-}$. The equivalence is seen by remembering that both definitions assume that $x_r$ is constant, so $dx_{r,-} = -dc_r$. The two definitions of the WTP for warm glow are equivalent, with one defined in terms of changes in $c$, and the other in terms of changes in $x_{r,-}$. The equivalence is seen by remembering that both definitions assume that $x_r$ is constant, so $dx_{r,-} = -dc_r$.
in equation [9] that \( P_{\text{wg}} + P_a = \gamma \) so that the first element of equation [11] can also be written \( P_a dx_p \). This term measures the marginal external effect of increasing the consumption of the poor. Unlike altruism, warm glow does not introduce an external effect. If \( P_a = 0 \) in equations [11] and [12], the change in a rich household’s utility is independent of the consumption or changes in consumption of others.\(^\text{11}\)

If households are pure altruists, or if one wishes to fully ignore warm glow, then \( P_{\text{wg}} = 0 \). The change in utility is

\[
[NB_{\text{PA}} = \gamma dx_p - (dR_p + WC_{\text{PA}})\]

where

\[
[WC_{\text{PA}} = -m_r \delta Wd\delta_r + dc_r,\]

and \( P_a = \gamma \). In equation [13] the marginal external value of another unit of \( x_p \) is equal to \( \gamma \) if households are in fact impure altruists, so that \( P_a = \gamma - P_{\text{wg}} < \gamma \) then equation [13] over-values the marginal external effect of increasing the consumption of the poor.

To focus on welfare cost, consider the case of a non–altruist. As mentioned in the introduction, most of the literature on the efficiency of marginal reforms assumes that households are non–altruists that do not give to charity. Equations [13] and [14] simplify to

\[
[NB_{\text{NA}} = -(dR_p + WC_{\text{NA}})\]

where welfare cost is

\[
[WC_{\text{NA}} = -m_r W \delta d\delta_r.\]

The rich household receives no additional benefit from the additional consumption of the poor because it is not altruistic. Consequently, the change in a rich household’s utility is simply the marginal cost of paying additional taxes, and any redistribution of income fails the Pareto test. Any redistribution also fails the cost–benefit test if labor supply is upward sloping because the aggregate change in utility is equal to minus aggregate welfare cost, \( NB^* = -WC^* = -(WC_r + nWC_c)\).\(^\text{12}\)

Welfare cost is a function of uncompensated elasticities, which implies there is no welfare cost if the elasticities are zero. If no redistribution is first–best, why is welfare cost zero if the uncompensated elasticity is zero?\(^\text{13}\) First, it is important to realize that welfare cost is not marginal excess burden. Marginal excess burden measures the difference in utility from using a distortionary tax versus a lump–sum tax. As such, the comparison is between two equilibria that involve the same amount of tax revenue. Both taxes involve an income effect, but only the distortionary tax involves a substitution effect. Consequently, the difference in welfare between the two tax instruments is determined by compensated changes in demand. Excess burden and marginal excess burden are appropriate measures for a “differential incidence” analysis with the intent of determining the efficiency of alternative ways of raising a given level of tax revenue.

Welfare cost is appropriate for a “balanced budget” analysis where interest lies in measuring the cost of raising an additional dollar of tax revenue for the purpose of funding a public project (in this case a cash transfer to the poor). Welfare cost is not intended to measure the change in utility in comparison to a hypothetical lump–sum tax. Instead,

\(^\text{11}\) To see this, consider a household that receives warm–glow utility but is not altruistic. Equation [11] becomes \( NB = -[dR_p - m_r (\delta Wd\delta_r - dc_r)] \), given that \( 1 - P_{\text{wg}} = m_r \) and \( P_a = 0 \).
\(^\text{12}\) The change in the utility of the poor when there is no giving to charity is \( NB_p = dR^* - (dR_p + WC_c) \), where welfare cost is defined similarly to equation [16]. Thus, \( NB^* = dR^* - (dR_p + WC_r) + n[-(dR_p + WC_{\text{PA}})] = -(WC_r + nWC_c) \).
\(^\text{13}\) I thank one of the referees for phrasing the question in this manner.
welfare cost modifies the Samuelson rule for providing public goods to reflect that the tax base typically shrinks as tax rates increase. The resulting “tax leakage,” or welfare cost, shows that the cost of raising one more dollar of tax revenue is greater than one dollar. Because welfare cost measures actual changes in tax revenue, it is measured with uncompensated elasticities. If, for example, the uncompensated elasticity is zero, then welfare cost is zero and the cost of raising an additional dollar of tax revenue is one. The Samuelson rule is passed as long as the public project generates a benefit at least as large as marginal tax revenue. On the other hand, a positive welfare cost (a positive uncompensated elasticity of labor supply) means that the cost of raising one more dollar of tax revenue is greater than one, and the public project must generate a benefit in excess of one dollar per dollar of tax revenue.14

Welfare cost in equation [16] arises from the under–provision of labor supply caused by the government’s taxation of labor income, and welfare cost is simply the loss in the tax revenue due to a declining tax base. Equation [14] shows that introducing altruism alters welfare cost, \( WC^{PA} = -m \delta \text{Widh}_i + (m_i + P_a)dc_i \), where equation [14] is rewritten using \( P_a = 1 - m_i \). The term \( m_i dc_i \) is tax leakage that arises because the tax lowers the price of charity and leads to over–consumption of charity. This is why \( m_i dc_i \) has the opposite sign of the labor supply term. Thus, welfare cost is larger if the reform crowds–in private charity. The second term \( (P_a dc_i) \) arises because the utility of rich, altruistic households increases net of their contribution to \( x_i \). The addition of warm–glow utility to the model further modifies welfare cost to include \(-P_{wg} dc_i\) (equation [12]). Welfare cost is still positively related to the change in charitable giving, but \( dc_i \) contributes less to welfare cost than it does for the pure altruist because the impure altruist values their private contribution.

The analysis illustrates how the equations for evaluating marginal tax and spending reforms are altered given alternative assumptions about preferences. Diamond (2006) considers the optimal tax implication of including or ignoring the utility effects generated by warm glow. Diamond (2006, p. 915) observes that, “The fact that warm glows improve the description of individual behavior does not necessarily imply that social welfare should be defined including warm glows.” He concludes that, in general, allocating resources to give warm glow “does not seem like a good use of resources” (Diamond, 2006, p. 917). Upon quick inspection one might conclude that following Diamond’s modeling prescription to ignore the social welfare effects of warm glow implies using equations [13] and [14] instead of equations [11] and [12]. That is, ignoring utility changes due to warm glow is incorporated by setting \( P_{wg} = 0 \). This is not exactly correct. The welfare cost term \( P_{wg} dc \) from equation [12] reflects changes in utility arising from how a particular outcome is achieved (i.e., the process). Dropping this term is consistent with Diamond’s arguments for ignoring warm glow. However, even if one chooses to “ignore” changes in social utility arising from changes in warm–glow utility, it is still the case that \( P_a = \gamma - P_{wg} \). Thus, \( \gamma - P_{wg} \) is the appropriate measure of the marginal external effect of changing the consumption of the poor. Using \( P_a = \gamma \) overstates the marginal external effect, and it leads to

14 The standard Samuelson rule for efficient provision of a public good is \( \Sigma MRS = MRT \), where \( \Sigma MRS \) is the marginal rates of substitution summed across households and \( MRT \) is the marginal rate of transformation. The Samuelson rule becomes \( \Sigma MRS = MRT (1 + MWC) \), where \( 1 + MWC \) is called the marginal cost of funds. For more on this topic see Triest (1990), Fullerton (1991), or Creedy (2000).

15 Analytically, Diamond (2006) specifies individual utility as strictly additive, and he inserts a parameter (\( \theta \) in his model) that multiplies utility arising from warm glow. This parameter is set to zero or one depending on whether he wishes to include or exclude warm glow in social welfare.
erroneous conclusions about the benefits of redistributing income.

NUMERICAL EVALUATION OF A MARGINAL REFORM

The discussion of the analytical model shows how the normative evaluation of a given set of behavioral changes is affected by different modeling decisions about altruism and warm glow. These modeling decisions extend to behavioral parameters as well. In particular, there are cross-price effects between labor supply and charitable giving and there is the effect of giving by others on charitable giving. The following numerical analysis is not intended to provide numbers to guide policy decisions, but it is intended to provide a better understanding of the magnitudes of the welfare effects introduced by impure altruism.

Positive Effects of Marginal Reforms

Before numerically calculating the model, equations for evaluating the marginal response to a reform must be determined. Taking the total derivatives of \( h_r(w_r, \gamma, I_r), h_p(w_p, I_p), \) and \( c_r(w_r, \gamma, I_r, x_{p,r}) \) gives

\[
\frac{dh_r}{I_r} = -\left( \eta'_w + \eta'_{\gamma} \right) \frac{dm_r}{1 - m_r} + \eta'_r \frac{dl_r}{I_r},
\]

\[
\frac{dh_p}{I_p} = -\eta'_w \frac{dm_p}{1 - m_p} + \eta'_r \frac{dl_p}{I_p},
\]

and

\[
\frac{dc_r}{c_r} = -\left( \zeta'_w + \zeta'_r \right) \frac{dm_r}{1 - m_r} + \hat{\zeta}'_{\gamma} \frac{dl_r}{I_r} + \zeta'_{\gamma} \frac{dx_{p,r}}{x_{p,r}}.
\]

where \( d(W/\delta W) = d\gamma = -dm_r \) and the \( \eta' \)'s and \( \zeta' \)'s are elasticities. For example, \( \eta'_{\gamma} \) in equation [17] is the elasticity of a rich household’s labor supply with respect to its own wage rate. Based on the empirical work of Triest (1992), leisure and charitable contributions are assumed to be substitutes, \( \eta'_{\gamma} < 0 = \) and \( \zeta'_{w} > 0, \) and \( c_r \) is assumed to be a normal good so \( \zeta'_r < 0 \) and \( \zeta'_r > 0. \)

Charitable giving is also a function of the egoism effect, \( x_{p,r}. \) How do charitable contributions change in response to changes in \( x_{p,r}. \) Andreoni (1990) shows that the demand for the public good \( f_r \) is positively related to a change in the egoism or warm–glow term, \( 0 > f_{rg} > 1, \) where \( f_{rg} = \partial f_r/\partial x_{p,r} \mid_{d(x_{p,r})}. \) This result shows that as the giving of others to the consumption of the poor increases by one dollar, the demand of the rich for \( x_r \) increases by less than one dollar. Equation [10] implies that

\[
\frac{c_{wg}}{f_{wg}} = f_{wg} - 1.
\]

If a rich household’s demand for \( x_p \) increases by less than a dollar when others increase their giving by a dollar (\( 0 > f_{wg} > 1 \)), then the rich household meets this demand by decreasing their giving by some amount less than one dollar, \(-1 < c_{wg} < 0. \) Thus, it is assumed that \( c_{wg} < 0. \)

Equations [17]–[19] introduce behavioral parameters not present in models that assume non–altruistic households. Giving to charity introduces the two cross–price effects captured in the elasticities \( \zeta'_{wg} > 0 \) and \( \eta'_{\gamma} < 0, \) and there is a warm–glow effect on charitable giving \( \zeta'_{wg} < 0. \) To illustrate the impact of these elasticities, assume that there are no cross–price effects \( (\zeta'_{w} = \eta'_{\gamma} = 0), \) and that

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16 See Appendix B for a derivation of equations [17]–[19].

17 To obtain the expression in equation [20], first assume that \( m_r = N_r = dl_r = db_r = 0. \) Next assume that there are no changes in parameters such as MTRs. As in Andreoni, assume the government has access to lump–sum taxes so that \( dM_r = dx_{p,r} = -1 \) (so that \( dl_r = 0. \)) Now take the total derivative of equation [10] to get equation [20].
Consider a reform that lowers the prices of charity and leisure by raising the MTR so that labor supply decreases ($dh_l < 0$) and charitable giving increases ($dc_r > 0$). If, in fact, leisure and charity are substitutes ($\zeta_w > 0$), then the increase in $c_r$ is smaller or, if the effect is large enough, crowding-out occurs ($dc_r < 0$). If charity and leisure are substitutes, then $\eta_l < 0$, and there is a smaller decline in labor supply or possibly an increase in labor supply. Lastly, a negative value for $\zeta_w$ means a smaller increase (or a decrease) in charitable giving if the reform increases the giving of others. Each of these additional behavioral effects reduces tax leakage (welfare cost) (equation [11]). Less tax leakage means a larger increase in tax revenue that is transferred to the poor.

**Data and Parameters**

Data on hours worked and the wage rate are taken from Triest (1994, Table 4a). The bottom three deciles of Triest’s data are aggregated to create the poor and the top seven for the rich. The number of rich households is set at 3. Tiehen’s (2001) mid-range estimates suggest setting $\zeta_r = 0.35$ and $\eta_l = –1$. Saez (2004, p. 17) uses similar values for $\zeta_r$ and $\zeta_r = 0$ in his simulations (0.5 to 1.5 for “the price elasticity of contributions”), but he assumes larger values for $\zeta_r$ (1 and 0.5). A wide range of labor supply elasticities are used in the literature on measuring the costs and benefits of marginal reforms (Allgood and Snow, 1998). Again, the mid-range values of $\eta_l = 0.2$ and $\eta_l = –0.1$ are used for both the poor and rich. The MTR for rich households is 0.4 and 0.2 for the poor household. These values are comparable to one of the sets of MTRs used by Allgood and Snow (2006, Table 1, Parameter Set B) and by Triest (1994). These MTRs may be small, especially for the poor. This issue is addressed later. The value of $\beta$ depends on what constitutes a transfer to the poor. Although slightly higher values could be used, $\beta = 0.15$ seems reasonable.

Triest (1992) reports that labor supply is highly elastic with respect to changes in the price of tax deductions. His preferred value for this elasticity is –0.632. This is much larger than the assumed labor supply wage elasticity. Of course, charitable contributions are only one type of deduction, and Triest does not separate charitable giving from other tax deductions in his analysis. In contrast, Saez (2004) assumes that earnings are independent of the price of charitable contributions. To see the influence of $\zeta_r$ on the calculations, reforms are evaluated with the elasticity set to 0, –0.2 ($= –\eta_l$), and –0.632. Estimates of $\zeta_r$ and $\zeta_r = 0$ could not be found in the literature. It is assumed that $\zeta_r = –\zeta_w$ for all reforms. In addition, reforms are evaluated with $\zeta_w = 0$, –0.35 ($= –\zeta_l$), and –1($ = \zeta_l$). More information on data and parameters is available in Appendix C.

**Numerical Calculations**

The marginal reform evaluated is a cash transfer to the poor household that is funded by a higher MTR on rich households. The MTR of the rich is increased 0.01 ($dm = 0.01$) and $d\beta > 0$ is chosen so that the government’s budget is balanced. Allgood (2003) evaluates a similar reform. While not identical to the demogrant analyzed by Browning and Johnson (1984), Ballard (1988), and Allgood and Snow (1998), it is very similar.

Table 1 provides the results of the numerical analysis. There are seven columns of numbers. The first column of numbers is computed assuming no cross-price effects between leisure and charity and that charitable giving is independent of warm glow ($\zeta_w = \eta_l = \zeta_r = 0$). Columns 2 and 3 allow for cross-price effects by assuming two sets of values for $\zeta_w$ and $\eta_l$. Next are two columns that introduce warm–glow effects on giving ($\zeta_r < 0$).
TABLE 1
CASH TRANSFER REFORM

<table>
<thead>
<tr>
<th></th>
<th>Non–Altruist</th>
<th>Cross–Price Effects Only</th>
<th>Warm–glow Effects Only</th>
<th>Cross–Price &amp; Warm–glow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$\eta^r_w$</td>
<td>0.00</td>
<td>-0.20</td>
<td>-0.632</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma^r_w$</td>
<td>0.00</td>
<td>0.20</td>
<td>0.632</td>
<td>0.00</td>
</tr>
<tr>
<td>$\eta^r_{rg}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.35</td>
</tr>
<tr>
<td>$\gamma^r_{rg}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.129</td>
</tr>
</tbody>
</table>

Note: For all reforms $dm = 0$ and $dm_1 = 0.01$, $\eta^r_w$ is the wage elasticity of labor supply for rich households, $\gamma^r_w$ is the wage elasticity of charitable giving, $\gamma^r_{rg}$ is the elasticity of charitable giving with respect to the giving of others, $NB^{PA}_r$ is the change in utility for a rich, purely altruistic household, $MWC^{PA}_r$ is marginal welfare cost if rich households are purely altruistic, $P^{PA}_r$ is the value of the willingness to pay for warm glow that sets net benefit for impurely altruistic households ($NB^{PA}_r$) to zero, $P_{rg}$ is the value of the willingness to pay for warm glow that sets $NB^{PA}_r$ to zero, $d\beta$ is the change in the proportion of government tax revenue given to the poor that generates a balanced budget, $dc$ and $dh$ are the changes in charitable giving and labor supply, respectively, for rich households, and $dR$ is the change in aggregate tax revenue. Column 1 is computed assuming no cross–price effects between leisure and charity and that charitable giving is independent of warm glow ($\gamma^r_w = \eta^r_w = \gamma^r_{rg} = 0$). Columns 2 and 3 allow for cross–price effects by assuming two sets of non–zero values for $\gamma^r_w$ and $\eta^r_w$. Columns 4 and 5 introduce warm–glow effects on giving using two values of $\gamma^r_{rg}$ while again assuming no cross–price effects ($\gamma^r_w = \eta^r_w = 0$). Columns 6 and 7 allow for both cross–price and warm–glow effects.

while again assuming no cross–price effects. The last two columns allow for both cross–price and warm–glow effects.

Before discussing the results in Table 1, suppose that households are non–altruists. Aggregate utility, $NB^e_r$, simply equals (minus) aggregate welfare cost, where welfare cost for the rich is defined by equation [16]. Redistributing income does not increase aggregate utility unless the reform increases aggregate labor supply. Marginal welfare cost (MWC) provides perspective on the size of aggregate welfare cost relative to the amount of additional tax revenue, where $MWC = WC/dR^e_r$. MWC is 0.12 for this case (this number is not reported in Table 1). Allgood and Snow (1998) report a marginal welfare cost of 0.129 using similar parameters for a demogrant reform. Allgood (2003) studies a similar redistributive reform and reports a MWC of 0.10 using smaller labor supply elasticities and MTRs, and a MWC of 0.30 using larger parameters.

Column 1 of Table 1 reports a $MWC^{PA}_r$ of 0.16, or about a third larger than the 0.12 reported for the model without charitable deductions. The increase in MWC occurs because the reform crowds–in charity ($dc_r = 8.20$) increasing welfare cost. $NB^{PA}_r$ is positive because these purely altruistic households value the gain in the consumption of the poor more than the cost imposed on them by the reform ($dx_p > dR + WC$). Given these parameter assumptions and the assumption that households are purely altruistic, the reform passes the Pareto test because the poor are made better off and the utility of the rich increases ($NB^{PA}_r > 0$). If the reform passes the Pareto test it must also pass the cost–benefit test.

One of Kaplow’s (2004) criticisms of research that measures the normative effects of tax and spending reforms is the assumption that there is no value to redistribution. While this is a valid criticism, the abstraction is employed because there are no empirical estimates of $P_r$ and...
Thus, the reform passes the Pareto test. On the other hand, $P^r_{wg} > 0.32$ implies that $P^r_r \leq 0.32$, and the marginal benefit is too small to compensate the rich household for the costs of the reform which now include lost warm-glow utility. For $P^r_{wg}$ to be less than 0.32, it must be the case that the marginal value of altruism exceeds the marginal value of warm glow ($P^r_r > P^r_{wg}$). As a result, the reform passes the Pareto test as long as rich households value altruism more than warm glow at the margin.

What about changes in aggregate utility? The row labeled $P^r_{w}$ is the value of $P^r_{wg}$ that sets $NB$ to zero.\(^{19}\) $P^r_{wg}$ is 0.67, but given the MTR of 0.4, any value larger than 0.6 implies that the WTP for altruism is negative. Thus, the reform passes the cost–benefit test for any permissible value of $P^r_{wg}$.

The numbers in column 2 are computed assuming $\zeta^r_w = -\eta^r_r = 0.2$ and column 3 assumes $\zeta^r_w = -\eta^r_r = -0.632$. Labor supply increases with both sets of parameters, and the reform crowds–out private charity. As $\eta^r_r$ and $\zeta^r_w$ become larger (in absolute value), the change in a rich household’s contribution becomes smaller relative to the change in $x^r_r$, meaning that the household is losing larger amounts of warm glow. As a result, $P^r_w$ and $P^r_{wg}$ become smaller as the elasticities become larger. In column 3, $P^r_{wg}$ is 0.18, or almost half the value reported in column 1. This means that the WTP for altruism must be more than twice as large as the WTP for warm glow for the reform to pass the Pareto test. Since $P^r_{wg}$ is 0.52 and thus less than the upper bound of 0.60, it is theoretically possible for the reform to fail the cost–benefit test.

It is documented that MWC is sensitive to the labor supply elasticity with respect to the wage ($\eta^r_r$) (Allgood and Snow, 1998). Across the first three columns of Table 1, MWC falls from 16 cents to −23 cents although $\eta^r_r$ is constant. Which elasticity, $\eta^r_r$ or $\zeta^r_w$, is most responsible for the reduction in MWC? MWC is 15 cents if $\zeta^r_w = 0.2$ and $\eta^r_r = 0$, which is only a one–cent change from when both elasticities are zero (column 1). In fact, MWC only falls to 12.5 cents if $\zeta^r_w = 0.632$ and $\eta^r_r = 0$. This implies that the different values of $\eta^r_r$ are responsible for 36 cents of the 40–cent change in MWC. This is not surprising because welfare cost is primarily tax leakage, and labor supply accounts for most of the tax revenue. As a result, welfare cost tends to reflect what is happening to labor supply. However, the $40 drop in $NB^{14}$ across columns 1, 2, and 3 is driven by both $\eta^r_r$ and $\zeta^r_w$. Again assuming that $\eta^r_r = 0$, $NB^{14} = 209.55$ if $\zeta^r_w = 0.2$ and 198.92 if $\zeta^r_w = 0.632$. Thus, $\zeta^r_w$ accounts for about 40 percent of the decline in net benefit.

Columns 4 and 5 introduce the behavioral effects of warm glow on charitable giving (assuming no cross–price effects). Net benefit is smaller in column 4 ($\zeta^r_{wg} = -0.35$) than in column 1 because the reform crowds–out private charity. While this causes a lower MWC, it also means an increase in the tax burden of the rich and a smaller amount is redistributed to the poor. None of the differences is large, and the net effect is a slightly smaller increase in the utility for the rich (compared to column 1). The larger value of $\zeta^r_{wg}$ in column 5 yields a large amount of crowding–out, a nearly zero MWC, and an even smaller

---

\(^{18}\) Subtract equation [11] from equation [13] to get $NB^r_{r} = NB^{14} - (dR_r + WC_r)$. Set $NB^r_{r}$ to zero and solve for $P^r_{w}$ to get $P^r_{w} = -NB^{14} / (dR_r + WC_r)$.

\(^{19}\) With the introduction of charitable giving, net benefit for a poor household is $NB_p = dT_r - (dR_r + WC_r)$. $dR_r + WC_r$ is negligible because $dm_r = 0$ and changes in the labor supply of the poor are small. As a result, $NB_p = dT_r = dR_r + ndc_r$.  

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230
Note that \( dh_r \) is constant across columns 1, 4, and 5 so that the differences in the numerical results are due to different values of \( dc_r \).

The value of \( P_{wg}^{cr} \) is 0.28 in column 4, which is almost the same as that found with cross–price effects in column 2. This seems surprising given that the cross–price elasticities used in column 2 generate crowding–in (\( dc_r = 6.36 \)) and increased labor supply (\( dh_r = 1.86 \)), and the warm–glow elasticity used in column 4 generates crowding–out (\( dc_r = -3.41 \)) and decreased labor supply (\( dh_r = -5.60 \)). Due to these differences, particularly the different values of \( dh_r \), tax revenue and the amount given to the poor are much larger in column 2 than in column 4. This has two effects on \( P_{wg}^{cr} \). First, \( NB_{w t}^{PA} \) is smaller in column 4 because of a smaller increase in the consumption of the poor, and this leads to a larger value of \( P_{wg}^{cr} \). Second, there is a smaller increase in the giving of others in column 4 so that the lost warm–glow utility is smaller. These two effects offset so that \( P_{wg}^{cr} \) is almost identical in columns 2 and 4.

The last two columns of Table 1 show that allowing for both cross–price effects and the warm–glow effect results in crowding–out and increased labor supply. The result is negative MWC. Although welfare cost is negative, net benefit is quite small in column 7. Furthermore, warm glow is lost because of crowding–out, so that \( P_{wg}^{cr} \) is only 0.23 in column 6 and 0.06 in column 7. The implication is that if cross–price and warm–glow behavioral effects are large, as in column 7, a reform does not pass the Pareto test unless the WTP for altruism is large relative to the WTP for warm glow.

As discussed earlier, Diamond (2006) suggests ignoring the utility effects of warm glow, which implies ignoring \( P_{wg}^{cr} dc_r \) in calculating the normative effects of marginal reforms. For the reform considered here, \( dc_r \) is small relative to \( dx_r \) so that following Diamond would have very little effect on the normative implications of the reform. For example, dropping \( P_{wg}^{cr} dc_r \) when computing \( P_{wg}^{cr} \) and \( P_{wg}^{cr} \) leaves the values reported in Table 1 unchanged, or changed by less than two cents. Unless the change in an individual’s giving is sizable relative to the aggregate change in the public good (consumption of the poor here), ignoring warm–glow utility does not have much effect on the normative evaluation of marginal reforms. If however, a researcher wishes to model behavior by ignoring all elements of warm glow (setting \( \zeta_{wg}^{cr} = 0 \) and \( P_{wg}^{cr} = 0 \)), then Table 1 shows that the modeling decision will have a large impact on the normative evaluation of the reforms.

Browning and Johnson (1984), Browning (1995), and Allgood (2003) calculate welfare cost using larger MTRs. Table 2

<table>
<thead>
<tr>
<th>Table 2</th>
<th>CASH TRANSFER REFORM WITH LARGER MARGINAL TAX RATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non–Altruist</td>
<td>Cross–Price Effects Only</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>NB_{w t}^{PA}</td>
<td>96.66</td>
</tr>
<tr>
<td>MWC_{w t}^{PA}</td>
<td>0.32</td>
</tr>
<tr>
<td>P_{wg}^{cr}</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Notes: For all reforms \( m_s = m_r = 0.6, dm_s = 0 \) and \( dm_r = 0.01 \). All other parameters are the same as those reported in Table 1. \( NB_{w t}^{PA} \) is the change in utility for a rich, purely altruistic household, \( MWC_{w t}^{PA} \) is marginal welfare cost if rich households are purely altruistic, and \( P_{wg}^{cr} \) is the value of the willingness to pay for warm glow that sets net benefit for impurely altruistic households (\( NB_{w t}^{PA} \)) to zero. Column 1 is computed assuming no cross–price–effects between leisure and charity and that charitable giving is independent of warm glow (\( \zeta_{wg}^{cr} = \eta_{wg}^{cr} = 0 \)). Columns 2 and 3 allow for cross–price effects by assuming two sets of non–zero values for \( \zeta_{wg}^{cr} \) and \( \eta_{wg}^{cr} \). Columns 4 and 5 introduce warm–glow effects on giving using two values of \( \zeta_{wg}^{cr} \) while again assuming no cross–price effects (\( \zeta_{wg}^{cr} = \eta_{wg}^{cr} = 0 \)). Columns 6 and 7 allow for both cross–price and warm–glow effects.
shows the normative effects of the reform if rich and poor households both have a MTR of 0.60. Net benefit for pure altruists is much smaller for rich households and is even negative in columns 3 and 7 when cross–price elasticities are large. This reflects that increasing the MTR from 0.4 to 0.6 decreases the WTP for altruism from 0.6 to 0.4. As a result, the utility gained from increasing the consumption of the poor is much smaller. Larger initial MTRs also mean a larger welfare cost for a given decline in labor earnings or increase in charitable giving. The net effect of these changes is that $P_{wg}$ is much smaller. Of course, the WTP for warm glow must be non–negative, so there are two cases where the reform fails the Pareto test for any permissible value of $P_{wg}$. In general, larger MTRs imply that reforms only pass the Pareto test for small values of the WTP for warm glow.

Are the differences between the results in Tables 1 and 2 due to increasing the MTR of the poor, the MTR of the rich, or both? A larger initial MTR for only the poor is unlikely to alter the results presented in Table 1. A larger MTR for the poor household will yield larger welfare cost for the poor, but the poor only account for a small amount of tax revenue and tax leakage, so the effect on MWC will be small. Most of the increase in the consumption of the poor arises from changes in charitable giving and additional tax revenue, neither of which is effected by changes in $m_p$. Thus, the normative effects of only increasing $m_p$ are small. As an illustration, the calculations were performed assuming a large MTR for only the poor household ($m_p = 0.6$ and $m_r = 0.4$). The results (not reported) are very similar to those reported in Table 1 where poor households have a smaller MTR. Using the larger cross–price effects (column 3), for example, $NB_{PA}^P$ is $166$, MWC is $-0.22$, and $P_{wg}$ is $0.17$. Each of these numbers is almost identical when the poor are assumed to face the lower MTR used to compute numbers in Table 1.

Lastly, consider the effect that the number of rich households ($n$) has on the welfare properties of the reform. More rich households mean more is redistributed to the poor so that $NB_{PA}^P$ is larger, and this tends to make $P_{wg}$ larger. Of course, there is also greater giving by others ($dx_{p,r}$), which lowers warm–glow utility and this tends to generate smaller values of $P_{wg}$. For example, suppose there are five rich households instead of the three assumed for the calculations in Table 1, and assume that MTRs are set to their original values of $m_p = 0.2$ and $m_r = 0.4$. Given the other parameters from column 1 of Table 1, $NB_{PA}^P$ is 483.45, over twice as large as the number reported in column 1 of Table 1. The reform passes the Pareto test as long as $P_{wg} \leq 0.4$, or equivalently $P_a \geq 0.16$. This compares to a value of $P_{wg} = 0.32$ in column 1. Put another way, if there are more rich households contributing to the consumption of the poor, then each rich household can value altruism less—at the margin—and the reform would still pass the Pareto test.

CONCLUSION

It is not surprising that altruism creates opportunities for Pareto–improving marginal redistributions of income. Hochman and Rogers (1969, 1974) argued this point years ago. Warm–glow preferences mean that households receive utility just from giving to charity and eases the concerns raised by Warr (1982) that giving by others will crowd–out private giving dollar–for–dollar. Consequently, it would seem that warm–glow utility, like altruism, creates more opportunities for marginal redistributions of income to pass the Pareto test. However, warm glow can also be interpreted in the following manner: for a given size of the public good, an individual receives less utility the greater the giving by others (where the consumption of the poor is the public good in the context of this paper). Reforms that generate greater giving by
others (public or private) will generate smaller utility gains if households are modeled as impure altruists rather than as pure altruists.

The numerical calculations illustrate the magnitudes of the welfare effects of different assumptions about preferences introduced by the analytical model. The results suggest that the utility gains of rich households are smaller if labor supply is more responsive to changes in the price of charitable giving. Utility gains are also smaller if charitable giving is more responsive to changes in the wage rate and changes in the giving of others. Furthermore, rich households must value altruism at least as much as warm glow (at the margin) for the reform and data considered here to generate Pareto improvements. Unfortunately, there is no current estimate of the WTP for warm glow, but the numerical calculations do highlight an interesting result. The reform can pass the Pareto test for larger values of the WTP for warm glow when it is assumed that there are five rich households versus when it is assumed that there are three rich households. This occurs because more rich households mean more contributors to the consumption of the poor and the positive externality more than offsets the lost warm glow.

Without a proper understanding of warm glow, this last result is somewhat of a paradox: as the poor comprise a larger fraction of the population, there is a smaller range of values for the WTP for warm glow for which the reform passes the Pareto criterion. Suppose there are five rich households so that the poor comprise one-sixth of the population and empirical estimates suggest that the WTP for warm glow is 0.44. The reform considered here passes the Pareto test. If, in fact, there are only three rich households, so that the poor comprise one-fourth of the population, the reform fails the Pareto test for the same WTP for warm glow. In an environment where there is a greater need for redistribution (the poor comprise a larger fraction of the population) there are fewer values of the WTP for warm glow that generate Pareto–improving redistributions.

Acknowledgments

The author acknowledges the helpful comments of Arthur Snow, Edgar O. Olsen, and two anonymous referees.

REFERENCES


APPENDIX A

Consider a rich household that is an impure altruist. To derive net benefit, totally differentiate the indirect utility function $V(γ, w, I, x_{p,γ})$, divide by $V_{I}$, use Roy’s identity, and substitute in for $dl$, from equation [8] to obtain

$$NB_{r}^{IA} = -γ_{d}dw + x_{d,γ} + Ldw + x_{γ,γ} + γdx_{p,γ} + dM_{I} - P_{w_{g}}dx_{p,γ},$$

where $dN_{r} = 0$, $P_{w_{g}} = -V_{w_{g}}/V_{I}$ is the WTP for warm glow, and $V_{w_{g}} = ∂V/∂x_{p,γ} | dI = 0$. Substitute

---

Boswick, Robin, Pierre Pestieau, and David Wildasin.  

Browning, Edgar.  

Browning, Edgar, and William R. Johnson.  

Corneo, Giacomo, and Hans Peter Gruner.  

Creedy, John.  

Dahlby, Bev.  

Dawes, Robyn M. and Richard H. Thaler.  

Diamond, Peter.  

Fullerton, Don.  

Hochman, Harold M., and James D. Rodgers.  

Hochman, Harold M., and James D. Rodgers.  

Kaplow, Louis.  

Roberts, Russell D.  

Rosen, Harvey S.  

Saez, Emmanuel.  

Tiehen, Laura.  

Triest, Robert K.  

Triest, Robert K.  

Warr, Peter G.  
into $NB_i^{IA}$ for $dw_i, d\gamma_i, dR_i,$ and $dx_{p,r} = dx_j - dc_j,$ which yields equations [11] and [12]. Set $P_{wg} = 0$ to obtain equations [13] and [14] for a pure altruist.

Suppose a household maximizes utility by its choice of $x_i$ and $\ell_i$ given preferences $u(x_i, \ell_i),$ where consumption $x_i + w_i \ell_i = I$ equals income $I = w_iT + N_i + M_i.$ The indirect utility function is $V(w_i, I_i).$ To derive net benefit, totally differentiate the indirect utility function, divide by $VI_i,$ use Roy’s identity, and use $dI_i = Tdw_i + dM_i$ to obtain

$$NB_i^{NA} = -\ell_i dw_i + Ldw_i + dM_i,$$

where $dN_i = 0.$ The change in a rich household’s tax liability is the total differential of equation $R_i = m_1 \delta W_{1i} - N_i, dR_i = \delta W_{1i} dm_i + m_1 \delta W_{1i}.$ Substitute into $NB_i^{NA}$ for $dw_i, d\gamma_i, dR_i,$ yielding equations [15] and [16].

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Hours Worked & Wage & Exogenous Income \\
(h) & (\delta W) & (M + N) \\
\hline
Poor & 1900 & 4.97 & 5,018.70 \\
Rich & 2300 & 11.12 & 4,394.00 \\
\hline
\end{tabular}
\caption{APPENDIX TABLE A}
\end{table}

\section*{APPENDIX B}

To derive equations [17] – [19], take the total differential of $h_i(w_i, \gamma_i, I_i), h_j(w_j, I_j),$ and $c_i(w_i, \gamma_i, l_i, x_{p,r}),$

$$dh_i = h_i dw_i + h_i d\gamma_i + h_i dI_i,$$

$$dc_i = c_i dw_i + c_i d\gamma_i + c_i dI_i + c_i dx_{p,r},$$

where $h_i$ is the partial derivative of $i$’s labor supply function with respect to argument $k,$ and $dh_i$ and $dc_i$ are defined similarly. Define the elasticity of $i$’s labor supply with respect to argument $k$ to be $\eta_i^k = (k/h_i)h_i.$ The elasticities for $h$ and $c,$ $\eta_i^h$ and $\eta_i^c,$ are similarly defined. Converting the partial derivatives of the three equations into elasticities and using the fact that $d(w_i/\delta W) = d\gamma_i$ completes the derivation.

\section*{APPENDIX C}

Data on hours worked and wage rates are from Triest (1994, Table 4a) as described in the text. Appendix Table A shows data for poor and rich households. Non–labor income for poor households is $T_p,$ where $\beta R$ is from government transfers ($\beta R$) and $1,816.68$ is from charitable contributions. Exogenous income for rich households is $N_i + M_i,$ where $N_i = (m_i - m_j)y$ and $M_i$ is endowed income. To determine $y,$ the tax brackets are defined as the average of the labor income of rich and poor households $y = (w_ih_i + w_jh_j)/2,$ as in Allgood (2003). $N_i$ is then calculated using the marginal tax rates in the text, and $M_i$ is set to $4,394. As in Saez (2004), charitable contributions are calculated assuming that households give 2.5 percent of labor and exogenous income to the poor. It is also assumed that households are endowed with 8,000 hours a year ($L = 8,000).