Optimal Taxation of Families with Endogenous Fertility

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Abstract

In this paper, we introduce an endogenous fertility choice into an optimal nonlinear taxation model. Households are heterogeneous along several dimensions: the family labor earning ability and preferences over family size. While existing works in the literature have considered the impact of fertility and family size on optimal tax systems, our discrete choice framework allows us to consider the question within a continuous-type Mirrleesian model which can be directly compared to the data. We investigate the properties of the optimal nonlinear labor income tax and nonlinear taxation of children-related expenses. In addition to the optimal tax problem, we also consider an inverse optimum approach to recover the Pareto weights on the Planners social welfare function that would rationalize the current tax system present in the United States.

JEL-Classification: D1, H21, H24, J13

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1 Introduction

The design and implementation of optimal nonlinear tax systems have been considered going back to the seminal work of Mirrlees (1971). Such problems naturally require the simultaneous consideration of governmental efficiency and society's preference for redistribution. Early works in the optimal nonlinear income tax literature have abstracted away from family concerns such as child-rearing and the labor supply of secondary earners, and have focused instead on the design of optimal tax policies among individuals (or families) with perfectly observable incomes (see Seade, 1977, Atkinson and Stiglitz, 1976, for example).[1] However, concerns regarding both the number and age of children, as well as the joint labor supply decisions of the family, which includes the labor supply decision of secondary income earners, are crucially important for determining the shape and progressivity of tax and benefit systems utilized throughout the developed world [2] In this paper, we account for these family considerations by considering the optimal nonlinear tax problem within a model where agents face an endogenous discrete choice regarding the number of children to have (e.g., endogenous fertility) and a continuous choice regarding their time allocation between market work, childcare, and leisure activities.[3]

Accounting for the number of children within a family in an optimal tax setting addresses two concerns. First, it provides an additional observable over which the government may condition the optimal tax system. By serving as an additional observable, the number of children within a household reduces the information problem for the government, thereby facilitating an easier (or at least a less costly) redistribution of resources across agents with unobserved heterogenous characteristics (Akerlof, 1978). Second, casual inspection of the tax and benefit systems for many developed economies makes clear that social preferences may be paternalistic and provide financial incentives for increased fertility among certain income groups within the population, or at the very least poverty alleviation measures for households

[1]However, questions of optimal population size, fertility and whether the government should interfere in these decisions have been of interest to economists for a long time and have been explored in the literature, e.g., Mirrlees (1972).

[2]For example, the earned income tax credit (EITC) represents one of the largest redistributive programs in the United States, and its generosity is directly tied to both household income levels and the number and age of dependents (e.g., children) living within the household.

[3]The inclusion of both fertility (e.g., number of children) and other continuous choices (e.g., time allocation) complicates the optimal tax problem as an empirically relevant model requires multidimensional heterogeneity across households. This then leads to issues related to multidimensional screening problems, see Armstrong (1996), and Rochet and Choné (1998).

with children. The United States' EITC and Child Tax Credit (CTC), Canada's Canada Child Benefit (CCB) and the United Kingdom's Child Tax Credit (UKCTC) are all examples of such transfer programs. By considering both endogenous fertility and time allocation across market work, child-rearing, and leisure, we are able to assess how these margins interact to shape the optimal tax function. Furthermore, and perhaps more importantly, we are also able to assess how adjustments in the tax function feedback on individuals and alter the fertility decisions observed across the income distribution.

A few works within the existing literature have considered the impact of heterogenous fertility on optimal nonlinear tax systems.[4] For example, Balestrino et al. (2002) incorporates endogenous fertility, as continuous choice of children, within their optimal tax model. However, their primary focus is on the information provided by the knowledge of family size when solving the optimal tax problem. Their analysis is restricted to a two household-type setting where households can differ in their wage rates and domestic productivity which are restricted to take on two possible values. This simplified framework allows the authors to manage the multidimensional screening problem but limits their ability to analyze, more broadly, the progressivity of the tax system along the whole income distribution and for different family sizes. In contrast, by assuming a discrete fertility choice in the present paper, we are able to tackle the multidimensional screening problem in a fashion similar to that presented in Kleven et al. (2009) while still allowing for a large number of agent-types which is necessary in generating empirically consistent income and family-size distributions. Blundell and Shephard (2012) also consider the impact of children on the optimal tax problem, but they treat the number of kids within a household as an exogenous assignment, rather than an endogenous choice. Thus, while they are able to perform a rigorous estimation exercise using U.K. data, they are unable to address how adjustments in the tax policy directly alter fertility decisions of individuals (or households) with various income levels.

Along with the optimal tax problem discussed above, we also consider a modified inverse optimum approach to recover the social welfare function that would "rationalize" the tax system in the United States given the empirically observed distribution of children across family income levels observed in the data. The general inverse optimum approach was introduced to the optimal tax literature by Bourguinon and Spadaro (2012) and extended to include both intensive and extensive margins of labor supply (e.g., both discrete and continuous choices)

[4]There exists a body of literature that considers the optimal linear tax system with either fixed or endogenous fertility e.g., Cigno and Pettini (2002).

by Zoutman et al. (2017). In this paper, we approach the inverse optimum in a constrained way following Heathcote and Tsujiyama (2017) and Moser and Silva (2017)[5]. Specifically, rather than solving for the social welfare function non-parametrically as in Bourguinon and Spadaro (2012), we specify a functional form for the social welfare function and then solve for the Pareto weighting function that would result in an optimal tax system that mimics the tax system observed for the U.S. economy. By recovering the Pareto weights in this way, we are able to assess the implications of such social preferences and consider if these preferences satisfy basic regularity conditions one would expect. We are also able to consider modified Pareto weighting schemes that alter the weights placed on particular agents in the economy and assess the fertility and labor supply response that results from such changes.

The remainder of the paper is organized as follows. Section 2 presents the model structure, highlighting the households decisions which primarily consist of labor supply, time allocation, and endogenous fertility decisions. Section 3 specifies the optimal tax problem using both household labor hours and number of children as observables. Section 4 provides a brief discussion of the data used to calibrate our model, and the rest is ongoing.

2 Model

There is a continuum of heterogeneous families that are heterogeneous along $N+2$ dimensions. First they differ in family earning ability w, which is distributed according to the following distribution function $F(w)$ with density $f(w)$. They also differ in their preferences towards family size. This is captured by the vector $\boldsymbol{\delta} = \{\delta_0, \delta_1, ..., \delta_n, ..., \delta_N\}$, where each δ_n giving the utility value of the specific number of kids n, with $n \in \{0, 1, 2, ..., N\}$. These preferences are distributed according to distribution function $H(\boldsymbol{\delta}|w)$, where it is assumed that the preference vector may be distributed differently for different family earning ability.

A family i with number of children n chooses family consumption c (or parents' consumption, see Balestrino et al. (2002)), labor supply l, which gives labor income $y \equiv wl$,

[5] Moser and Silva (2017) consider the possibility of paternalistic social welfare functions that are crucial for the determents of mandatory retirement systems, such as social security. Paternalism is also relevant in our case as the government may place higher weight on the number and quality of children than the household themselves. This paternalism can be interpreted as a first-order approximation of richer models that incorporate lifecycle effects and future benefits to society, or to the government, of a greater number of "quality" individuals.

how much time to spend with their children per child, e, and also how much children related expenses per child k. Labor income and child expenses lead to tax burden $T(y,k,n)$.

Following the quantity-quality framework of the economics of the family (Becker, 1991), a child's quality in a family of n children is determined by the following function

$$Q(k,e,n), \text{ with } \frac{\partial Q}{\partial m} > 0, \ \frac{\partial^2 Q}{\partial^2 m} \leq 0 \text{ for } m = k, e,$$

with cross-partial being of any sign. Alternatively Q can be viewed as the "parental perception of a child's lifetime utility" (Balestrino et al., 2002).

The utility function of a family i given a number n of children is

$$\mathscr{U}(c,l,e,k;n) \equiv u(c) + z\big(Q(k,e,n);n\big) - h(l,ne),$$

where u is a strictly concave function, $z'' \leq 0 < z'$, $z(Q;0) = 0$, $h_l', h_l'', h_e', h_e'' > 0$ with $\frac{\partial^2 h(l,e)}{\partial l \partial e} > 0$.

Let the utility obtained for a family of type $(w, \boldsymbol{\delta})$ that is maximizing its intensive choices, i.e. y, e, k, given a number of children n be

$$U(w;n) \equiv \max_{y,e,k} \mathscr{U}\Big(y - nk - T(y,k,n), \frac{y}{w}, e, k; n\Big).$$

Thus a family of type $(w, \boldsymbol{\delta})$ with n children will have total utility

$$U(w;n) + \delta_n$$

2.1 The choice of number of children

A family of type $(w, \boldsymbol{\delta})$ will decide to have n children if

$$U(w;n) + \delta_n \geq U(w;\tilde{n}) + \delta_{\tilde{n}} \quad \forall \tilde{n} \neq n.$$

This will lead families with earnings ability w to be divided into different groups of family sizes. The fraction of families of skill w with n children is $P(\boldsymbol{U};n,w)$, where the vector

$\boldsymbol{U} = \{U(w;1), U(w;2), ..., U(w;n), ..., U(w;N)\}$ is the vector of "intensive" utility given a determined number of children. Note that the following equation

$$\sum_{n=0}^{N} P(\boldsymbol{U}; n, w) = 1,$$

must hold.

The number of families in the economy will sum to 1, i.e.

$$\int_{\underline{w}}^{\bar{w}} \sum_{n}^{N} P(\boldsymbol{U}; n, w) f(w) dw = 1.$$

Note that it will be convenient to define $p(n;w) \equiv P(\boldsymbol{U}; n, w) f(w)$ as the total number of families of skill w with n children.

2.2 Behavior of families

Solving the problem of the families given a number of children n give the following first-order conditions

$$(y): \qquad u'(c)[1 - T'_y(y,k,n)] - \frac{h'_l(y/w, ne)}{w} = 0, \tag{2.1}$$

$$(k): \qquad -u'(c)[n + T'_k(y,k,n)] + z'_Q(Q;n)Q'_k(k,e,n) = 0, \tag{2.2}$$

$$(e): \qquad z'_Q(Q;n)Q'_e(k,e,n) - h'_e(y/w, ne)n = 0. \tag{2.3}$$

Note for families with no children, (2.2) and (2.3) do not apply. Equations (2.1) and (2.2) can be used to obtain the tax wedges

$$T'_y(y,k,n) = 1 - \frac{h'_l(y/w, ne)}{u'(c)w}, \tag{2.4}$$

$$T'_k(y,k,n) = \frac{z'_Q(Q;n)Q'_k(k,e,n)}{u'(c)} - n. \tag{2.5}$$

The elasticity of the fraction of families of skill w with n children with respect to a change in consumption of families of skill w with $\tilde{n}$ children, $c(w;\tilde{n})$, is

$$\kappa_{\tilde{n}}(w;n) \equiv \frac{P'_{U\tilde{n}}(\boldsymbol{U};n,w)}{P(\boldsymbol{U};n,w)}u'\big(c(w;\tilde{n})\big) \tag{2.6}$$

where

$$P'_{U\tilde{n}}(\boldsymbol{U};n,w) \equiv \frac{\partial P(\boldsymbol{U};n,w)}{\partial U(w;\tilde{n})} \text{ for } \tilde{n}=0,..,N.$$

This elasticity measures the substitution behavior of families of skill w across different family sizes. Note that $\kappa_n(w;n)$ will capture the increase in the fraction of families of skill w who chooses to have n children as a response to an increase in consumption of families of skill w with n children, i.e. $c(w;n)$.

2.3 The government

The budget constraint of the government is

$$\int_{\underline{w}}^{\bar{w}}\sum_{n=0}^{N}T(y(w;n),k(w;n),n)\,P(\boldsymbol{U};n,w)f(w)dw = R,$$

where R is an exogenous amount of public expenditures. Let $y(w;n)$ and $k(w;n)$ be the intensive choices, that are observable to the government, of a family of skill w and number of children n, and let $c(w;n)=y(w;n)-nk(w;n)-T(y(w;n),k(w;n),n)$ be the corresponding consumption level. This allows to write the government budget constraint as

$$\int_{\underline{w}}^{\bar{w}}\sum_{n=0}^{N}[y(w;n)-nk(w;n)-c(w;n)]P(\boldsymbol{U};n,w)f(w)dw = R. \tag{2.7}$$

The government's preferences are written in a very general way so that it could suit different types of preferences (see Saez and Stantcheva (2016)):

$$W=\int_{\underline{w}}^{\bar{w}}\sum_{n=0}^{N}\left\{\omega(w,n)E\left[U(w;n)+\delta_n|n\right]+n\gamma\left(Q(k(w;n),e(w;n),n);n\right)\right\}P(\boldsymbol{U};n,w)f(w)dw, \tag{2.8}$$

where $\omega(w,n)$ is a welfare weight given to families to earning ability w and number of children

n. The function $\gamma\left(Q(k(w;n),e(w;n),n)\right)$ captures potential positive externalities children bring to society. Therefore, these preferences, whenever $\gamma(\cdot)\geq 0$, are paternalistic in nature and give value to children in excess of the value families give to children. If $\gamma(Q)=1\ \forall Q$, then the planner only puts weight on the number of children. However, if $\gamma''\leq 0<\gamma'$, then the government cares about the quality of the children and not just their number. Here the positive externality of children are increasing in their quality.

3 Optimal Tax Policy

First define the value to the government of a dollar given to a family with earning w and number of children n,

$$g(w;n)=\frac{\omega(w;n)u'(c(w;n))}{\lambda}.$$

Also define the social value of a child from a family that has earning ability w and number of children n as

$$g^c(w;n)\equiv\frac{\gamma\left(Q(k(w;n),e(w;n),n);n\right)}{\lambda}.$$

Also define the following "partial" elasticities

$$\alpha_y(w;n)\equiv\frac{h'_l(y/w,ne)}{h''_{ll}(y/w,ne)l},\quad\text{and}\quad\alpha_e(w;n)\equiv\frac{h'_l(y/w,ne)}{h''_{le}(y/w,ne)n}.$$

Also consider the following definitions

$$\chi_Q=-\frac{z''_{QQ}Q}{z'_Q}\geq 0,\quad \xi_{ke}=\frac{Q''_{ke}Q}{Q'_kQ'_e}\geq 0\text{ and }\xi_{ee}=\frac{Q''_{ee}Q}{[Q'_e]^2}\leq 0$$

3.1 Observable family earning skill

This subsection is similar to Jacquet et al. (2013) where they investigate the first-and-half best setting, i.e. the case where the government is able to observe wage levels but not the preferences of the workers. In this section, we also suppose that the planner can't observe the choice of time spend with children e. Thus the government seeks to maximize (2.8) subject to (2.7) and the first-order condition of the families with children with respect to time spent with their children, i.e.

$$z'_Q(Q;n)Q'_e(k,e,n) - h'_e(y/w,ne)n = 0,$$

and that for all n and skill types. It does this by choosing allocation $\{\boldsymbol{c(w)}, \boldsymbol{y(w)}, \boldsymbol{e(w)}, \boldsymbol{k(w)}\}$ where $\boldsymbol{c(w)} = \{c(w;0), ...c(w;N)\}$, $\boldsymbol{y(w)} = \{y(w;0), ...y(w;N)\}$, $\boldsymbol{e(w)} = \{e(w;0), ...e(w;N)\}$ and $\boldsymbol{k(w)} = \{k(w;0), ...k(w;N)\}$.

Lemma 1. *At the Observable family earning skill optimum, the optimal allocation verifies*

$$\frac{T'_y(y(w;n),k(w;n),n)}{1-T'_y(y(w;n),k(w;n),n)} = \frac{n\gamma'(Q(k,e,n))Q'_e(k(w;n),e(w;n),n)}{\frac{z'_Q\left(Q\left(k,e\right);n\right)[Q'_e]^2}{Q}[\chi_Q - \xi_{ee}] + h''_{ee}\left(\frac{y}{w},ne\right)n^2} \frac{u'(c(w;n))}{\lambda\alpha_e(w;n)} \geq 0 \ \forall w,n \tag{3.1}$$

$$T'_k(y(w;n),k(w;n),n) = -\left[1 - \frac{\chi_Q - \xi_{ke}}{[\chi_Q - \xi_{ee}] + \frac{h''_{ee}\left(\frac{y}{w},ne\right)n^2 Q}{z'_Q\left(Q\left(k,e\right);n\right)[Q'_e]^2}} \frac{1}{\lambda}\right] n\gamma'(Q(k,e,n))Q'_k(k,e,n) \tag{3.2}$$

$$\sum_{\tilde{n}\neq n}^{N}[T(y,k,\tilde{n}) - T(y,k,n)]\kappa_n(w;\tilde{n}) = 1 - g(w;n) - \sum_{\tilde{n}\neq n}^{N}\left[\tilde{n}g^c(w;\tilde{n}) - ng^c(w;n)\right]\kappa_n(w;\tilde{n}). \tag{3.3}$$

Proof: See Appendix.

Under this information scenario, marginal taxes on labor income or spending on children are only for pigouvian reason. But the tax system is still redistributive by the presence of the social marginal utility of the household $g(w;n)$ in (3.3), but it also puts weight on the value he puts on children and their quality.

Lemma 2. *At the Observable family earning skill optimum without any paternalistic values towards children, i.e.* $\gamma(Q(k,e,n)) = 0 \ \forall n > 0$ *, the optimal allocation verifies*

$$T'_y(y(w;n),k(w;n),n) = 0 \quad \textit{and} \quad T'_k(y(w;n),k(w;n),n) = 0 \ \forall w,n, \tag{3.4}$$

$$T(y(w;n),k(w;n),n) = \frac{1-g(w;n)}{\kappa_n(w;n)} - \sum_{\tilde{n}\neq n}^{N} T(y,k,\tilde{n})\frac{\kappa_n(w;\tilde{n})}{\kappa_n(w;n)} \ \forall w,n \tag{3.5}$$

Proof: Take equations from lemma 1 and use the fact that $\gamma(Q(k,e,n)) = 0$ and that the sum of the elasticities $\kappa_n(w;\tilde{n})$ must equal $-\kappa_n(w;n)$.

Without paternalistic motives, the government no longer needs to distort the choice of families. The tax burden of families has a similar formula to those found in the extensive margin literature, e.g. Saez(2002) and Jacquet et al. (2013).

3.2 Second-Best: Unobservable labor and time towards children

3.2.1 Incentive Comptability

In this subsection, we assume that the government is unable to observe labor and the time spend with children, however he is able to observe labor income y and spending on children, the number of children n and thus the spending per children k. The problem of the government is to find the optimal tax schedule $T(y,k,n)$ to maximize the social objective (2.8) subject to the budget constraint (2.7) and the behavior of the families.

We adopt a Mirrleesian approach and solve the government's problem by choosing the allocation that maximizes his preferences subject to the budget constraint and a series of incentive constraints. The model of this paper allows to limit the number of ways families can mimic since the number of children is observable. Hence, a family with a number of children n can only mimic other families with number of children n, this leads to the following set of incentive constraints

$$\begin{aligned}U(w;n) = \mathscr{U}\left(c(w;n), \frac{y(w;n)}{w}, e(w;n), k(w;n); n\right) \\ \geq \mathscr{U}\left(c(\tilde{w};n), \frac{y(\tilde{w};n)}{w}, e(\tilde{w};n), k(\tilde{w};n); n\right) \ \forall\ w, \tilde{w} \in [\underline{w}, \bar{w}], \text{for } n = 0, ..., N.\end{aligned} \tag{3.6}$$

On the other hand, these sets of incentive constraints are not enough to ensure incentive compatibility. Recall that the choice e is unobservable to the planner, therefore the allocation must also satisfy the family's first-order condition with respect to the choice of time spent per children, i.e.

$$z'_Q\big(Q\big(k(w;n),e(w;n)\big);n\big)Q'_e\big(k(w;n),e(w;n),n\big)-h'_e\left(\frac{y(w;n)}{w},ne(w;n)\right)n=0\;\forall w,\;\text{for}\;n=1,...N. \tag{3.7}$$

Also note that this only applies to families with children, i.e. $n \geq 1$.

Because there is a continuum of family earning ability levels, this would cause an infinity of constraints and that for every number of children. Thus we adopt the first-order approach where the incentive constraints are replaced by an envelope condition, i.e.

$$\frac{\partial U(w;n)}{\partial w}=\frac{h'_l\left(\frac{y(w;n)}{w},ne(w;n)\right)y(w;n)}{w^2}. \tag{3.8}$$

However, this is not sufficient to insure incentive compatibility. A monotonicity constraint would also be required. We ignore this constraint for now and assume it is satisfied.[6] In addition, the extra incentive constraints (3.7) add a complication to the first-order approach as it may no longer be valid. However, as shown in Abraham and Pavoni (2008), there are many cases in which it remains valid and thus we will assume it is valid in this situation and it will be verified *ex-post* numerically. Thus we will call the "relaxed problem" the problem where the incentive constraints are replaced by envelope conditions without adding a monotonicity constraint.

[6]In numerical simulations, the monotonicity condition is verified ex-post.

3.2.2 The Government's Problem

We write the government's relaxed problem the following way

$$\max_{\{\boldsymbol{c(w)},\boldsymbol{y(w)},\boldsymbol{e(w)},\boldsymbol{k(w)}\}_{\forall w}}$$

$$\int_{\underline{w}}^{\bar{w}} \sum_{n=0}^{N} \{\omega(w,n)E\left[U(w;n)+\delta_n|n\right]+n\gamma\left(Q(k(w;n),e(w;n),n);n\right)\} P(\boldsymbol{U(w)};n,w)f(w)dw$$

$$\begin{aligned}
s.t. \quad & \int_{\underline{w}}^{\bar{w}} \sum_{n=0}^{N}[y(w;n)-nk(w;n)-c(w;n)]P(\boldsymbol{U};n,w)f(w)dw = R \\
& U(w;n) = u(c(w;n)) + z\Big(Q(k(w;n),e(w;n),n);n\Big) - h\left(\frac{y(w;n)}{w}, ne(w;n)\right) \\
& \dot{U}(w;n) = \frac{h_l'\left(\frac{y(w;n)}{w}, ne(w;n)\right) y(w;n)}{w^2} \\
& z_Q'\Big(Q\big(k(w;n),e(w;n)\big);n\Big)Q_e'\big(k(w;n),e(w;n),n\big) - h_e'\left(\frac{y(w;n)}{w}, ne(w;n)\right)n = 0,
\end{aligned}$$

with $\boldsymbol{c(w)} = \{c(w;0),...c(w;N)\}$, $\boldsymbol{y(w)} = \{y(w;0),...y(w;N)\}$, $\boldsymbol{e(w)} = \{e(w;0),...e(w;N)\}$ and $\boldsymbol{k(w)} = \{k(w;0),...k(w;N)\}$. The government's problem is then an optimal control problem with $c(w;n), y(w;n), e(w;n), k(w;n)$ for each number of children as control variables and $U(w;n)$'s as state variables with the envelope condition being the "state equations" (law of motion). Also note that

$$c(w;n) = u^{-1}\left(U(w;n) - z\Big(Q(k(w;n),e(w;n),n);n\Big) + h\left(\frac{y(w;n)}{w}, ne(w;n)\right)\right)$$

This leads to the following Hamiltonian:

$$
\begin{aligned}
&\mathcal{H}(\boldsymbol{y},\boldsymbol{k},\boldsymbol{e},\boldsymbol{U},w,\boldsymbol{\mu},\boldsymbol{\nu}) = \\
&\sum_{n=0}^{N}\left\{\omega(w,n)E\left[U(w;n)+\delta_n|n\right]+n\gamma\left(Q(k(w;n),e(w;n),n);n\right)\right\}P(\boldsymbol{U(w)};n,w)f(w) \\
&+\lambda\sum_{n=0}^{N}\Big\{y(w;n)-nk(w;n) \\
&\quad -u^{-1}\Big(U(w;n)-z\big(Q(k(w;n),e(w;n),n);n\big)+h\big(y(w;n)/w,ne(w;n)\big)\Big)\Big\}P(\boldsymbol{U(w)};n,w)f(w) \\
&+\sum_{n=0}^{N}\mu(w;n)\left[\frac{h_l'\left(\dfrac{y(w;n)}{w},ne(w;n)\right)y(w;n)}{w^2}\right] \\
&+\sum_{n=1}^{N}\nu(w;n)\left[z_Q'\big(Q\big(k(w;n),e(w;n)\big);n\big)Q_e'\big(k(w;n),e(w;n),n\big)-h_e'\left(\frac{y(w;n)}{w},ne(w;n)\right)n\right]
\end{aligned}
\tag{3.9}
$$

3.2.3 Optimal Tax Formulas

Assumption 1. *The solution to the relaxed problem is incentive-comptatible and solves the government's problem. In addition, the costate variables $\mu(w;n)$ are negative for some $w \in (\underline{w},\bar{w})$ and that for all family size n.*

Assumption 2. *There is no bunching for all family earning ability $w \in [\underline{w},\bar{w}]$.*

Proposition 1. *Under Assumptions 1 and 2, the optimal second-best tax system verify the* *following conditions:*

$$\frac{T_y'^*(y(w;n),k(w;n),n)}{1-T_y'^*(y(w;n),k(w;n),n)} = \left[\frac{-\mu(w;n)}{w}\left(1+\frac{1}{\alpha_y(w;n)}\right)+\frac{\nu(w;n)}{\alpha_e(w;n)}\right]\frac{u'(c(w;n))}{\lambda p(n,w)}\ \forall w,n \tag{3.10}$$

$$\begin{aligned} T_k'^*(y(w;n),k(w;n),n) &= -n\gamma'(Q(k,e,n))Q_k'(k(w;n),e(w;n),n) \\ &+\frac{\nu(w;n)}{\lambda p(n,w)}\frac{z_Q'\big(Q\big(k,e\big);n\big)Q_k'(k,e,n)Q_e'(k,e,n)}{Q\big(k,e\big)}[\chi_Q-\xi_{ke}]\ \forall w,n>0 \end{aligned} \tag{3.11}$$

where

$$\begin{aligned} -\mu(w;n) = \int_w^{\bar{w}} \Bigg\{ & 1-g(\tilde{w};n)-\sum_{\tilde{n}\neq n}^{N}\left[\tilde{n}g^c(\tilde{w};\tilde{n})-ng^c(\tilde{w};n)\right]\kappa_n(\tilde{w};\tilde{n}) \\ & -\sum_{\tilde{n}\neq n}^{N}[T(y,k,\tilde{n})-T(y,k,n)]\kappa_n(\tilde{w};\tilde{n})\Bigg\}\frac{\lambda p(n;\tilde{w})}{u'(c(\tilde{w};n))}d\tilde{w} \qquad \forall w,n>0 \end{aligned} \tag{3.12}$$

$$\begin{aligned} \nu(w;n) = \Bigg\{ & \frac{n\gamma'(Q(k,e,n))Q_e'(k(w;n),e(w;n),n)}{\frac{z_Q'\left(Q\left(k,e\right);n\right)[Q_e']^2}{Q}[\chi_Q-\xi_{ee}]+h_{ee}''\left(\frac{y}{w},ne\right)n^2} \\ & +\frac{\mu(w;n)\left[\frac{nh_{le}''\left(\frac{y}{w},ne\right)y}{w^2}\right]}{\frac{z_Q'\left(Q\left(k,e\right);n\right)[Q_e']^2}{Q}[\chi_Q-\xi_{ee}]+h_{ee}''\left(\frac{y}{w},ne\right)n^2}\frac{1}{p(n;w)}\Bigg\}p(n;w)\ \forall w,n>0, \end{aligned} \tag{3.13}$$

and with transversality conditions $\lim_{w\to\underline{w}}\mu(w;n)=0$ *and* $\lim_{w\to\bar{w}}\mu(w;n)=0.$ (3.14)

Proof: See Appendix

Corollary 1.a. *Under Assumptions 1 and 2, if the distribution of* w *is bounded from above* $\bar{w}<\infty$*, bounded from below* $\underline{w}>0$*, and the government has paternalistic preferences over number of children and quality of children, i.e.* $\gamma\left(Q(k(w;n),e(w;n),n);n\right)>0\ \forall n>0$*, then the marginal labor income tax rates at the top and bottom for families with children is positive* $T_y'^*(y(w;n),k(w;n),n)>0$*. If the government puts no weight on children besides the weight put by their own parent, i.e.* $\gamma\left(Q(k(w;n),e(w;n),n);n\right)=0\ \forall n>0$*, then the marginal labor income tax rates at the top and bottom are nil for all family size,* $T_y'^*(y(w;n),k(w;n),n)=0.$

Proof: Use the transversality conditions and insert them into (3.10). This is a pigouvian result where increasing the relative cost of labor effort compared to time spent with children leads to more children quality.

Corollary 1.b. *Under Assumptions 1 and 2, and assuming that the government puts no additional weight on children, i.e.* $\gamma\left(Q(k(w;n), e(w;n), n); n\right) = 0\ \forall n > 0$,*, for the families with* $\mu(w;n) < 0$ *the marginal tax rate on spending on children will have the following features under different scenarios:*

- *If subutility function* $z(\cdot)$ *is linear, i.e.* $\chi_Q = 0$*, then* $T_k'^*(y(w;n), k(w;n), n) \geq 0\ \forall\ w \in (\underline{w}, \bar{w}),\ n > 0$,
- *If subutility function* $z(\cdot)$ *is non-linear, i.e.* $\chi_Q > 0$*, and* $\xi_{ke} = 0$*, then* $T_k'^*(y(w;n), k(w;n), n) < 0\ \forall\ w \in (\underline{w}, \bar{w}),\ n > 0$,
- *if subutility function* $z(Q) = \ln Q$ *i.e.* $\chi_Q = 1$*, then* $T_k'^* \gtreqless 0$ *if* $\xi_{ke} \lesseqgtr 1\ \forall\ w \in (\underline{w}, \bar{w}),\ n > 0$*. If* $\xi_{ke} = 1$*, e.g. a Cobb-Douglas production function, then* $T_k'^* = 0\ \forall\ w \in (\underline{w}, \bar{w}),\ n > 0$.

Proof: Use equation (3.11), equation (3.13) and the assumptions to obtain the results. Corollary 1.b is related to the Atkinson-Stiglitz theorem. Since labor effort and time spent with children are not separable, the government will wish to subsidize spending on children when child spending and time spent with children are more subsitutes whereas he will wish to tax spending on children when they are complements. Here the government wishes to make spending time with children less desirable to mimickers so that families can work more and he is able to extract more ressources to redistribute.

3.2.4 ABC Formulas

In progress...

4 Data

In order to perform numerical simulations and conduct our inverse optimum exercise, we must calibrate our model to data. We restrict attention to the United States, and gather data on the demographic characteristics, labor market activities, and family related time-use and expenses for a nationally representative sample of U.S. families. Our data on family income, age of parents, tax liability after credits, family labor hours, and the number and age of children within the household comes from the Consumer Population Survey's (CPS) March 2006 Supplement. The 2006 American Time Use Survey (ATUS) and Consumer Expenditure Survey (CE) are used to obtain information on hours and income devoted to childcare, respectively. These datasets are merged along a standard vector of characteristics, including age, income, number of children, and education level. Our final sample consists of 15,946 households.[7]

Distribution of number of children

Figure 1 shows the fraction of married couples with different number of children below the age of 15, with 5 children capturing all married couples with five or more children in the household. Married couples are first divided by their socio-economic status, with one group being households with family income below or at the 2005 poverty line, adjusted for the number of persons in the household, the second group of married couples having family income between the poverty line and 250 percent of the poverty line, and the third group comprised all married couples who are in the top 5 percent of the income distribution of married couples whose head of household is below 45 years of age. Looking at Figure 1, it is possible to see that the distribution of families across number of children is more even for household below the poverty line than for households in the top 5 percent, which is skewed towards a lower numbers of kids. This would make sense as richer household may face a greater opportunity costs of child-rearing due to their high wages. This pattern appears to be robust to further divisions by age, as evidenced by Figure 2.

Labor hours and hours devoted to children

Figure 3 demonstrates the impact of children on the labor hours supplied by the household. The average weekly labor hours of households at a given number of children decreases with

[7] Given our interest on endogenous fertility decisions, we restrict our sample to include households whose head is between age 20 and 45. We also drop any observations where labor hours are reported as 0, but where positive labor earnings is reported.

the number of children. This relationship appears to be almost linear and indicates that as the number of children increases, labor hours appear to be less attractive. Figure 4 shows the relationship between the age of the youngest child in the household with labor hours of the family. The figure indicates that younger children most likely require more time to tend to their need and wellbeing, making labor hours more costly.

Taxation of the family

To calibrate the model, we require an approximation of the US tax system that incorporates the impact of children on the tax burden of married households. To do this, we use the information in the CPS which simulates the tax burden of household surveyed. We limit the observations to households for which labor income represents 95 to 105 percent of the total family income. From there, we follow Heathcote et al. (forthcoming) and use the following tax function

$$T(y,n) = y - \lambda(n) y^{1-\tau(n)},$$

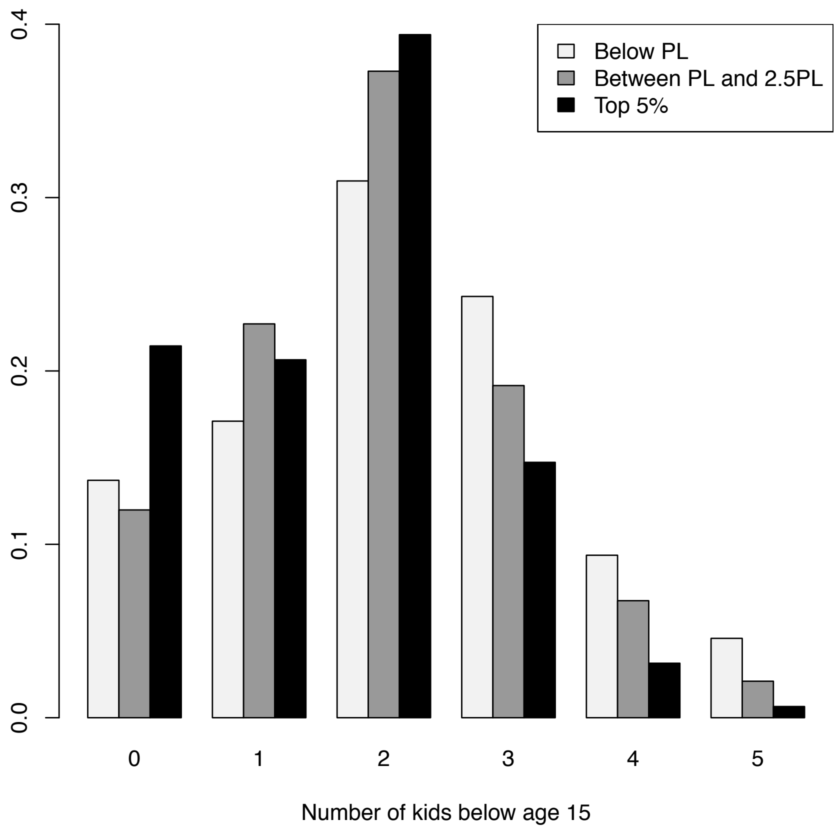
where parameters $\lambda(n)$ and $\tau(n)$ are allowed to vary with the number of children. Briefly, $\tau(n)$ measures the degree of progressivity of the tax system, whereas $\lambda(n)$ partly determines for which income levels the tax liability is a transfer. More precisely, households with income $y \in \left(0, \lambda(n)^{\frac{1}{\tau(n)}}\right)$ receive a transfer from the government.

Looking at Figure 5, we can see that US tax system is not very generous towards married couples without children as they start having a positive tax burden at roughly \$17,000 in 2006. This changes quickly as the number of children increases. For a married couple with one child, the tax burden becomes positive around \$30,000. For a married couple with two children, the tax burden becomes positive at around \$35,000. This pattern continues as the number of children increases. Figure 6 shows the approximation of the tax function based on the estimated parameters for each number of children. The approximation captures the pattern of the taxation for married couples with no children, but appears to be slightly more generous for married couples with children.

5 Inverse Optimum

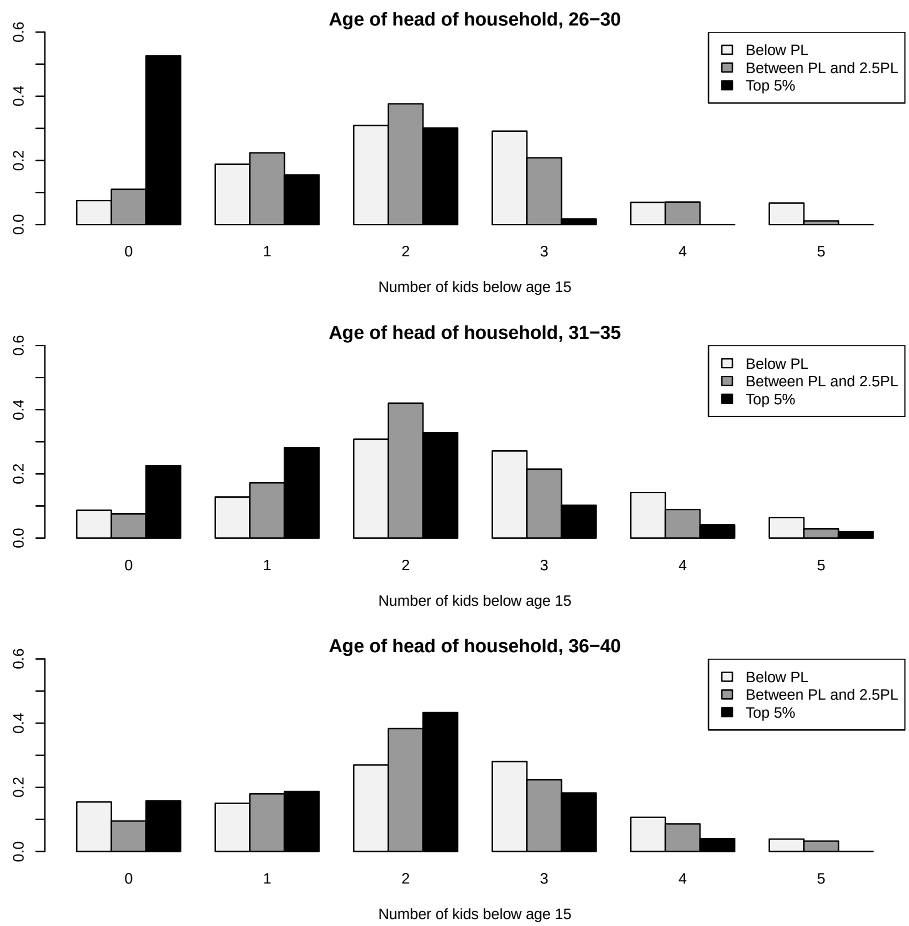
In progress...

Figure 1: Percentage of families at different income threshold with a given number of children



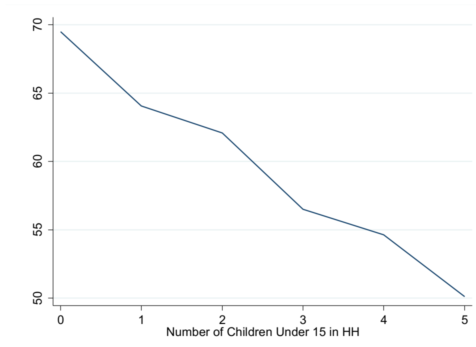
Source: Current Population Survey, 2006 March Supplement

Figure 2: Percentage of families at different income threshold with a given number of children, by age groups



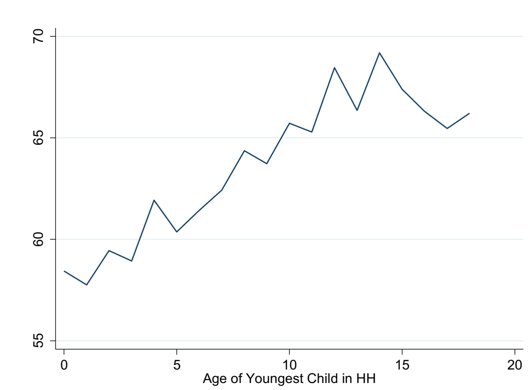
Source: Current Population Survey, 2006 March Supplement

Figure 3: Average Family Labor Hours by Number of Children



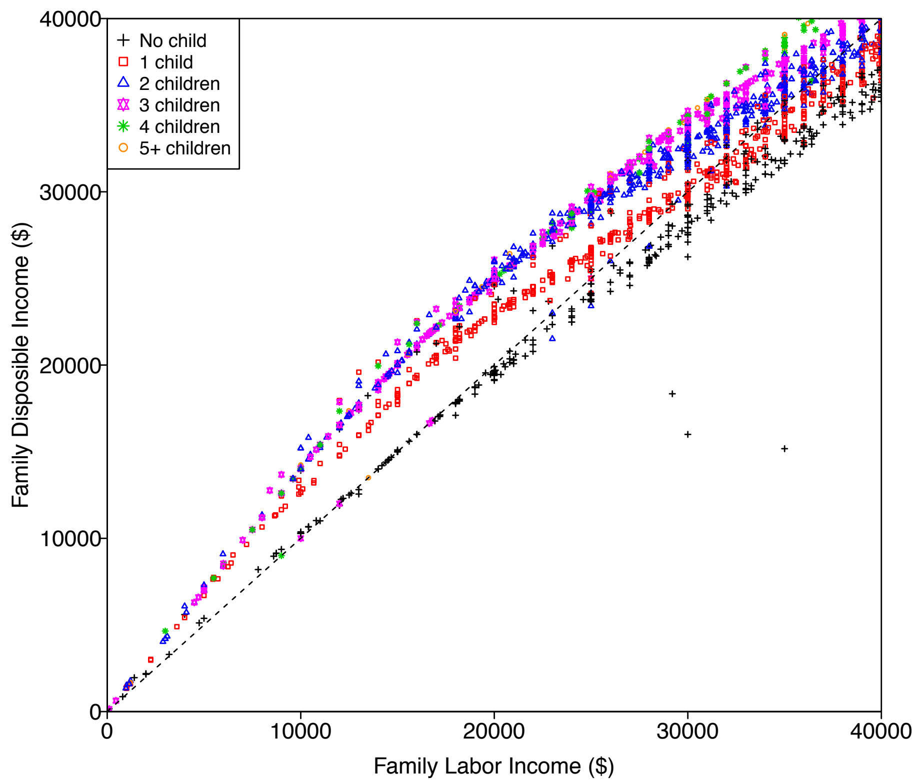
Source: Current Population Survey, 2006 March Supplement

Figure 4: Average Family Labor Hours by Age of Youngest Child



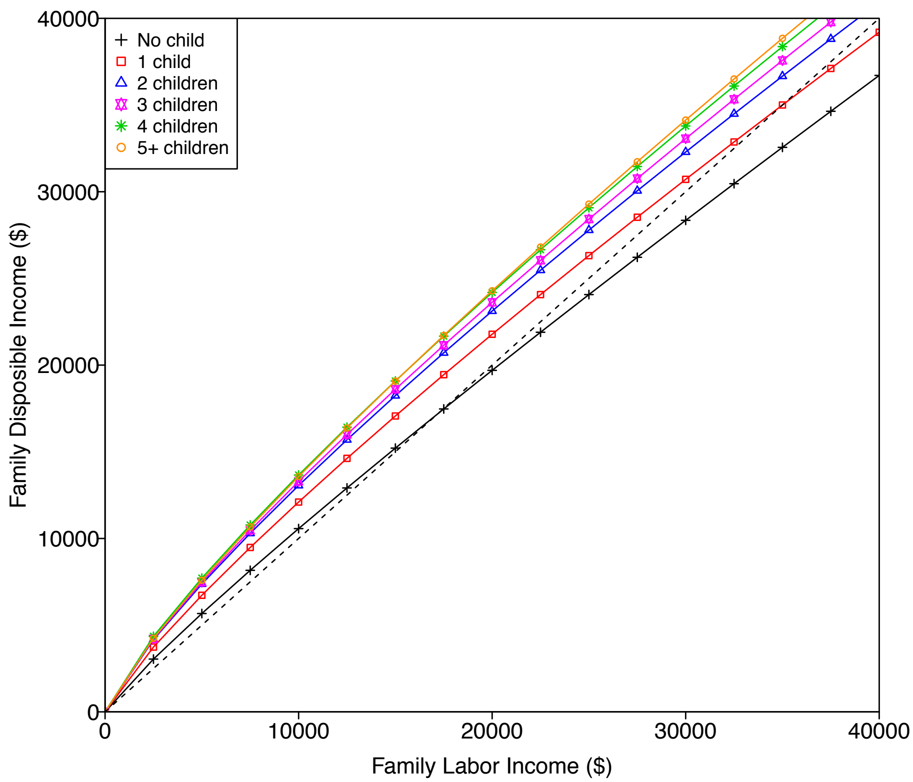
Source: Current Population Survey, 2006 March Supplement

Figure 5: Disposable Income of Married Households on Household Labor Income (Head of household age 45 and under)



Source: Current Population Survey, 2006 March Supplement

Figure 6: Tax Function Approximation: Disposable Income of Married Households on Household Labor Income



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Appendix

A Solving Hamiltonian

$$
\begin{aligned}
&\mathcal{H}(\boldsymbol{y},\boldsymbol{k},\boldsymbol{e},\boldsymbol{U},w,\boldsymbol{\mu},\boldsymbol{\nu}) = \\
&\sum_{n=0}^{N}\left\{\omega(w,n)E\left[U(w;n)+\delta_n|n\right]+n\gamma\left(Q(k(w;n),e(w;n),n)\right)\right\}P(\boldsymbol{U(w)};n,w)f(w) \\
&+\lambda\sum_{n=0}^{N}\Big\{y(w;n)-nk(w;n) \\
&\quad -u^{-1}\Big(U(w;n)-z\big(Q(k(w;n),e(w;n),n);n\big)+h\big(y(w;n)/w,ne(w;n)\big)\Big)\Big\}P(\boldsymbol{U(w)};n,w)f(w) \\
&+\sum_{n=0}^{N}\mu(w;n)\left[\frac{h'_l\left(\frac{y(w;n)}{w},ne(w;n)\right)y(w;n)}{w^2}\right] \\
&+\sum_{n=1}^{N}\nu(w;n)\left[z'_Q\big(Q\big(k(w;n),e(w;n)\big);n\big)Q'_e\big(k(w;n),e(w;n),n\big)-h'_e\left(\frac{y(w;n)}{w},ne(w;n)\right)n\right]
\end{aligned}
\tag{A.1}
$$

I argue that change to $U(w;n)$ will bring no welfare effects coming from changes in families changing the number of children. This is an enveloppe condition argument [**Note**: to prove once I have a better idea on process determining $P(U,w,n)$].

The FOC with respect to $y(w;n)$

$$
\begin{aligned}
\mathcal{H}_y = {} & \lambda\left[1-\frac{h\left(\frac{y(w;n)}{w},ne(w;n)\right)}{wu'(c(w;n))}\right]P(\boldsymbol{U(w)};n,w)f(w) \\
& +\mu(w;n)\left[\frac{h''_{ll}\left(\frac{y(w;n)}{w},ne(w;n)\right)y(w;n)}{w^3}+\frac{h'_l\left(\frac{y(w;n)}{w},ne(w;n)\right)}{w^2}\right] \\
& -\nu(w;n)\frac{h''_{el}\left(\frac{y(w;n)}{w},ne(w;n)\right)n}{w}=0.
\end{aligned}
\tag{A.2}
$$

The FOC with respect to $k(w;n)$

$$\begin{aligned}\mathcal{H}_k &= n\gamma'(Q(k,e,n))Q'_k(k(w;n),e(w;n),n)P(\boldsymbol{U(w)};n,w)f(w)\\ &+\lambda\left[-n+\frac{z'_Q(Q(k,e,n))Q'_k(k(w;n),e(w;n),n)}{u'(c(w;n))}\right]P(\boldsymbol{U(w)};n,w)f(w)\\ &+\nu(w;n)\Big[z''_{QQ}(Q(k,e,n))Q'_k(k,e,n)Q'_e(k,e,n)+z'_Q\big(Q\big(k,e\big);n\big)Q''_{ke}\big(k,e,n\big)\Big]=0.\end{aligned} \tag{A.3}$$

The FOC with respect to $e(w;n)$

$$\begin{aligned}\mathcal{H}_e &= n\gamma'(Q(k,e,n))Q'_e(k(w;n),e(w;n),n)P(\boldsymbol{U(w)};n,w)f(w)\\ &+\lambda\left[\frac{z'_Q(Q;n)Q'_e(k(w;n),e(w;n),n)-h'_e(y(w;n)/w,ne(w;n))n}{u'(c(w;n))}\right]\\ &+\mu(w;n)\left[\frac{nh''_{le}\left(\frac{y(w;n)}{w},ne(w;n)\right)y(w;n)}{w^2}\right]\\ &+\nu(w;n)\left[z''_{QQ}(Q(k,e,n))[Q'_e(k,e,n)]^2+z'_Q\big(Q\big(k,e\big);n\big)Q''_{ee}\big(k,e,n\big)-h''_{ee}\left(\frac{y(w;n)}{w},ne(w;n)\right)n^2\right]\\ &=0.\end{aligned} \tag{A.4}$$

The FOC with respect to $U(w;n)$ [I do not add the changes to social welfare from people moving from number of children, they cancel out, envelope argument]

$$\begin{aligned}\mathcal{H}_U &= \omega(w;n)P(\boldsymbol{U(w)};n,w)f(w)+\sum_{\tilde{n}=0}^{N}\tilde{n}\gamma\left(Q(k(w;\tilde{n}),e(w;\tilde{n}),\tilde{n})\right)\frac{\partial P(\boldsymbol{U(w)};\tilde{n},w)}{\partial U(w;n)}f(w)\\ &+\lambda\left[-\frac{P(\boldsymbol{U(w)},w;n)}{u'(c(w;n))}+\sum_{\tilde{n}=0}^{N}T(y(w;\tilde{n}),k(w;\tilde{n}),\tilde{n})\frac{\partial P(\boldsymbol{U(w)};\tilde{n},w)}{\partial U(w;n)}\right]f(w)\\ &=-\dot{\mu}(w;n)\end{aligned} \tag{A.5}$$

Rearranging $\mathcal{H}_y$ and using the families FOC, we obtain (omitting $(w;n)$):

$$T'_y(y,k,n)=\frac{-\mu(w;n)}{\lambda P(\boldsymbol{U(w)};n,w)f(w)}\left[\frac{h''_{ll}\left(\frac{y}{w},ne\right)y}{w^3}+\frac{h'_l\left(\frac{y}{w},ne\right)}{w^2}\right]+\frac{\nu(w;n)}{\lambda P(\boldsymbol{U(w)};n,w)f(w)}\frac{h''_{el}\left(\frac{y}{w},ne\right)n}{w}$$

Rearranging $\mathcal{H}_k$ and using the families FOC, we obtain (omitting $(w;n)$):

$$
\begin{aligned}
T'_k(y,k,n) = &-n\gamma'(Q(k,e,n))Q'_k(k(w;n),e(w;n),n) \\
&+ \frac{\nu(w;n)}{\lambda P(\boldsymbol{U(w)};n,w)f(w)} \frac{z'_Q\big(Q\big(k,e\big);n\big)Q'_k(k,e,n)Q'_e(k,e,n)}{Q\big(k,e\big)}[\chi_Q - \xi_{ke}], \quad \text{(A.6)}
\end{aligned}
$$

where

$$
\chi_Q = -\frac{z''_{QQ}Q}{z'_Q} \text{ and } \xi_{ke} = \frac{Q''_{ke}Q}{Q'_kQ'_e},
$$

who are a measure of concavity of utility function z and the hicksian complementarity measure of the production function Q. [**Note**: This last two is probably replaceable by an elasticity measure.] Clearly if $z(Q(k,e,n);n) = Q(k,e,n)$ then $T'^*_k(y,k,n) < 0$ if $\nu(w;n) > 0$ or $T'^*_k(y,k,n) > 0$ if $\nu(w;n) < 0$. If $z(Q(k,e,n);n) = \ln Q(k,e,n)$ and there are no externalities, then $T'^*_k(y,k,n) \lesseqgtr 0$ if $1 \lesseqgtr \xi_{ke}$ and $\nu(w;n) > 0$, or the opposite if $\nu < 0$ In this case, a Cobb-Douglass would lead to no marginal tax on k [I dont think n would matter].

We can use $\mathcal{H}_e$ and families FOC to obtain information on the sign of $\nu(w;n)$ (under the assumptions that e and l are not separable in the disutility of effort and/or there are positive externalities to the quality of children)

$$
\begin{aligned}
\nu(w;n) = &\frac{n\gamma'(Q(k,e,n))Q'_e(k(w;n),e(w;n),n)P(\boldsymbol{U(w)};n,w)f(w)}{\frac{z'_Q\left(Q\left(k,e\right);n\right)[Q'_e]^2}{Q}[\chi_Q - \xi_{ee}] + h''_{ee}\left(\frac{y}{w},ne\right)n^2} \\
&+ \frac{\mu(w;n)\left[\frac{nh''_{le}\left(\frac{y}{w},ne\right)y}{w^2}\right]}{\frac{z'_Q\left(Q\left(k,e\right);n\right)[Q'_e]^2}{Q}[\chi_Q - \xi_{ee}] + h''_{ee}\left(\frac{y}{w},ne\right)n^2}, \quad \text{(A.7)}
\end{aligned}
$$

where $\xi_{ee} = \frac{Q''_{ee}Q}{[Q'_e]^2} < 0$. Thus whenever $h''_{le}\left(\frac{y(w;n)}{w}, ne(w;n)\right) > 0$, and $\mu(w;n) > 0$ and no externalities $\nu(w;n) > 0$ since the denominator is always positive. Although in most optimal tax papers $\mu(w;n)$ would be negative, or $-\mu(w;n) > 0$.

Note that in this problem the transversality conditions for each family size are

$$
\lim_{w \to \underline{w}} \mu(w;n) = 0 \text{ and } \lim_{w \to \bar{w}} \mu(w;n) = 0.
$$

This implies that

$$
-\mu(w;n) = \int_w^{\bar{w}} -\mathcal{H}_U(\tilde{w})d\tilde{w}
$$

Rearranging $\mathcal{H}_U$, we obtain

$$-\mathcal{H}_U = \Bigg\{1 - \omega(w;n)\frac{u'(c(w;n))}{\lambda} - \sum_{\tilde{n}=0}^{N} \frac{\tilde{n}\gamma(Q(k,e,n);n}{\lambda}\kappa_n(w;\tilde{n})$$
$$- \sum_{\tilde{n}=0}^{N} T(y,k,\tilde{n})\kappa_n(w;\tilde{n})\Bigg\}\frac{\lambda P(\boldsymbol{U(w)}, w; n)f(w)}{u'(c(w;n))}. \tag{A.8}$$

Notice that since

$$\sum_{n=0}^{N} P(\boldsymbol{U(w)}, n; w) = 1,$$

we have

$$\sum_{\tilde{n}=0}^{N} \frac{\partial P(\boldsymbol{U(w)}, w; \tilde{n})}{\partial U(w;n)} u'(c(w;n)) = 0,$$

and thus

$$T(y(w;n), k(w;n), n)\kappa_n(w;n) = -\sum_{\tilde{n}\neq n}^{N} T(y(w;n), k(w;n), n)\kappa_n(w;\tilde{n}).$$

Therefore

$$\sum_{\tilde{n}=0}^{N} T(y,k,\tilde{n})\kappa_n(w;\tilde{n}) = \sum_{\tilde{n}\neq n}^{N} [T(y,k,\tilde{n}) - T(y,k,n)]\kappa_n(w;\tilde{n}).$$

By the same logic, we have

$$\sum_{\tilde{n}=0}^{N} \frac{\tilde{n}\gamma(Q(k,e,n);n)}{\lambda}\kappa_n(w;\tilde{n}) = \sum_{\tilde{n}\neq n}^{N} \left[\frac{\tilde{n}\gamma(Q(k,e,\tilde{n});\tilde{n})}{\lambda} - \frac{n\gamma(Q(k,e,n);n)}{\lambda}\right]\kappa_n(w;\tilde{n})$$

We can rewrite

$$-\mathcal{H}_U = \Bigg\{1 - g(w;n) - \sum_{\tilde{n}\neq n}^{N} \left[\tilde{n}g^c(w;\tilde{n}) - ng^c(w;n)\right]\kappa_n(w;\tilde{n})$$
$$- \sum_{\tilde{n}\neq n}^{N} [T(y,k,\tilde{n}) - T(y,k,n)]\kappa_n(w;\tilde{n})\Bigg\}\frac{\lambda p(n;w)}{u'(c(w;n))}. \tag{A.9}$$

Using the transversality condition, $\lim_{w\to\bar{w}} \mu(w;n) = 0$ we have [**Note**: I think this is wrong]

$$1 = \sum_{n=0}^{N} \left\{ \int_{\underline{w}}^{\bar{w}} \left(g(w;n) + \sum_{\tilde{n}\neq n}^{N} \left[\tilde{n} g^c(w;\tilde{n}) - n g^c(w;n)\right] \kappa_n(w;\tilde{n}) \right.\right.$$
$$\left.\left. + \sum_{\tilde{n}\neq n}^{N} [T(y,k,\tilde{n}) - T(y,k,n)]\kappa_n(w;\tilde{n}) \right) \frac{p(n;w)}{u'(c(w;n))} \right\} dw. \tag{A.10}$$

A.0.1 Behavioral Elasticities

We do a tax reform around $y(w:n)$ and $k(w;n)$ where

$$T(y,k,n) - \tau_y[y - y(w;n)] - \tau_k[k - k(w;n)] - \rho$$

So solve following problem:

$$\max_{y,e,k} \mathscr{U}\Big(y - nk - T(y,k,n) + \tau_y[y - y(w;n)] + \tau_k[k - k(w;n)] + \rho, \frac{y}{w}, e, k; n\Big).$$

This gives the following FOCs

$$\begin{aligned}
(M_y):\ & u'\big(y - nk - T(y,k,n) + \tau_y[y - y(w;n)] + \tau_k[k - k(w;n)] + \rho\big)[1 - T'_y(y,nk,n) + \tau_y] \\
& - \frac{h'_l(y/w, ne)}{w} = 0, \\
(M_k):\ & - u'\big(y - nk - T(y,k,n) + \tau_y[y - y(w;n)] + \tau_k[k - k(w;n)] + \rho\big)[n + T'_k(y,k,n) - \tau_k] \\
& + z'_Q(Q;n)Q'_k(k,e,n) = 0, \\
(M_e):\ & z'_Q(Q;n)Q'_e(k,e,n) - h'_e(y/w, ne)n = 0.
\end{aligned}$$

Let M_x be the vector of FOCs, i.e. $M = (M_y, M_k, M_y)$. Let M_{xx} be the square matrix of derivative of the perturbed FOCs evaluated at $y = y(w;n)$ and $k = k(w;n)$. Use this to get elasticities.