Think Globally, Cap Locally, and Trade Often: The Critical Importance of Virtual Markets for Efficient Decentralized Policymaking in the Presence of Spillovers

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When externalities and public goods impose spillovers on agents that they do not anticipate, the allocation selected by the price mechanism is unlikely to be efficient since markets for externalities and public goods are missing or incomplete. Virtual markets, such as cap and trade systems, have been proposed to restore efficiency. Prior literature suggests that when spillovers cross borders, that decentralized governments are unlikely to select policies that restore efficiency. However, this paper shows that decentralized governments through their atomistic cap or mandate choices may be able to establish an efficient system of virtual markets even when spillovers are trans-boundary. By creating an explicit link between virtual and existing markets through the general equilibrium price system, decentralized governments can restore efficiency by competing in caps or mandates, so long as they allow free trade in virtual permits and 'think globally'—that is, consider the external benefits or damages their policy choices impose on others. This result is strikingly robust and has important implications for the ability for decentralized governments to address significant externality and public good challenges, such as global climate change.

Keywords: Virtual Markets, Externalities, Public Goods, Policy Competition

I. Introduction

When markets are incomplete or missing for some commodities the allocation chosen by the price mechanism is not Pareto optimal and thus inefficient. Externalities, such as greenhouse gas (GHG) emissions, are a canonical example of such a missing market. Although fossil fuels are priced and transacted in markets, their combustion releases GHG emissions, which are not. The anthropogenic release of these emissions is a major contributor to numerous alterations to the biophysical earth system known colloquially as climate change. These alterations affect the welfare of economic agents who have no direct means by which to express their preferences for GHG emissions when a market for emissions is not available. As such, the combustion of fossil fuels imposes spillovers on other agents which they are not able to completely internalize, and the price mechanism achieves an allocation in which too much GHG emissions are released. Similarly, public goods also impose spillovers on economic agents. Even when a market for the public good exists, these markets are incomplete as the individual choices of agents who do not fully account for spillovers leads to under-provision of the public good. The presence of spillovers arising from missing and incomplete markets not only affects market efficiency, but also the ability for decentralized governments to restore efficiency through their uncoordinated policy choices. When spillovers cross government borders, it is often understood that decentralized governments are unlikely to achieve the efficient allocation (see, e.g., Wallace E. Oates (1972)).

The extent to which decentralized governments' atomistic policy choices will be inefficient depends upon the instruments available to them and whether they are willing to consider the benefits or costs on other jurisdictions from the trans-boundary spillovers their policy choices unleash—i.e., in the context of a global pollutant such as GHG emissions, whether governments 'think globally' by adopting a global estimate of the social cost of carbon when making policy decisions.[1] Government solutions to the missing or incomplete market problem have tended to focus on two alternatives. Provide the missing price; i.e., levy a Pigouvian tax in the case of a negative externality such as GHG emissions (Pigou, 1920), or subsidies in the case of a public good. Alternately, the government may directly specify the amount of the externality or public good. In

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[1]Equivalently, whether governments behave as pure altruists (Andreoni, 1990).

the context of an externality generated by many agents, a common method for doing this has been for the government to introduce a *virtual market*, such as a cap and trade system.[2] As noted by Kenneth J. Arrow (1969) such systems amount to an expansion of the commodity space to include the externality, thus bringing the externality under the control of the general equilibrium price system. A correctly set cap then allows the price system to identify the Pareto efficient allocation.[3] If spillovers are purely a domestic concern, then the central government may be able to select the Pareto efficient cap directly. However, when spillovers cross government borders, it is unclear whether the caps decentralized governments select will achieve the Pareto efficient level of emissions.

In this paper, I show that when decentralized governments use caps or mandates to address trans-boundary spillovers generated by externalities or public goods, allow free trade in permits among governments affected by the spillovers[4] (thus establishing a virtual market aligned with the spatial extent of spillovers), and also 'think globally' then it is possible for the uncoordinated cap or mandate choices of heterogeneous governments to achieve the Pareto efficient allocation. Our central result applies very broadly to a wide range of externality and public good contexts in which governments are heterogeneous in nearly every way and is robust to many extensions.

If governments instead use taxes or subsidies, use caps or mandates but do not allow free trade in permits between governments, or do not consider how the spillovers generated by their policy choices alter external benefits or costs in affected jurisdictions, then decentralized policymaking is unlikely to achieve an efficient allocation. In such instances, governments' decentralized policy choices may be distorted for four reasons. When a government's policy choice imposes trans-boundary spillovers on others but they only internalize external damages or benefits upon their own citizens, their policy choice will reflect a *damage internalization distortion*. When external spillovers arise

[2] In a cap and trade system the government specifies the cap or total amount of the externality to be generated within a specific time period, requires that agents have a property right or permit for each unit of the externality they generate over that time period, and allow agents to buy and sell permits as needed for compliance (Crocker, 1966; Dales, 1968; Baumol and Oates, 1971). A cap and trade system presupposes a negative externality. In the case of a positive externality, or, as we discuss below, a public good, the analogue is mandate and trade. Moreover, Lindahl (1919) considers virtual personalized markets in public goods.

[3] Several non-trivial assumptions are also required for such systems to be cost-effective, and, hence, efficient: zero transaction costs, full information, perfectly competitive markets, and cost minimization behavior (Hahn and Stavins, 2011). See Schmalensee and Stavins (2017) for a review of major cap and trade systems.

[4] Although many cap and trade systems only allow permit trades by agents located within the jurisdiction of the government that has proposed them, some governments do accept permits from other jurisdictions. For example, some states allow free trade in Renewable Energy Credits with other states for compliance with their state level Renewable Portfolio Standards. Offsets have also been considered in some markets for GHG emissions, although typically these have reflected trades from governments without cap and trade systems to those with them, akin to voluntary provision of a public good (Kotchen, 2009).

from a mobile factor, governments' policy choices will reflect a *spillback distortion* as they anticipate how their policy choices affect other jurisdictions' generation of spillovers (conditional on those jurisdictions' policy choices) and which in turn spillback to their own jurisdiction. When spillovers do not align with the extent to which jurisdictions internalize external damages then a *spillover-damage misalignment distortion* can result. Finally, government attempts to address spillovers may alter their net imports and exports across a whole range of commodities, and governments may use these efforts to distort trade flows to their own advantage causing a *terms of trade distortion*.

This paper contributes to three literatures. Of immediate relevance is the tax competition literature in the presence of spillovers. In a seminal paper, Wallace E. Oates and Robert M. Schwab (1988) show that when jurisdictions impose spillovers on other jurisdictions, that decentralized governments are unlikely to achieve the Pareto efficient allocation of resources. However, an important recent paper by Hikaru Ogawa and David E. Wildasin (2009) suggests this need not be the case. They show that self-interested decentralized governments that are heterogeneous across multiple dimensions may be capable of achieving the Pareto efficient level of emissions reductions even when those emissions spillover onto other jurisdictions. Although their analysis is quite general, their result rests on several important assumptions. They assume that jurisdictions have identical emissions intensities, that spillovers are uniform across all jurisdictions, that the mobile factor is inelastically supplied and completely utilized, and that end-of-pipe abatement is not possible. Some of these assumptions have been challenged. Thomas Eichner and Marco Runkel (2012) show that if the mobile factor is elastically supplied, then the canonical result of decentralized policymaking leading to an inefficient allocation of resources is restored, even when jurisdictions are exactly identical. Harrison Fell and Daniel T. Kaffine (2014) show that when an emissions generating mobile factor can be retired or when end-of-pipe abatement is available then the allocation is again inefficient.[5]

We extend Eichner and Runkel (2012) to allow for the possibility that governments may consider only damages to their own citizens or, alternately, to all citizens. Using this framework we show that the Nash equilibrium of decentralized taxes is unlikely to be efficient as a result of the four distortions discussed above. We then compare this tax equilibrium to the case when governments instead compete in caps but do not allow

[5]Unlike Oates and Schwab (1988), Ogawa and Wildasin (2009) also assume that emissions are released in proportion to the amount of a mobile factor consumed in a given jurisdiction. This is not unrealistic for many pollutants.

free trade in permits between jurisdictions. Cap competition without trading is likely to significantly reduce the spillback and spillover-damage misalignment distortions and may completely eliminate them if all governments select binding caps. We then show that the Nash equilibrium of caps when governments allow free trade in permits completely eliminates the spillback, spillover-damage misalignment, and terms of trade distortions. The first part of this statement is not surprising. All jurisdictions will select binding caps since there is always an incentive for hold-outs to select very slack caps, sell more permits than needed given the emissions they actually produce, and therefore achieve a windfall in permit revenue. Therefore, when permit trading is allowed the spillback distortion is completely eliminated as each unit of emissions reduction achieved by a jurisdiction's cap choice corresponds to an equivalent reduction in overall emissions. The spillover-damage misalignment distortion can be eliminated so long as the boundaries of the new virtual market correspond to the jurisdiction in which damages are realized. When spillovers are global this is always the case, but when spillovers are not global multiple markets may instead be needed.

That the terms of trade distortion can also be completely eliminated is a startling result. It emerges because free trade in permits introduces a new unified virtual market in which governments are now empowered to seek strategic advantage through trade in permits. In the Nash equilibrium governments select caps such that this new permit market terms of trade effect arising from the virtual market completely offsets the terms of trade effects across all other commodities for which markets already exist. Effectively the inclusion of the new virtual market eliminates arbitrage opportunities that governments may otherwise seek to exploit were free trade in permits not allowed. The cumulative sum of caps are then determined by the average of damages internalized by jurisdictions when making their cap selections. Therefore, if jurisdictions *all* voluntarily internalize the damages their policy choices impose on others—thus eliminating the damage internalization distortion—then the resulting Nash equilibrium of decentralized caps will achieve the Pareto efficient level of global emissions reductions. This result withstands the critiques raised by Eichner and Runkel (2012) and Fell and Kaffine (2014). Moreover, our result still stands even when spillovers are not uniform, the emissions intensity of the mobile factor is not identical across jurisdictions, when multiple mobile factors are present, and when end-of-pipe abatement and private adaptation are possible.

This paper also contributes to a rich literature that has looked at the strategic implications of alternative instruments to address environmental externalities on trade flows.[6] Of particular relevance is the paper by Brian R. Copeland and M. Scott Taylor (1995), who examine a Nash equilibrium of cap competition with and without permit trading between two regions who differ only in their income using a Heckscher-Ohlin trade model. Similar to Ogawa and Wildasin (2009), emissions are co-produced from an inelastically supplied factor, although Copeland and Taylor (1995) restrict their analysis to a global pollutant under certain functional form assumptions. Moreover, they assume governments do not anticipate how their policy choices affect emissions from other jurisdictions. Thus cap choices do not reflect spillback and spillover-damage misalignment distortions and cap competition is equivalent to tax competition. Given the strong assumptions of their model, when all outputs, permits, and the input can be freely traded, large governments' cap choices only reflect a permit market terms of trade effect.[7] Finally, their analysis does not consider the possibility for altruistic preferences. We relax all of these assumptions here and thus identify intuitive new results regarding policy competition across different instruments and the precise circumstances in which decentralized governments can achieve the Pareto optimal allocation.

Finally, this paper also contributes to the literature that has examined the inefficiency arising from the decentralized provision of public goods (Samuelson, 1954, 1955; Bergstrom et al., 1986; Cornes and Sandler, 1996). Our central result is not just applicable for spillovers generated by externalities, which, when they generate spillovers, exhibit non-rivalry in consumption similar to public goods, but can also be applied to examine public goods directly, even when incomplete private markets for the public good are already present. While cap and trade systems can be used to address public bads, mandate and trade systems can be used for public goods. These results provide additional evidence in favor of standards for addressing incomplete markets related to public goods when agents are heterogeneous (Jacobsen et al., 2017).

Our results suggest an important benefit from virtual markets that has heretofore been underappreciated. The First Fundamental Welfare Theorem formalized Adam Smith's "invisible hand" (Smith, 1776): the individual choices of a multitude of atomistically optimizing economic agents, considering only their own private information

[6] Rauscher (1997) provides a useful synthesis of this literature in a general equilibrium setting of two countries under perfect and imperfectly competitive markets. See also Phaneuf and Requate (2016).

[7] Since they consider preferences which exhibit heterogeneous marginal utilities of income between regions and incomes differ between the two regions the permit market terms of trade effect is not zero.

and posted prices, gives rise to a vector of equilibrium prices which support the Pareto efficient allocation when markets are perfectly competitive and complete, so long as preferences are non-satiated locally. I show that when a market is missing or incomplete that the individual choices of a multitude of atomistically optimizing governments can give rise to a vector of equilibrium caps or mandates (conditional on the resulting economic equilibrium) which supports the Pareto optimal allocation of the good for which the market is missing or incomplete. This depends upon governments own private information, information on global benefits or damages generated by the externality or public good, as well as the ability of governments to calculate or intuit how equilibrium prices change from their policies choices. This phenomenal result suggests that one solution to the missing or incomplete market problem raised by externalities and public goods may be for a centralized authority to require that decentralized governments compete in caps or mandates with free trade in permits and that decentralized governments 'think globally.' Decentralized governments can then compete with each other to establish a new virtual market that is linked through the general equilibrium price mechanism to existing markets which in conjunction will support the Pareto efficient allocation.

The rest of this paper is organized as follows. Section II introduces the analytic model. Section III examines the efficiency of three alternative forms of decentralized policy competition: competition in taxes which has received the bulk of attention in the prior literature, competition in caps without permit trading, and competition in caps with permit trading. Section IV reviews several extensions and Section V concludes.

II. Analytic Model

A. The Economy Conditional on Decentralized Policies

Following Eichner and Runkel (2012), consider a model of $j = 1, ..., J$ jurisdictions, where $J \geq 2$. Jurisdictions can reflect multiple state or provincial governments in which case the model captures the economic activity of a nation that is assumed to be closed with respect to the rest of the global economic system. Alternately, jurisdictions can reflect nations in which case it accounts for all economic activity globally. Each jurisdiction has an endowment of a mobile factor, $\bar{F}_j$, and an immobile factor, $\bar{L}_j$, such as labor. Each jurisdiction produces a private numeraire good, X_j, whose price is normalized to one.

We assume that the mobile factor used to produce the private good, f_j, generates emissions at a rate of $\alpha_j > 0$. Since the consumption of the mobile factor releases emissions, it is most straightforward to think of the mobile factor as fossil fuels, although Ogawa and Wildasin (2009), Eichner and Runkel (2012), and Fell and Kaffine (2014) refer to the mobile factor as capital. As shown in the Appendix, the model can be extended to consider multiple mobile factors, such as physical, human, or financial capital which does not release emissions, and fossil fuel 'capital', which does. The central results of the paper are unaffected by this extension, although some of the economic mechanisms discussed below become more difficult to sign due to the emergence of cross-price effects. As these cross-price effects are likely small in many contexts, our preference is to refer to the mobile factor as fossil fuels.[8] α_j is allowed to vary across jurisdictions reflecting innate differences in the technical capacity to convert fossil fuels into energy across jurisdictions, differences in pre-existing abatement technology across jurisdictions, or differences between jurisdictions in the extant thermal efficiency of the existing capital stock (i.e., power plants).

Initially, I assume a government located in each jurisdiction can only implement policies to restrict the source emissions generated in their own jurisdiction, $\alpha_j f_j$. Yet the emissions generated by producers within a jurisdiction need not equal the emissions consumed by the representative consumer in a jurisdiction as emissions may spillover from other jurisdictions. Therefore, we specify the destination emissions experienced by a representative consumer located in each jurisdiction as:

$$e_j = \alpha_j f_j + \beta \sum_{t \neq j}^{J} \alpha_t f_t = (1 - \beta) \alpha_j f_j + \beta \sum_{t=1}^{J} \alpha_t f_t, \tag{1}$$

where β reflects the degree of emissions spillovers across jurisdictions. β lies between zero and one, with a value of one implying a pollutant such as greenhouse gas (GHG) emissions which mixes uniformly globally and a value of zero an exclusively local pollutant. Although I initially consider a common β and thus uniform spillovers across jurisdictions, later in Section IV I relax this assumption, and allow for non-uniform spillovers.

Consumer Demand

A representative consumer located in each district receives utility from consuming a final private good across two periods and emissions. We assume that the consumer's

[8]For instance, the first-order mechanisms that determine the spillback distortion are the elasticities of supply and demand for fossil fuels and not capital when cross-price effects are small.

preferences can be represented by the following quasi-linear utility function:[9]

$$(2) \qquad u_j = W_j\left(\bar{F}_j - F_j\right) + x_j + v_j\left(g_j, \mathbf{e}\right),$$

where x_j is the amount of the numeraire consumed in j, g_j is a local public good which only provides benefits to citizens within j, and $\mathbf{e} = \{e_t\}_{t=1}^{J}$ is a vector of jurisdictions' received emissions given (1). $W_j(\cdot)$ is the utility received from first period consumption of fossil fuels, with $W_{iF} > 0$, and $W_{iFF} < 0$.[10] In the first period the consumer receives an endowment of fossil fuels, $\bar{F}_j$, which they can save, F_j, or consume, $\bar{F}_j - F_j$.[11] In the second period, which is the period in which the rest of the model is enumerated, the consumer receives fossil fuel income from foregone consumption in period one, wF_j, where w is the pre-tax price of fossil fuels.

In addition to income from the fossil fuel endowment, the consumer also receives profits from producing the final good, π_j, as income. Thus the consumer's private budget constraint is given by: $x_j = \pi_j + wF_j - T_j$, where T_j is a lump-sum transfer (possibly negative), and x_j is second period consumption of the final composite numeraire. The consumer maximizes (2) subject to this constraint, taking the amount of local public good provided and emissions as exogenous. This provides the Walrasian demand for the private good, $x_j\left(w, T_j\right)$, and the amount of fossil fuels supplied, $F_j\left(w\right)$, where $F_{jw} = \frac{-1}{W_{jFF}} > 0$. We assume that a government was elected by a median voter with preferences identical to those of the representative consumer.

$v_j\left(g_j, \boldsymbol{e}\right)$ in (2) is similar to Eichner and Runkel (2012), although it is more general in one important respect. In particular, we allow the consumer located in j to only consider emissions realized in j, e_j, or possibly emissions realized within *all* jurisdictions, $\boldsymbol{e}$. As will become evident below, the latter is important for decentralized policymaking to

[9] As we show in the Appendix, our central result is retained even if preferences are instead defined as $u_j\left(\bar{F}_j - F_j, x_j, g_j, \boldsymbol{\gamma}_j \mathbf{e}\right)$ with $u_{jF} > 0$, $u_{jx} > 0$, $u_{jg} > 0$, and $u_{jgg} < 0$ so long as jurisdictions exhibit consistency in their assessment of marginal damages across all jurisdictions, i.e., $\frac{u_{te_t}}{u_{tx}} = \left(\frac{1}{J}\right)\left(\sum_{l=1}^{J} \frac{u_{le_t}}{u_{lx}}\right)$ for all $t = 1, ..., J$. In the case of GHG emissions, this amounts to the requirement that each jurisdiction's estimate of global marginal damages from emissions, $\sum_{t=1}^{J} \frac{u_{je_t}}{u_{jx}}$, is the same and equals the global social cost of carbon. A special case when this occurs is when $u_{te_j} = u_{e_j}$ and $u_{tx} = u_x$ for all $t = 1, ..., J$; that is, all consumers experience the same dis-utility from a unit of emissions received by j and all consumers have identical marginal utilities of income.

[10] We adopt the following convention when referring to partial derivatives, $W_{jF} = \frac{\partial W_j}{\partial F_j}$ and $W_{jFF} = \frac{\partial^2 W_j}{\partial F_j^2}$.

[11] Since emissions are enumerated on the basis of fossil fuel consumption in the second period and not supply, when $\beta = 1$ total global emissions need not be fixed in the absence of an abatement technology as is the case examined in Ogawa and Wildasin (2009). We abstract from emissions and polices affecting emissions in the first period. Future policies may affect consumption decisions in earlier periods, reflecting inter-temporal leakage; see, e.g., van der Ploeg (2016).

be able to achieve the efficient policy outcome when $\beta > 0$. We assume the following preferences for the local public good and emissions in the analysis that follows:

$$v_j \left(g_j, \boldsymbol{e}\right) = V_j \left(g_j\right) - \phi \left(\boldsymbol{\gamma}_j \mathbf{e}\right), \tag{3}$$

where $V_j(\cdot)$ is j's preferences for the local public good with $V_{jg} > 0$, $-\phi_{te} \gtreqless 0$ is the marginal external damages that accrue to jurisdiction t from emissions realized in t,[12] and $\boldsymbol{\gamma}_j = \left\{\gamma_j^t\right\}_{t=1}^J$ is a vector of distributional weights where γ_j^t reflects the extent to which jurisdiction j values damages realized in jurisdiction t. Given (3), marginal external damages can be defined as $v_{je} = -\sum_{t=1}^J \gamma_j^t \phi_{te}$.

One special case is when each jurisdiction internalizes only their own external damages. This occurs when $\gamma_j^j = 1$ and $\gamma_j^t = 0$ for all j and $t \neq j$ and thus $v_{je} = -\phi_{je}$. In the context of climate change when $\beta = 1$, this can be understood as the jurisdiction's *domestic* or *own social cost of carbon*. However, consumers may also be concerned with the damages incurred to others outside of their own borders. If $\gamma_j^t > 0$ for some t, then the consumer in j will be concerned with the external damages incurred in those other t jurisdictions. A second special case is when each jurisdiction internalizes damages across all districts using equal distributional weights. This occurs when $\gamma_j^t = 1$ for all j and t, in which case total external damages reflect the utilitarian sum of external damages across all jurisdictions or $v_{je} = -\sum_{t=1}^J \phi_{te}$.[13] In the context of climate change, these marginal damages can be understood as the *global social cost of carbon*.[14] The global

[12]Technically, these reflect the marginal disutilty realized from consumption of the externality. However, in light of our quasi-linear specification of the utility function, the marginal utility of income is one and so these can be directly interpreted as monetized damages. Moreover, we do not restrict ϕ_{e_t} to be non-negative to allow the possibility that emissions may provide gains to some jurisdictions and losses to others. For example, climate change may yield net benefits for some agricultural regions and net losses for other agricultural and non-agricultural regions.

[13]An alternative interpretation of the two special cases comes from the literature that has examined voluntary provision of public goods when consumers exhibit altruistic preferences; see, e.g., Andreoni (1990). When $\gamma_j^j = 1$ and $\gamma_j^t = 0$ for all $t \neq j$, then the government is *purely egotistic*. When $\gamma_j^t = 1$ for all t and j, then the government is *purely altruistic*. One could also consider the case when $\gamma_j^j = 1$ and $\gamma_j^t = \gamma$ for all $t \neq j$, reflecting efficiency preferences as in Charness and Rabin (2002). Following Andreoni (1990), one could also consider the *impurely altruistic* case, by allowing both $\boldsymbol{\gamma}_j \mathbf{e}$, and $\gamma_j^j e_j$ to enter $\phi(\cdot)$. Altruistic distributional weights are only supportable when government's social welfare functions reflect them. Since γ_j^t is unrestricted, it is also possible that some jurisdictions may care more about external damages to other jurisdictions, e.g., when $\gamma_j^t > \gamma_j^j$. For example, climate change is expected to have disproportionate impacts on less developed countries and some nations may be especially sensitive to those impacts rather than direct impacts to themselves. Since we do not impose any restrictions on the sign and magnitude of γ_j^t, we also permit that consumers may have inaccurate assessments regarding their own or others' external damages.

[14]Some have argued on normative grounds that the global social cost of carbon should not assume equal welfare weights, but should instead reflect equity considerations. We take no stand on this issue here, but define the global social cost of carbon as assuming equal distributional weights as this is how it is typically defined.

social cost of carbon has been used by national and sub-national governments when setting climate policy.[15] In this context, it is well established that governments will need to internalize global rather than own damages in order to have any hope of achieving the Pareto efficient allocation of GHG emissions (see, e.g., IWGSCC (2015), page 31). When this is not the case, a damage internalization distortion will emerge.

Final Good Production

A representative final good producer located in each jurisdiction produces the final good using labor and fossil fuels as inputs according to the following production function: $X_j = h_j(f_j, l_j)$. Since labor is fixed in each jurisdiction, we can impose market clearing in the labor market, $l_j = \bar{L}_j$, to obtain $X_j = h_j(f_j)$, which is analogous to specifying a decreasing returns to scale production function in fossil fuels where $h_{jf} > 0$ and $h_{jff} < 0$. Prior to policies the producer simply selects fossil fuels to maximize profits, $h_j(f_j) - wf_j$, taking all prices as exogenous. Emissions are released linearly from the amount of fossil fuels demanded as discussed above and are initially unpriced, leading to too many of them being produced in the realized economic equilibrium when $\phi_{je} > 0$ for all $j = 1, ..., J$.

Initially, we consider two policies that decentralized governments may use to regulate the source emissions generated by the producer located in their district: an emissions tax/subsidy, τ_j, and an emissions cap, $\bar{e}_j$. When a government imposes an emissions tax on their private good producer, the producer maximizes profits according to:

$$\max_{f_j \geq 0} \quad h_j(f_j) - (w + \alpha_j \tau_j) f_j. \tag{4}$$

The first-order condition to (4) is given by:

$$h_{jf} = w + \alpha_j \tau_j. \tag{5}$$

(5) provides the inverse demand for fossil fuels: $w = h_{jf} - \alpha_j \tau_j$. Conceptually, τ_j causes a parallel shift of the firm's inverse fossil fuels demand function as scaled through by the jurisdiction's emissions intensity of fossil fuels, α_j. The unconditional demand for fossil

[15] With respect to federal rule-making in the United States, until recently the Interagency Working Group on the Social Cost of Carbon (IWG) recommended that the federal government use the global estimate of the social cost of carbon when evaluating new federal regulations targeting GHG emissions (Nordhaus, 2017; IWGSCC, 2015). However, the current presidential administration has disbanded the IWG. Moreover, in it's proposed repeal of the Clean Power Plan, the U.S. Environmental Protection Agency suggests that a domestic social cost of carbon should be used rather than a global estimate (EPA, 2017).

fuels is given by $f_j(w, \tau_j)$. We also obtain the supply of the final good, $X_j(w, \tau_j)$, and profits, $\pi_j(w, \tau_j) = h_j(f_j(w, \tau_j)) - (w + \alpha_j \tau_j) f_j(w, \tau_j)$. Finally, the source emissions released by the jurisdiction are given by $\alpha_j f_j(w, \tau_j)$.

When a government imposes an emissions cap, but does not allow permit trading with other jurisdictions, the final good producer maximizes profits by solving:

$$\max_{f_j \geq 0} \quad h_j(f_j) - w f_j$$

$$\text{subject to:}$$

$$\alpha_j f_j \leq \bar{e}_j \ (\lambda_j) . \tag{6}$$

The first-order conditions to (6) are given by:

$$h_{jf} = w + \alpha_j \lambda_j, \text{ and}$$

$$(\bar{e}_j - \alpha_j f_j) \lambda_j = 0, \alpha_j f_j \leq \bar{e}_j, \lambda_j \geq 0. \tag{7}$$

(7) implies the unconditional factor demand for fossil fuels, $f_j(w, \bar{e}_j)$, the supply of the final good, $X_j(w, \bar{e}_j)$, and firm profits, $\pi_j(w, \bar{e}_j) = h_j(f_j(w, \bar{e}_j)) - w f_j(w, \bar{e}_j)$. $f_j(w, \bar{e}_j)$ is a piece-wise linear function which equals the $f_j(w)$ that solves $h_{jf} = w$ when $w < h_{jf}\left(\frac{\bar{e}_j}{\alpha_j}\right)$ (i.e. when $\lambda_j = 0$), and equals $\frac{\bar{e}_j}{\alpha_j}$ when $w \geq h_{jf}\left(\frac{\bar{e}_j}{\alpha_j}\right)$ (i.e. when $\lambda_j > 0$).[16] Conceptually, the firm's inverse demand for fossil fuels is unconstrained for values of w greater than $h_{jf}\left(\frac{\bar{e}_j}{\alpha_j}\right)$ and then completely vertical at $\frac{\bar{e}_j}{\alpha_j}$ for values of w less than $h_{jf}\left(\frac{\bar{e}_j}{\alpha_j}\right)$.

While the first-order conditions in (7) to (5) suggest an inherent equivalency between a non-negative tax and cap, the two are not equivalent when governments anticipate the policy choices of others. For a fixed, exogenous emissions target, i.e., for $\alpha_j f_j(w, \tau_j) = \bar{e}_j$, τ_j does indeed equal $\lambda_j(w, \bar{e}_j)$ when $\tau_j > 0$. This is expected since we have abstracted from uncertainty in our model.[17] Yet when policies are endogenous and governments anticipate how their policy choices affect prices in the resulting economic equilibrium, the Nash equilibrium of taxes need not equal the λ_j realized from the Nash equilibrium

[16] It follows that this function can also be written as: $f_j(w, \bar{e}_j) = f_j(w)\, i(\lambda_j(w, \bar{e}_j) = 0) + \left(\frac{\bar{e}_j}{\alpha_j}\right) i(\lambda_j(w, \bar{e}_j) > 0)$, where $i(s)$ is an indicator function that equals 1 if s is true and equals 0 otherwise; this is useful for many of the derivations that follow.

[17] When uncertainty is present, it is well known that the welfare equivalence between price and quantity instruments can break down (Weitzman, 1974).

of caps. While τ_j is fixed from the perspective of the search for w that identifies the economic equilibrium, λ_j is not. A tax causes a parallel shift down of the inverse fossil fuel demand curve whereas the cap causes the inverse demand curve to pivot to vertical at a kink point equal to $\frac{\bar{e}_j}{\alpha_j}$. The marginal cap lowers the elasticity of the total demand for fossil fuels whereas a marginal tax only shifts the total demand curve. They thus cause the equilibrium price of fossil fuels to be altered in fundamentally different ways and in turn the incentives facing governments when making their policy choices.

When a government imposes an emissions cap and also allows permits to be used for compliance, y_j, the representative producer maximizes profits by solving:

$$\max_{f_j \geq 0, y_j} \quad h_j(f_j) - w f_j - z y_j$$
$$\text{subject to:}$$
$$\alpha_j f_j - y_j \leq \bar{e}_j \ (\mu_j), \tag{8}$$

where z is the price of permits. The first order conditions to (8) are given by:

$$h_{jf} = w + \alpha_j \mu_j,$$
$$\mu_j = z,$$
$$(\bar{e}_j - \alpha_j f_j + y_j)\,\mu_j = 0, \alpha_j f_j - y_j \leq \bar{e}_j, \mu_j \geq 0. \tag{9}$$

(9) generates the unconditional demand for fossil fuels $f_j(w, z)$. We also obtain the supply of the final good, $X_j(w, z)$, the amount of permits supplied/demanded, $y_j(w, z, \bar{e}_j) = \alpha_j f_j(w, z) - \bar{e}_j$, profits, $\pi_j(w, z, \bar{e}_j) = h_j(f_j(w, z)) - (w + \alpha_j z) f_j(w, z) + z\bar{e}_j$, and source emissions, $\alpha_j f_j(w, z)$.

Similar to the tax, when permit trading is allowed a jurisdiction's choice of cap causes a parallel shift in that jurisdiction's inverse demand for fossil fuels equal to z times their emissions intensity of fossil fuels. Unlike the tax, a jurisdiction's cap choice also causes all other jurisdictions' inverse demand curves to shift in the same fashion. Thus, unlike the tax, cap competition with permit trading provides an opportunity for a single government to simultaneously lower emissions across all jurisdictions on a one-for-one basis. However, allowing permit trading comes at a loss of control of the ability to directly regulate the release of emissions in one's own jurisdiction. Eichner and Runkel (2012) showed how critical the assumption of inelastic fossil fuel supply is to the ability for tax competition to achieve the Pareto optimal allocation. The ability for cap competition with permit trading to achieve the Pareto optimum is not sensitive to this assumption, since cap competition with permit trading creates a virtual market for emissions in which the total demand of emissions is perfectly inelastic, even if fossil fuel supply itself need not be.

When permit trading is allowed each jurisdiction's choice of cap also affects the value of that jurisdiction's net permit holdings. These net permit holdings therefore reflect a type of cash transfer received by (in the case of a jurisdiction that is a net seller of permits) or paid to (in the case of a jurisdiction that is a net buyer of permits) other jurisdictions. Thus, cap competition with permit trading potentially introduces an additional distortionary terms of trade effect when governments compete in their policy choices. However, as we show below the vector of permit market terms of trade distortions that are realized in the resulting Nash equilibrium of caps when permit trading is allowed exactly cancels the corresponding vector of fossil fuel terms of trade distortions that emerge.

Characterization of the Economic Equilibrium

A *competitive equilibrium* is the vector of prices and government expenditures on local public goods, $\left(w, \{g_i\}_{i=1}^{J}\right)$, and resulting quantities that solve all consumers' utility maximization problems and all firms' profit maximization problems, conditional on governments' policies, $\boldsymbol{\theta} = \{\theta_j\}_{j=1}^{J}$ (where $\theta_j = (\tau_j, T_j)$ in the case of a tax and $\theta_j = (\bar{e}_j, T_j)$ in the case of a cap), such that all markets clear:

$$\sum_{j=1}^{J} f_j\left(w, \theta_j\right) \leq \sum_{j=1}^{J} F_j\left(w\right),$$

$$\sum_{j=1}^{J} x_j\left(w, \theta_j\right) + \sum_{j=1}^{J} g_j\left(\theta_j\right) \leq \sum_{j=1}^{J} X_j\left(w, \theta_j\right), \tag{10}$$

and each government's budget constraint is balanced. If an emissions tax is possible this occurs when: $T_j + \tau_j \alpha_j f_j\left(w, \tau_j\right) = g_j$, and otherwise when: $T_j = g_j$. In the case of caps with permit trading, we also search over z such that the permit market clears, i.e., $\sum_{j=1}^{J} y_j\left(w, z, \bar{e}_j\right) \leq 0$.

B. First Best Pigouvian Taxes

As shown in the Appendix the vector of first-best Pigouvian taxes are given by:

$$\tau_j^{PO} = (1-\beta)\,\phi_{je} + \beta \sum_{l=1}^{J} \phi_{le}, \text{ for all } j = 1, ..., J. \tag{11}$$

Put simply, each jurisdiction j's tax should equal the marginal damages to themselves, ϕ_{je}, times one minus the amount of emissions that spillback from other jurisdictions, $(1-\beta)$, plus the sum of marginal damages across all jurisdictions, $\sum_{l=1}^{J} \phi_{le}$, times the spillback parameter, β. In the case of a local pollutant ($\beta = 0$), each jurisdiction only internalizes damages to their own district, and, in the case of a global pollutant ($\beta = 1$), each jurisdiction should internalize the marginal damages accruing to all districts. In the context of GHG emissions, the Pareto optimal carbon tax for each jurisdiction equals the global estimate of the social cost of carbon.

III. Decentralized Decisionmaking

Letting θ_j be the policy chosen by jurisdiction j and $\boldsymbol{\theta}_{\sim j} = \{\theta_t\}_{t\neq j}^{J}$ the vector of all other $t \neq j$ jurisdictions' policy choices, each jurisdiction selects policies, θ_j, that solve:

$$\max_{\theta_j} \quad u_j\left(\theta_j, \boldsymbol{\theta}_{\sim j}\right), \tag{12}$$

conditional on $\boldsymbol{\theta}_{\sim j}$ and the resulting competitive equilibrium in (10). The solution to (12) is jurisdiction j's conditionally optimal policy choice, $\theta_j\left(\boldsymbol{\theta}_{\sim j}\right)$. A *decentralized policy equilibrium* is the vector of policies, $\boldsymbol{\theta} = \{\theta_j\left(\boldsymbol{\theta}_{\sim j}\right)\}_{j=1}^{J}$, that solves (12) for all $j = 1, ..., J$ conditional on the prices and allocation that solve the competitive equilibrium in (10).

We are interested in comparing the first-best vector of Pigouvian taxes in (11) to the policies and/or shadow prices from three decentralized decisionmaking equilibria: 1. the *decentralized tax equilibrium* when $\theta_j = (\tau_j, T_j)$, 2. the *decentralized cap equilibrium without permit trading* when $\theta_j = (\bar{e}_j, T_j)$, and 3. the *decentralized cap equilibrium with permit trading* when $\theta_j = (\bar{e}_j, T_j)$. We next review the conditionally optimal policies for each of these policy equilibria in turn.

Decentralized Competition in Taxes

Under decentralized tax competition each government solves (12) where $\theta_j = (\tau_j, T_j)$, conditional on all other jurisdictions' policy choices, $\boldsymbol{\theta}_{\sim j} = \{\tau_t, T_t\}_{t\neq j}^{J}$, and the appropriate competitive equilibrium. The conditionally optimal lump-sum transfer,

$T_j(\boldsymbol{\theta}_{\sim j})$ is obtained by solving $\frac{du_j}{dT_j} = 0 \Leftrightarrow V_{jg} = 1$, which is the Samuelson condition for the optimal provision of local public goods. Thus, when lump-sum transfers are available to governments the provision of local public goods is undistorted. However, when lump-sum transfers are not possible then the amount of local public goods will also be distorted, as is well-established by the tax competition literature (see, e.g., Zodrow and Mieszkowski (1986) and Oates and Schwab (1988)).

Jurisdiction j's optimal emissions tax conditional on other jurisdictions' tax choices, $\tau_j(\boldsymbol{\theta}_{\sim j})$, is obtained by solving $\frac{du_j}{d\tau_j} = 0$, which provides:[18]

$$\tau_j(\boldsymbol{\theta}_{\sim j}) = \tau_j^{E,O} + \tau_j^{E,S} + \tau_j^{E,X} + \tau_j^{T}, \tag{13}$$

where:

$$\begin{aligned}
\tau_j^{E,O} &= \left(\gamma_j^j \phi_{je} + \beta \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}\right), \\
\tau_j^{E,S} &= \left[\left(\gamma_j^j \phi_{je} + \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}\right)\left(\beta \sum_{l \neq j}^{J} \frac{d\alpha_l f_l^{\tau_j}}{d\alpha_j f_j}\right)\right], \\
\tau_j^{E,X} &= \left[(1-\beta) \sum_{t \neq j}^{J} \gamma_j^t \phi_{te} \frac{d\alpha_t f_t^{\tau_j}}{d\alpha_j f_j}\right], \\
\tau_j^{T} &= \left(f_j^{\tau_j} - F_j^{\tau_j}\right)\left(\frac{dw^{\tau_j}}{d\alpha_j f_j}\right),
\end{aligned}$$

and given: $\frac{dw^{\tau_j}}{d\alpha_j f_j} = \frac{1}{\alpha_j\left(\sum_{t=1}^{J} F_{tw}^{\tau_j} - \sum_{t \neq j}^{J} f_{tw}^{\tau_j}\right)} = \left(\frac{w^{\tau_j}}{\alpha_j f_j^{\tau_j}}\right)\left(\frac{1}{\eta_{j,\hat{F}}^{\tau_j}}\right) > 0$, $\frac{d\alpha_t f_t^{\tau_j}}{d\alpha_j f_j} = \alpha_t f_{tw}^{\tau_j}\left(\frac{dw^{\tau_j}}{d\alpha_j f_j}\right) = \left(\frac{\alpha_t f_t^{\tau_j}}{\alpha_j f_j^{\tau_j}}\right)\left(\frac{\eta_{t,f}^{\tau_j}}{\eta_{j,\hat{F}}^{\tau_j}}\right) < 0$, $\eta_{j,\hat{F}}^{\tau_j} = \left(\sum_{t=1}^{J} F_{tw}^{\tau_j} - \sum_{t \neq j}^{J} f_{tw}^{\tau_j}\right)\left(\frac{w^{\tau_j}}{f_j^{\tau_j}}\right) > 0$, $\eta_{t,f}^{\tau_j} = f_{tw}^{\tau_j}\left(\frac{w^{\tau_j}}{f_t^{\tau_j}}\right) < 0$, $F_{tw}^{\tau_j} = \left(-\frac{1}{W_{tFF}}\right) \geq 0$[19], and $f_{tw}^{\tau_j} = \left(\frac{1}{h_{tff}}\right) < 0$.

Conditional on other jurisdiction's tax choices, jurisdiction j's optimal emissions tax, $\tau_j(\boldsymbol{\theta}_{\sim j})$, is the sum of four terms: the *own emissions Pigouvian correction*, $\tau_j^{E,O}$, the *spillback emissions Pigouvian correction*, $\tau_j^{E,S}$, the *other jurisdictions' emissions*

[18] See Appendix for derivation.

[19] Superscripts denote the economic datum observed for the competitive equilibrium associated with a particular policy case; e.g., $F_{tw}^{\tau_j}$ is the partial derivative realized at the decentralized tax equilibrium and the resulting competitive equilibrium. If $F_{tw}^{\tau_j} = 0$ for all t, then fossil fuel supply is completely inelastic as in Ogawa and Wildasin (2009).

Pigouvian correction, $\tau_j^{E,X}$, and the own jurisdiction *terms of trade effect*, τ_j^T.

The own emissions Pigouvian correction, $\tau_j^{E,O} \geq 0$,[20] equals the sum of the weighted marginal external damages from j's emissions to its own district, $\gamma_j^j \phi_{je}$, plus the weighted sum of marginal external damages resulting from j's emissions that spillover from j onto other $t \neq j$ jurisdictions, $\beta \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}$, and which are only a concern to jurisdiction j when $\gamma_j^t \neq 0$ for some $t \neq j = 1, ..., J$. For the case of a global pollutant (i.e., $\beta = 1$), when $\gamma_j^t = 1$ for all $t, j = 1, ..., J$ then the own emissions Pigouvian correction will equal the first-best tax level from (11) for jurisdiction j, $\sum_{t=1}^{J} \phi_{te}$. Therefore, if all jurisdictions internalize global damages, the J vector of decentralized taxes will equal the Pareto efficient vector of taxes so long as the last three terms sum to zero for all $j = 1, ..., J$. However, when $\gamma_j^t = 0$ for all $t \neq j = 1, ..., J$, then the own emissions Pigouvian correction will only equal jurisdiction j's own marginal external damages, ϕ_{je}. Therefore, in the case of a global pollutant, own internalization of damages will generate a *damage internalization distortion*[21] equal to $\sum_{t \neq j}^{J} \phi_{te}$. The magnitude of this distortion depends upon the share of jurisdiction j's marginal damages to global marginal damages.

The spillback emissions Pigouvian correction, $\tau_j^{E,S} \leq 0$, equals the weighted sum of marginal external damages across all jurisdictions, $\gamma_j^j \phi_{je} + \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}$, times the change in emissions in all other $t \neq j$ jurisdictions that spillback in response to j's tax, $\beta \sum_{l \neq j}^{J} \frac{d\alpha_l f_l^{\tau_j}}{d\alpha_j f_j}$. Put simply, this term reflects the emissions damages that result from emissions that return or *spillback* from other jurisdictions as a result of jurisdiction j's choice of tax and conditional on other jurisdictions' tax choices. When $\tau_j^{E,S} < 0$, the government's decentralized tax choice generates a *spillback distortion*. This distortion will be smaller for those jurisdictions who consume a larger share of fossil fuels, when the own-price elasticity of total fossil fuel supply is more elastic relative to the own-price elasticity of fossil fuel demand for all jurisdictions except j, and when the emissions intensity of jurisdiction j is larger (j is dirtier) relative to other jurisdictions.

To further understand $\tau_j^{E,S}$ in light of the previous literature, suppose that all jurisdictions have identical emissions intensities of fossil fuels, i.e., $\alpha_t = \alpha$ for all

[20] The signing of terms in this section assumes $\gamma_j^t \geq 0$ and $\phi_{te} \geq 0$ for all $t, j = 1, ..., J$.

[21] 'Distortion' as used here and below reflect the deviation of $\tau_j^{E,O}$ from τ_j^{PO}, and $\tau_j^{E,S}$, $\tau_j^{E,X}$, and $\tau_j^{E,T}$, respectively, from zero, for a given j. If the sum of these distortions do not equal zero for at least one $j = 1, ..., J$ this will imply a vector of policies and resulting competitive equilibrium allocation that is not Pareto optimal. It is possible, although highly unlikely, that the sum of these distortions exactly equals zero in every jurisdiction in which case no distortion would be present and the Pareto optimal allocation would be observed.

$t = 1, ..., J$. Consistent with Ogawa and Wildasin (2009), $\tau_j^{E,S}$ now equals $\beta\phi_{je}(-1) < 0$, when fossil fuel supply is perfectly inelastic, i.e., $F_{tw} = 0$ and jurisdictions internalize own damages. Put simply, for each unit of emissions that jurisdiction j reduces through their choice of tax *exactly* β emissions spillback to jurisdiction j from all other jurisdictions. In the case of a global pollutant ($\beta = 1$), under these assumptions and the additional assumption that jurisdictions do not anticipate that their tax choices will alter the equilibrium price of fossil fuels (and, hence, $\tau_j^T = 0$), this implies that jurisdictions' conditionally optimal tax choices will equal the zero vector. Put simply, if a tax were to reduce emissions by one unit, fossil fuels would migrate to other jurisdictions and cause emissions to increase by exactly one unit across all other jurisdictions. Since a jurisdiction's tax causes 100% emissions leakage, the optimizing government will select a Pigouvian tax of zero. Yet, in spite of this, the decentralized tax equilibrium will achieve the Pareto efficient level of emissions reductions under these assumptions so long as all fossil fuel supply is exhausted (Fell and Kaffine, 2014). Of course, under these assumptions the Pareto efficient level of emissions reductions is itself zero. The Ogawa and Wildasin (2009) model assumptions are especially unrealistic in the context of GHG emissions, since they suggest that centralized and decentralized governments' policy choices will have zero effect on total emissions and all fossil fuels will be used up.

As noted by Eichner and Runkel (2012), that $\beta(-1)$ emissions spillback is an artifact of the assumption of perfectly inelastic fossil fuel supply. When jurisdictions internalize own damages and fossil fuel supply is instead elastic, $\tau_j^{E,S}$ now equals $\beta\phi_{je}\left(\frac{\sum_{t\neq j}^{J} f_{tw}}{\sum_{t=1}^{J} F_{tw} - \sum_{t\neq j}^{J} f_{tw}}\right) < 0$. Since $\left|\frac{\sum_{t\neq j}^{J} f_{tw}}{\sum_{t=1}^{J} F_{tw} - \sum_{t\neq j}^{J} f_{tw}}\right| < 1$, when fossil fuel supply is elastic, each unit of emissions reduced by j through their choice of tax will cause *less than* β emissions to spillback. In the case of a global pollutant, there may again a role for decentralized governments to reduce emissions since the emissions leaked and which spillback from their choice of tax are less than one. Taxes induce spillback since they only allow jurisdictions to shift their own demand for fossil fuels and do not constrain emissions directly.[22] As shown above, if jurisdictions internalize own damages, in the case of a non-local pollutant the taxes they choose are likely to introduce a damage internalization distortion. Internalization of global damages can lower the damage

[22] Consider the case of the jth government contemplating a marginal change in its tax upon observing a vector of pre-existing taxes across all other t jurisdictions. For those jurisdictions with relatively steep inverse demand curves, the incidence of those pre-existing taxes is borne primarily on the demand side, entailing smaller declines in f_t from a marginal change in j's tax rate, and hence smaller spillbacks from those t jurisdictions.

internalization distortion, although it will amplify the spillback distortion, which now equals $\beta\left(\sum_{t=1}^{J}\phi_{te}\right)\left(\frac{\sum_{t\neq j}^{J}f_{tw}}{\sum_{t=1}^{J}F_{tw}-\sum_{t\neq j}^{J}f_{tw}}\right)$, when the emissions intensities of fossil fuels are identical across all jurisdictions.

The other jurisdictions' emissions Pigouvian correction, $\tau_j^{E,X}\leq 0$, equals the weighted sum of marginal damages to all other jurisdictions $t\neq j$ from the marginal release of emissions in all other jurisdictions as a result of j's choice of tax and conditional on all other jurisdictions tax choices, $\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\frac{d\alpha_t f_t^{\tau_j}}{d\alpha_j f_j}$, times one minus the amount of emissions that spillback from other jurisdictions. This term equals zero in the case of a global pollutant or when jurisdiction j internalizes own damages. However, when neither is the case $\tau_j^{E,X}<0$, reflecting a *spillover-damage misalignment distortion*. This distortion arises from the mismatch between the rate by which emissions spillover onto other jurisdictions when $\beta<1$ and the extent to which jurisdictions regard damages outside of their borders. To understand this, consider the case when $\beta=0$ but $\gamma_j^t=1$ for all $t\neq j$. Jurisdictions internalize the general equilibrium damages to other jurisdictions even when their policy choices induce no spillovers on those jurisdictions, which causes them to choose policies that deviate from the first-best tax. The magnitude of this distortion is determined by the ratio of emissions in other $t\neq j$ jurisdictions to jurisdiction j's emissions and the ratio of the own-price elasticity of fossil fuel demand for other jurisdictions $t\neq j$ to the own-price elasticity of net fossil fuel supply to jurisdiction j.

The own jurisdiction *terms of trade effect*[23], $\tau_j^T \gtreqless 0$, equals the sum of the change in consumer ($f_j^{\tau_j}\left(\alpha_j\tau_j+\left(\frac{dw^{\tau_j}}{d\alpha_j f_j}\right)\right)$) and producer ($-F_j^{\tau_j}\left(\frac{dw^{\tau_j}}{d\alpha_j f_j}\right)$) surpluses in the market for fossil fuels and the tax revenue raised by the tax ($\tau_j\alpha_j f_j^{\tau_j}$). Intuitively, if j is a fossil fuel importer (i.e. $f_j^{\tau_j}>F_j^{\tau_j}$) then the government can raise more revenue then it loses in producer and consumer surpluses from imposing a positive tax at the margin and $\tau_j^T>0$. Conversely, if the jurisdiction is a fossil fuel exporter they will have a marginal incentive to introduce a negative tax (subsidy) since $\tau_j^T<0$. In either case, the *ceteris paribus* gain to j will be less than the sum of the changes in producer and consumer surpluses in all other jurisdictions, reflecting a *terms of trade distortion* arising from jurisdiction j's decision to use their emissions tax to introduce a trade barrier.[24]

[23]This effect is well-known in the tax competition literature on asymmetric jurisdictions (see Eichner and Runkel (2012) and citations therein for additional discussion), as well as the trade and environment literature (see, e.g., Markusen (1975)).

[24]This distortion is present even when $\phi_{te}=0$ for all $t=1,...J$, in which case the model identifies the Nash equilibrium vector of trade barriers which is well-known to be inefficient (Johnson, 1954).

Finally, $\tau_j^T = 0$ and thus imposes no distortion on the decentralized tax equilibrium only when jurisdictions are symmetric (Eichner and Runkel, 2012) or if jurisdictions do not anticipate the impact of their policy choices on the price of fossil fuels realized in the economic equilibrium (i.e. they assume $\frac{dw^{\tau_j}}{d\alpha_j f_j} = 0$).[25] The magnitude of this distortion depends upon the magnitude by which jurisdiction j is a fossil fuel importer/exporter, $f_j^{\tau_j} - F_j^{\tau_j}$, and the own-price elasticity of net fossil fuel supply facing jurisdiction j, $\eta_{j,\hat{F}}^{\tau_j}$, with a less elastic net supply curve facing jurisdiction j resulting in a larger distortion.

Decentralized Competition in Caps Without Permit Trading

Under decentralized cap competition without permit trading each government solves (12) where $\theta_j = (\bar{e}_j, T_j)$, conditional on all other jurisdictions' policy choices, $\boldsymbol{\theta}_{\sim j} = \{\bar{e}_t, T_t\}_{t \neq j}^J$, and the appropriate competitive equilibrium. As was the case for decentralized tax competition, the conditionally optimal lump-sum transfer is again identified by the Samuelson rule for public goods.[26] The conditionally optimal cap $\bar{e}_j(\boldsymbol{\theta}_{\sim j})$ selected by each jurisdiction one-to-one corresponds to a Lagrange multiplier on the emissions constraint in (6) that is zero if a jurisdiction selects a binding cap and positive valued otherwise. For ease of comparison with the emissions tax, we characterize the optimal cap in terms of this multiplier, $\lambda_j(\boldsymbol{\theta}_{\sim j})$ (assuming $\lambda_j(\boldsymbol{\theta}_{\sim j}) > 0$):

$$\lambda_j(\boldsymbol{\theta}_{\sim j}) = \lambda_j^{E,O} + \lambda_j^{E,S} + \lambda_j^{E,X} + \lambda_j^{T}, \tag{14}$$

[25] In many models of tax competition the 'small jurisdiction' assumption that $\frac{dw}{d\tau_j} = 0$ is imposed (see, e.g., Zodrow and Mieszkowski (1986)). Ogawa and Wildasin (2009) implicitly make a similar assumption. As Eichner and Runkel (2012) show, the decentralized taxes and central proposition of Ogawa and Wildasin (2009) remains without this assumption when capital supply is fixed and the capital market is always binding, so long as all jurisdictions are symmetric.

[26] Although not the focus of our analysis, the absence of lump-sum taxation in this case would imply that zero local public goods are provided.

where:

$$\lambda_j^{E,O} = \left(\gamma_j^j \phi_{je} + \beta \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}\right),$$

$$\lambda_j^{E,S} = \left[\left(\gamma_j^j \phi_{je} + \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}\right)\left(\beta \sum_{l \neq j}^{J} \frac{d\alpha_l f_l^{\lambda_j}}{d\alpha_j f_j}\right)\right],$$

$$\lambda_j^{E,X} = \left[(1-\beta) \sum_{t \neq j}^{J} \gamma_j^t \phi_{te} \frac{d\alpha_t f_t^{\lambda_j}}{d\alpha_j f_j}\right],$$

$$\lambda_j^T = \left(f_j^{\lambda_j} - F_j^{\lambda_j}\right)\left(\frac{dw^{\lambda_j}}{d\alpha_j f_j}\right),$$

and given: $\frac{dw^{\lambda_j}}{d\alpha_j f_j} = \frac{1}{\alpha_j\left(\sum_{t=1}^{J} F_{tw}^{\lambda_j} - \sum_{t \neq j}^{J} f_{tw}^{\lambda_j}\right)} = \left(\frac{w^{\lambda_j}}{\alpha_j f_j^{\lambda_j}}\right)\left(\frac{1}{\eta_{j,\hat{F}}^{\lambda_j}}\right) > 0$, $\frac{d\alpha_t f_t^{\lambda_j}}{d\alpha_j f_j} = \alpha_t f_{tw}^{\lambda_j}\left(\frac{dw^{\lambda_j}}{d\alpha_j f_j}\right) = \left(\frac{\alpha_t f_t^{\lambda_j}}{\alpha_j f_j^{\lambda_j}}\right)\left(\frac{\eta_{t,f}^{\lambda_j}}{\eta_{j,\hat{F}}^{\lambda_j}}\right) \leq 0$, $\eta_{j,\hat{F}}^{\lambda_j} = \left(\sum_{t=1}^{J} F_{tw}^{\lambda_j} - \sum_{t \neq j}^{J} f_{tw}^{\lambda_j}\right)\left(\frac{w^{\lambda_j}}{f_j^{\lambda_j}}\right) > 0$, and $\eta_{t,f}^{\lambda_j} = f_{tw}^{\lambda_j}\left(\frac{w^{\lambda_j}}{f_t^{\lambda_j}}\right) \leq 0$.

Conditional on other jurisdictions' cap and lump-sum transfer choices, jurisdiction j's optimal emissions cap will be selected such that the shadow price on the cap in (6), $\lambda_j(\boldsymbol{\theta}_{\sim j})$, equals the sum of four terms: the *own emissions Pigouvian correction*, $\lambda_j^{E,O}$, the *spillback emissions Pigouvian correction*, $\lambda_j^{E,S}$, the *other jurisdictions' emissions Pigouvian correction*, $\lambda_j^{E,X}$, and the own jurisdiction *terms of trade effect*, λ_j^T. Each of these terms are exactly analogous to the conditionally optimal taxes in (13) with the sign and magnitude of each term affected by many of the same factors discussed before.

The main difference between the two decentralized policy equilibria is how the cap and tax policy equilibria differentially distort the demand for fossil fuels and the implications of this on the conditionally optimal policies that jurisdictions select. In particular, the partial derivative of fossil fuel demand with respect to the price of fossil fuels now equals $f_{tw}^{\lambda_j} = \left(\frac{1}{h_{tff}}\right) i(\lambda_t = 0) \leq 0$, where $i(x)$ is an indicator function that evaluates to 1 if x is true, and 0, otherwise. For comparison, in the tax case, $f_{tw}^{\tau_j} = \left(\frac{1}{h_{tff}}\right) < 0$. On the supply side, $F_{tw}^{\lambda_j} = F_{tw}^{\tau_j} = \left(-\frac{1}{W_{tFF}}\right) \geq 0$. Given the assumptions of the model a jurisdiction is indifferent between a cap and a tax conditional on all other jurisdictions arbitrary policy choice, $\boldsymbol{\theta}_{\sim j}$ (Phaneuf and Requate, 2016). However, this is not to say that a cap and tax are strategically equivalent. When jurisdiction j selects an emissions cap they restrict the amount of emissions that can spillback from j to zero for when other jurisdictions make their own policy choices.

If all jurisdictions select binding caps, then the spillback and spillover-damage misalignment distortions can be completely eliminated. To see this, define K as the number of jurisdictions selecting binding caps. When $K = J$, then $\lambda_j^{E,S} = 0$ and $\lambda_j^{E,X} = 0$. Suppose h_{tff} were a constant for all $t = 1, ..., J$. As $K \to J$, then both $\lambda_j^{E,S} \to 0$ and $\lambda_j^{E,X} \to 0$.[27] As was the case for decentralized tax competition, a damage internalization distortion can emerge through $\lambda_j^{E,O}$ if jurisdictions internalize own damages in the case of a non-local pollutant. Finally, unlike tax competition, decentralized cap competition also eliminates the possibility for subsidization since $\lambda_j(\boldsymbol{\theta}_{\sim j})$ cannot be negative. Therefore, decentralized cap competition implicitly sets a lower bound on how far decentralized policies can deviate from τ_j^{PO} to $-\tau_j^{PO}$. Although decentralized competition in caps without permit trading can eliminate the spillback and spillover-damage misalignment distortions when all jurisdictions select binding caps, it cannot eliminate the terms of trade distortion unless jurisdictions are symmetric. Moreover, it cannot eliminate the damage internalization distortion. As we next show, when permit trading is allowed the terms of trade distortion can also be eliminated, but the latter challenge will persist.

Decentralized Competition in Caps With Permit Trading

Under decentralized cap competition with permit trading each government solves (12) where $\theta_j = (\bar{e}_j, T_j)$, conditional on all other jurisdictions' policy choices, $\boldsymbol{\theta}_{\sim j} = \{\bar{e}_t, T_t\}_{t \neq j}^J$, and the competitive equilibrium with permit market clearing. The conditionally optimal lump-sump transfer is again identified by the Samuelson rule for public goods. As before, the conditionally optimal cap $\bar{e}_j(\boldsymbol{\theta}_{\sim j})$ selected by each jurisdiction one-to-one corresponds to a Lagrange multiplier, $\mu_j(\boldsymbol{\theta}_{\sim j})$, in this case on the emissions constraint in (6):

$$\mu_j(\boldsymbol{\theta}_{\sim j}) = \mu_j^{E,O} + \mu_j^{E,S} + \mu_j^{E,X} + \mu_j^{T,F} + \mu_j^{T,Z}, \tag{15}$$

[27] Moreover as $K \to J$, $\frac{dw^{\lambda_j}}{d\alpha_j f_j} \to \frac{1}{\alpha_j \sum_{t=1}^J F_{tw}^{\lambda_j}}$. On its own this would suggest a larger terms of trade distortion as $K \to J$; however, $f_j^{\lambda_j} - F_j^{\lambda_j}$ may also adjust given the resulting competitive equilibrium and so it is not possible *a priori* to make statements about where λ_j^T tends relative to τ_j^T.

where:

$$
\begin{aligned}
\mu_j^{E,O} &= \left(\gamma_j^j \phi_{je} + \beta \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}\right)\left(\frac{d\alpha_j f_j^{\mu_j}}{d\bar{e}_j}\right), \\
\mu_j^{E,S} &= \left[\left(\gamma_j^j \phi_{je} + \sum_{t \neq j}^{J} \gamma_j^t \phi_{te}\right)\left(\beta\left(1 - \frac{d\alpha_j f_j^{\mu_j}}{d\bar{e}_j}\right)\right)\right], \\
\mu_j^{E,X} &= \left[(1-\beta)\sum_{t \neq j}^{J} \gamma_j^t \phi_{te} \frac{d\alpha_t f_t^{\mu_j}}{d\bar{e}_j}\right], \\
\mu_j^{T,F} &= \left(f_j^{\mu_j} - F_j^{\mu_j}\right)\left(\frac{dw^{\mu_j}}{d\bar{e}_j}\right), \\
\mu_j^{T,Z} &= \left(\alpha_j f_j^{\mu_j} - \bar{e}_j^{\mu_j}\right)\left(\frac{dz^{\mu_j}}{d\bar{e}_j}\right),
\end{aligned}
$$

and given: $\frac{dw^{\mu_j}}{d\bar{e}_j} = \frac{\left(\sum_{t=1}^{J} \alpha_t f_{tw}^{\mu_j}\right)}{\left(\sum_{t=1}^{J}\left(F_{tw}^{\mu_j} - f_{tw}^{\mu_j}\right)\right)\left(\sum_{t=1}^{J} \alpha_t^2 f_{tw}^{\mu_j}\right) + \left(\sum_{t=1}^{J} \alpha_t f_{tw}^{\mu_j}\right)^2} > 0$, $\frac{dz^{\mu_j}}{d\bar{e}_j} = \frac{\left(\sum_{t=1}^{J}\left(F_{tw}^{\mu_j} - f_{tw}^{\mu_j}\right)\right)}{\left(\sum_{t=1}^{J}\left(F_{tw}^{\mu_j} - f_{tw}^{\mu_j}\right)\right)\left(\sum_{t=1}^{J} \alpha_t^2 f_{tw}^{\mu_j}\right) + \left(\sum_{t=1}^{J} \alpha_t f_{tw}^{\mu_j}\right)^2} < 0$, $\frac{d\alpha_t f_t^{\mu_j}}{d\bar{e}_j} = \alpha_t f_{tw}^{\mu_j}\left[\left(\frac{dw^{\mu_j}}{d\bar{e}_j}\right) + \alpha_t\left(\frac{dz^{\mu_j}}{d\bar{e}_j}\right)\right] \gtreqless 0$,[28] $\eta_F^{\mu_j} = \left(\sum_{t=1}^{J} F_{tw}^{\mu_j}\right)\left(\frac{w^{\mu_j}}{\sum_{t=1}^{J} F_t^{\mu_j}}\right) > 0$, $\eta_f^{\mu_j} = \left(\sum_{t=1}^{J} f_{tw}^{\mu_j}\right)\left(\frac{w^{\mu_j}}{\sum_{t=1}^{J} f_t^{\mu_j}}\right) < 0$, $\eta_{t,f}^{\mu_j} = f_{tw}^{\mu_j}\left(\frac{w^{\mu_j}}{f_t^{\mu_j}}\right) < 0$, $F_{tw}^{\mu_j} = \left(-\frac{1}{W_{tFF}}\right) \geq 0$, and $f_{tw}^{\mu_j} = \left(\frac{1}{h_{tff}}\right) < 0$.

Conditional on other jurisdictions' cap and lump-sum transfer choices, jurisdiction j's optimal emissions cap will be selected such that the shadow price on that cap in (8), $\mu_j(\boldsymbol{\theta}_{\sim j})$, equals the sum of five terms: the *own emissions Pigouvian correction*, $\mu_j^{E,O}$, the *spillback emissions Pigouvian correction*, $\mu_j^{E,S}$, the *other jurisdictions' emissions Pigouvian correction*, $\mu_j^{E,X}$, the own jurisdiction *fossil fuel market terms of trade effect*, $\mu_j^{T,F}$, and the own jurisdiction *permit market terms of trade effect*, $\mu_j^{T,Z}$.

Many of these terms have similarities to (13) and (14). Unlike the tax and cap without permit trading cases, the three Pigouvian correction terms now have an indeterminate sign. However, when all jurisdictions have identical emissions intensities of fossil fuels,

[28] When $\alpha_t = \alpha$ for all $t = 1, ..., J$: $\frac{dw^{\mu_j}}{d\bar{e}_j} = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\sum_{t=1}^{J} F_{tw}^{\mu_j}}\right) = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\eta_F^{\mu_j}}\right)\left(\frac{w^{\mu_j}}{\sum_{t=1}^{J} F_t^{\mu_j}}\right)$; $\frac{dz^{\mu_j}}{d\bar{e}_j} = \left(\frac{1}{\alpha^2}\right)\left(\frac{\sum_{t=1}^{J}\left(F_{tw}^{\mu_j} - f_{tw}^{\mu_j}\right)}{\left(\sum_{t=1}^{J} F_{tw}^{\mu_j}\right)\left(\sum_{t=1}^{J} f_{tw}^{\mu_j}\right)}\right) = \left(\frac{1}{\alpha^2}\right)\left(\frac{\eta_F^{\mu_j} - \eta_f^{\mu_j}}{\eta_F^{\mu_j}\eta_f^{\mu_j}}\right)\left(\frac{w^{\mu_j}}{\sum_{t=1}^{J} F_t^{\mu_j}}\right)$; and, $\frac{d\alpha f_t^{\mu_j}}{d\bar{e}_j} = \left(\frac{f_{tw}^{\mu_j}}{\sum_{t=1}^{J} f_{tw}^{\mu_j}}\right) = \left(\frac{\eta_{j,f}^{\mu_j}}{\eta_f^{\mu_j}}\right)\left(\frac{f_t^{\mu_j}}{\sum_{t=1}^{J} f_t^{\mu_j}}\right) > 0$. It's also the case that $\frac{d\alpha f_t^{\mu_j}}{d\bar{e}_j} < 1$. Although $\frac{d\alpha_t f_t^{\mu_j}}{d\bar{e}_j}$ cannot be signed for the general case, for dirtier jurisdictions with high α_t, this term is more likely to be positive.

$\mu_j^{E,O} \geq 0$, $\mu_j^{E,S} \geq 0$, and $\mu_j^{E,X} \geq 0$. The own emissions Pigouvian correction has the same sign as in the tax and cap without permit trading cases although it is smaller in magnitude, since $0 < \frac{d\alpha_j f_j^{\mu_j}}{d\bar{e}_j} < 1$.[29] Intuitively, when permit trading is allowed the government in j selects a cap that alters $\alpha_j f_j^{\mu_j}$ to affect $\sum_{l=1}^{J} \alpha_l f_l^{\mu_j}$. To understand this, observe that the total derivative of the permit market clearing condition with respect to a change in jurisdiction j's emissions cap implies that $\sum_{t=1}^{J} \frac{d\alpha_t f_t^{\mu_j}}{d\bar{e}_j} = 1$. Put simply, when jurisdiction j reduces emissions by one unit through their choice of cap, total emissions produced across all jurisdictions falls by exactly one unit. Thus a cap with permit trading allows each jurisdiction to receive a one-for-one unit reduction in produced emissions by imposing a marginally more stringent cap, reflecting the fact that the sum of caps selected by all governments generates a perfectly inelastic demand for emissions in a new virtual market for emissions. In sharp contrast, in the case of emissions taxes, $\sum_{t=1}^{J} \frac{d\alpha_t f_t^{\tau_j}}{d\alpha_j f_j} < 1$, and in the case of caps without permit trading, $\sum_{t=1}^{J} \frac{d\alpha_t f_t^{\lambda_j}}{d\alpha_j f_j} \leq 1$, with equality only when all jurisdictions select binding caps. In the current version of the model which only considers a single permit market, competition in caps with permit trading allows each jurisdiction to affect total emissions produced, but requires that jurisdictions forfeit their capacity to directly target their own produced emissions.[30] As such, only when emissions are global (i.e., $\beta = 1$) will the former be unambiguously preferred to the latter. When emissions are not global a spillover-damage misalignment distortion will emerge, i.e., $\mu_j^{E,X} \neq 0$. However, below when I extend the model to account for non-uniform spillovers this trade-off can be eliminated by allowing separate virtual markets on destination emissions realized in each jurisdiction. This is because the introduction of a separate virtual market for each jurisdiction allows for damages to align with spillovers.

The fossil fuel market terms of trade effect, $\mu_j^{T,F} \gtreqless 0$, is similar to the cap without permit trading case when all jurisdictions select binding caps. In that case, when all jurisdictions have identical emissions intensities, the change in the equilibrium price of

[29]The sign on the the spillback Pigouvian correction and the other jurisdictions' emissions Pigouvian correction are reversed relative to the tax and cap without permit trading cases. This difference emerges because the tax and cap without permit trading case target the actual emissions produced in jurisdiction j ($\alpha_j f_j^{\tau_j}$ in the case of the tax and $\alpha_j f_j^{\lambda_j} = \bar{e}_j$ in the case of the cap without permit trading), whereas when permit trading is allowed, the jurisdiction targets a cap, $\bar{e}_j$, which need not equal the emissions directly produced in jurisdiction j in this case, $\alpha_j f_j^{\mu_j}$.

[30]This trade-off is also evident by observing the $\mu_j = z$ for all $j = 1, ..., J$. In contrast, λ_j and τ_j need not be constant across all j. Thus the first order conditions to the profit maximization problem for the final good producer in (9) imply fewer degrees of freedom for the government to affect $f_j^{\mu_j}$.

fossil fuels equals the slope of the inverse total fossil fuel supply curve (evaluated at each respective policy equilibria) normalized by the emissions intensity of fossil fuels. Thus when total fossil fuel supply is more elastic, the fossil fuel market terms of trade effect is likely to be smaller. The key difference between $\lambda_j^{T,F}$ and $\mu_j^{T,F}$ is that for the permit trading case, $f_j^{\mu_j}$ is not constrained to equal $\frac{\bar{e}_j}{\alpha_j}$ as it is in the no permit trading case. Instead it is allowed to vary with the price of permits realized in the competitive equilibrium conditional on the Nash equilibrium vector of decentralized caps with permit trading.

In addition, a second permit market terms of trade effect emerges when permit trading is allowed, $\mu_j^{T,Z} \gtreqless 0$. This term is positive when the jurisdiction is a permit seller and negative when the jurisdiction is a permit buyer. Whether a jurisdiction is a permit buyer or seller will depend upon a number of factors in the cap and resulting competitive equilibria. When the emissions intensity of fossil fuels is identical across all jurisdictions, the change in the permit price from a change in jurisdiction j's cap, $\frac{dz^{\mu_j}}{d\bar{e}_j}$, will be positive and depend upon the elasticities of total fossil fuel supply and total fossil fuel demand. When either or both are inelastic, a change in j's cap will induce a larger change in the equilibrium permit price.

For the case of a global pollutant such as GHG emissions, the other jurisdictions' emissions Pigouvian correction, $\mu_j^{E,X} = 0$ for all $j = 1, ..., J$. Moreover, $\mu_j^{E,O} + \mu_j^{E,S} = \left(\gamma_j^j \phi_{je} + \sum_{t \neq j}^J \gamma_j^t \phi_{te}\right)$. The central result of this paper can be obtained by summing (15) across all $j = 1, ..., J$. In this case, we have:

$$Jz^\mu = \sum_{j=1}^J \left(\gamma_j^j \phi_{je} + \sum_{t \neq j}^J \gamma_j^t \phi_{te}\right) + \sum_{j=1}^J \left(f_j^\mu - F_j^\mu\right)\left(\frac{dw^\mu}{d\bar{e}}\right) + \sum_{j=1}^J \left(\alpha_j f_j^\mu - \bar{e}_j^\mu\right)\left(\frac{dz^\mu}{d\bar{e}}\right) \Leftrightarrow$$

$$z^\mu = \left(\frac{1}{J}\right)\sum_{j=1}^J \left(\gamma_j^j \phi_{je} + \sum_{t \neq j}^J \gamma_j^t \phi_{te}\right) \Rightarrow$$

(16)

$$z^\mu = \sum_{j=1}^J \phi_{je} = \tau_j^{PO}, \text{ when } \gamma_j^t = 1 \text{ for all } t, j = 1, ..., J.$$

The second line emerges from imposing market clearing in both the fossil fuel and permit markets, and given that $z^\mu = \mu_j$, $\frac{dw^\mu}{d\bar{e}} = \frac{dw^{\mu_j}}{d\bar{e}_j}$, and $\frac{dz^\mu}{d\bar{e}} = \frac{dz^{\mu_j}}{d\bar{e}_j}$ for all $j = 1, ..., J$. The last line is obtained when all jurisdictions internalize global damages. The last line of (16) states that the Nash equilibrium of decentralized caps also achieves the Pareto efficient global emissions level when permit trading is allowed and all jurisdictions internalize global damages. This remarkable result is far more general than Ogawa and Wildasin (2009) as we show in Section IV.[31] Below, we show that this efficiency result is not unique to the case of global spillovers, but also applies to the case of non-uniform spillovers so long as governments internalize global damages and compete in setting caps on destination emissions across J permit markets and allow free trade in permits within each J virtual market.

If all jurisdictions do not internalize global damages then the cumulative emissions level under decentralized cap competition with permit trading within a single virtual market will diverge from the Pareto efficient emissions level, reflecting a damage internalization distortion. Unlike the tax and the cap without permit trading cases, the damage internalization distortion will be the only distortion when emissions are global and this distortion will be identical across all jurisdictions. For example, suppose instead that all jurisdictions instead internalize only their own damages. In this case, $z^{\mu} = \left(\frac{1}{J}\right)\left(\sum_{j=1}^{J} \phi_{je}\right) = \left(\frac{1}{J}\right)\tau_j^{PO}$ for all $j = 1, ..., J$. Thus as the number of jurisdictions increases, the further will the cap competition with permit trading policy equilibrium deviate from the Pareto efficient emissions level. This result is consistent with theoretical models which only consider the damage internalization distortion (Nordhaus, 2015). Thus our central result, rests on the knife's edge that all jurisdictions internalize global damages. The U.S. EPA's recent decisions to use a domestic as opposed to global estimate of the social cost of carbon for evaluating the GHG emissions reductions benefits from federal rules targeting GHG emissions illustrates the sensitivity of our central result to this critical assumption (EPA, 2017).

Although the assumption that all jurisdictions must internalize global damages is non-trivial, our central result is still remarkable as it highlights the strategic benefits of permit trading. Internalization of global damages is what corrects the damage internalization distortion. Cap competition itself, even in the absence of permit trading as shown above, can neutralize the spillback distortion. Cap competition with permit trading allows individual jurisdictions to unilaterally reduce the sum of produced emissions

[31] In addition, our result clarifies for the tax competition literature with spillovers the importance of the assumption that all jurisdictions internalize global damages, which has been a key component of the non-cooperative theoretical and empirical models examined by the climate economics literature (see, e.g., Nordhaus (2015) and Nordhaus and Yang (1996)).

across all jurisdictions. This only occurs when permit trading is not allowed when all jurisdictions select binding caps. This is unlikely to be the case, since terms of trade distortions which induce some jurisdictions to select negative taxes (subsidies) in the case of tax competition, would likely induce many of the same jurisdictions to select non-binding caps in the case of cap competition without permit trading. However, when permit trading is allowed, all jurisdictions now have incentives to select binding caps. Jurisdictions that would prefer to subsidize emissions may select a cap that exceeds their produced emissions, effectively claiming a cash transfer from other jurisdictions as reflected in their permit market terms of trade effect. Other jurisdictions will participate for the benefits from emissions reductions and/or from fuel market terms of trade gains.

Unexpectedly, when permit trading is allowed, what our result suggests is that the permit market terms of trade exactly offsets the fuel market terms of trade effect. This is because the introduction of a single virtual market allows for the general equilibrium price mechanism to auto-correct for the terms of trade distortions self-interested governments introduce through policy competition. For the case of a global pollutant and global internalization of damages, substituting the last line of (16) into (15) implies:

$$\begin{aligned}\left(f_j^\mu - F_j^\mu\right)\left(\frac{dw^\mu}{d\bar{e}}\right)(-1) &= \left(\alpha_j f_j^\mu - \bar{e}_j^\mu\right)\left(\frac{dz^\mu}{d\bar{e}}\right) \Leftrightarrow \\ \left(\frac{F_j^\mu - f_j^\mu}{\alpha_j f_j^\mu - \bar{e}_j^\mu}\right) = \left(\frac{dz^\mu}{dw}\right) &= \left(\frac{F_t^\mu - f_t^\mu}{\alpha_t f_t^\mu - \bar{e}_t^\mu}\right) \text{ for all } t, j = 1, ..., J,\end{aligned} \tag{17}$$

where $\left(\frac{dz^\mu}{dw}\right) = \left[\frac{\sum_{l=1}^J\left(F_{lw}^\mu - f_{lw}^\mu\right)}{\sum_{l=1}^J \alpha_l f_{lw}^\mu}\right] < 0$. The first line states that at the Nash equilibrium of caps when permit trading is allowed, that jurisdictions will select caps such that their fossil fuel market terms of trade effect times minus one exactly equals their permit market terms of trade effect.[32] In fact, as shown by the last line of (17), this implies that all jurisdictions select caps such that the ratio of their net supply of fossil fuels to their net supply of permits equals $\frac{dz^\mu}{dw}$. This roughly corresponds to the ratio of the own price total supply elasticity minus the own price total demand elasticity to the own price total demand elasticity.[33] Since all jurisdictions do so at the Nash equilibrium of caps with

[32]As shown in the Appendix, this statement can be generalized to any of number of traded commodities. In the general case the sum of terms of trade effects across all traded commodities (excluding permits) will be set equal to the permit market terms of trade effect.

[33]If the emissions intensity of fossil fuels is the same across all jurisdictions, then $\left(\frac{F_j^\mu - f_j^\mu}{\alpha_j f_j^\mu - \bar{e}_j^\mu}\right) =$

permit trading, this ratio is identical across all jurisdictions.

An alternate interpretation of the last line of (17) is that, conditional on some given level of global emissions, each jurisdiction selects a cap until the marginal rate of substitution between their net fossil fuel exports to their net demand for permits just equals the ratio of the change in permit prices to the change in the price of fossil fuels. Each jurisdiction does this in isolation given the same observed ratio of price changes. Effectively the price change hyper-plane 'supports' each jurisdictions' decentralized cap choices, analogous to the way in which the price ratio between any pairs of goods simultaneously supports the decentralized production and consumption decisions of all firms and consumers in the model of perfectly competitive markets. That jurisdictions' decentralized cap choices are supported by the ratio of price changes and that this simultaneously identifies the Pareto efficient level of emissions when emissions are global and jurisdictions internalize global damages, is an extraordinary result, and suggests another important benefit from cap and trade systems. Even when all jurisdictions do not internalize global damages the last line of (17) will still ensure that the sum of the term of trade effects equal zero. While the *sum of caps* or the Pareto efficient emissions level is identified by the last line of (16) and the fact that jurisdictions internalize global damages, the *distribution of caps* that eliminates the terms of trade distortions across all jurisdictions is identified by the last line of (17).[34]

As discussed above, the sum of caps is identified by the shadow price on each jurisdictions' emissions constraint which equals the average of damages internalized across all jurisdictions in the case of a global pollutant. Since caps with permit trading completely eliminate the spillback distortion, each jurisdiction can lower global emissions by exactly one unit by submitting a cap that reduces emissions by one unit. When all jurisdictions internalize global damages, each jurisdiction receives a marginal benefit equal to the global estimate of the social cost of carbon from imposing a marginally more stringent cap, and thus they are willing to select caps that at the margin will sum to the Pareto efficient emissions level. Functionally every jurisdiction is equally willing to perform this task.

$\left(\frac{1}{\alpha}\right)\left(\frac{\eta_F^\mu - \eta_f^\mu}{\eta_f^\mu}\right) < 0$ for all $j = 1, ..., J$.

[34] When jurisdictions do not all internalize global damages, permit trading still insures the elimination of the total terms of trade distortion. Unfortunately, when that is the case a damage internalization distortion again emerges. Thus it is conceivable that if permit trading were not possible, that these multiple distortions could result in a cumulative emissions level that is closer to the Pareto efficient emissions level than if permit trading is not permitted.

Concurrently, permit trading also yields an implicit vector of lump-sum transfers across jurisdictions that does not necessitate explicit *a priori* coordination among jurisdictions. Jurisdictions implicitly compete through their choice of caps for a total pie of permit revenue that equals z^{μ} times the Pareto efficient emissions level, $\sum_{t=1}^{J} \alpha_j f_j^{\mu} = \sum_{t=1}^{J} \bar{e}_t^{\mu}$. Across all jurisdictions, the last line of (17) suggests that fossil fuel exporters will also be permit sellers and thus receive a positive lump-sum transfer from other jurisdictions. Likewise, fossil fuel importers will be permit buyers and thus pay a lump-sum tax to other jurisdictions. Therefore permit trading is akin to the introduction of an additional policy instrument that can be used to compensate winners and losers in terms of their fossil fuel terms of trade effects. What is remarkable is that this can be achieved absent explicit centralized coordination by establishing a virtual market for emissions. It is well established that permit trading allows for equalization of marginal abatement costs across jurisdictions (here z^{μ}) which is a necessary condition for a global reduction in emissions to be cost-effective. What our analysis suggests is that permit trading also provides a strategic advantage when governments compete in setting caps in a non-cooperative setting.

IV. Extensions

Non-Uniform Spillovers

The prior result regarding the Pareto efficiency of decentralized policymaking when jurisdictions internalize global damages, compete in emissions caps, and allow free trade in permits, at first glance appears to only hold for the case of a global pollutant. When spillovers are not global (i.e., $\beta < 1$), (16) suggests that decentralized cap competition with permit trading may still be distortionary. This occurs, not because of the presence of global spillovers, but because we have so far only considered a single virtual market for emissions. Because of this, when a non-global pollutant generates non-uniform spillovers, a single virtual market causes marginal abatement costs across jurisdictions to be equalized when heterogeneous marginal abatement costs are necessary to achieve the Pareto optimal emissions level. Next, I show that if instead: J virtual markets on *delivered* emissions exist, jurisdictions compete in caps on delivered emissions to each jurisdiction with free trade in permits, and all jurisdictions internalize global damages, that decentralized competition in caps will again achieve the Pareto efficient allocation of emissions in each jurisdiction. This occurs because when decentralized governments compete in caps with free trade across J permit markets and internalize global damages, they establish a virtual market on the emissions delivered to each jurisdiction where the cumulative sum of caps on delivered emissions to each jurisdiction reflects a perfectly inelastic demand for emissions in each virtual market. J permit prices provide sufficient degrees of freedom to allow for the distribution of the emissions generating mobile factor to be distributed efficiently across jurisdictions. As with the single market case, this coincides with jurisdictions' selections of caps such that the sum of terms of trade effects from all virtual and real markets equal zero.[35]

In our previous analysis, we considered caps on *produced emissions* (alternately, *source emissions*), $\alpha_j f_j$. This is the volume of emissions just before departing from jurisdiction j's boundaries. It is also the volume of emissions which we continue to assume has uniform impacts within jurisdiction j's boundaries. Now, instead suppose that each jurisdiction j can impose caps on *delivered emissions* (alternately, *destination emissions*) for each t, which reflect the emissions from j that actually arrive at t, $\beta_j^t \alpha_j f_j$ given complicated physical chemical processes as reflected in $0 \leq \beta_j^t \leq 1$. Other jurisdictions again generate emissions in their home jurisdictions which spillback to j, such that the total emissions delivered to j from j and all other jurisdictions are now, $e_j = \sum_{t=1} \beta_t^j \alpha_t f_t$, where we assume $\beta_j^j = 1$ for all $j = 1, ..., J$.

Suppose each jurisdiction has a permit market which requires a permit be obtained for each unit of emissions that are delivered to that jurisdiction or produced internally by that jurisdiction. We denote the permit price for emissions delivery to t, as z_t, and the total number of permits bought/sold by j for delivery to t as y_j^t. Under cap competition each jurisdiction now selects a lump-sum transfer, T_j, and a vector of caps on the emissions they deliver to other jurisdictions, $\bar{\boldsymbol{e}}_j = \left\{\bar{e}_j^t\right\}_{t=1}^J$. As such, the representative producer now maximizes profits by solving:

$$\max_{f_j \geq 0, \left\{y_j^l\right\}_{l=1}^J} \quad h_j\left(f_j\right) - w f_j - \sum_{l=1}^J z_l y_j^l$$

subject to:

$$\beta_j^l \alpha_j f_j - y_j^l \leq \bar{e}_j^l \quad \left(\delta_j^l\right) \text{ for all } l = 1, ..., J. \tag{18}$$

[35] Our findings mirror the early theoretical work on cap and trade systems. Baumol and Oates (1971) show that in the case of a uniformly mixed pollutant, only the level and not the location of emissions matter for cost-effectiveness. Montgomery (1972), reviews the conditions under which cap and trade systems can be cost-effective for non-uniformly mixed pollutants in which both the location and level of emissions matters. The multi-jurisdictional cap and trade system considered by Montgomery (1972) focuses on destination emissions, which guides the approach here.

The first order conditions to (18) are given by:

$$h_{jf} = w + \alpha_j \sum_{l=1}^{J} \beta_j^l \delta_j^l,$$

$$\delta_j^l = z_l, \text{ for all } l = 1, ..., J, \text{ and}$$

$$\left(\bar{e}_j^l - \beta_j^l \alpha_j f_j + y_j^l\right) \delta_j^l = 0, \beta_j^l \alpha_j f_j - y_j^l \leq \bar{e}_j^l, \delta_j^l \geq 0, \text{ for all } l = 1, ..., J. \tag{19}$$

(19) yields the unconditional demand for fossil fuels, $f_j(w, \mathbf{z})$, where $\mathbf{z} = \{z_t\}_{t=1}^J$. We also obtain the supply of the final good, $X_j(w, \mathbf{z})$, the amount of permits supplied/demanded for each $l = 1, ..., J$, $y_j^l(w, \mathbf{z}, \bar{\mathbf{e}}_j) = \beta_j^l \alpha_j f_j(w, \mathbf{z}) - \bar{e}_j^l$, and profits, $\pi_j(w, \mathbf{z}, \bar{\mathbf{e}}_j) = h_j(f_j(w, \mathbf{z})) - \left(w + \sum_{l=1}^J \beta_j^l \alpha_j z_l\right) f_j(w, \mathbf{z}) + \mathbf{z}\bar{\mathbf{e}}_j$. The competitive equilibrium is similar to the case with a single virtual market, except now we search over $\mathbf{z}$ such that $t = 1, ..., J$ permit markets clear: $\sum_{l=1}^J y_l^t(w, \mathbf{z}, \bar{\mathbf{e}}_l) = 0$.

Under decentralized cap competition with free trade in permits within each of the J permit markets, the j government solves (12) where $\theta_j = (\bar{\mathbf{e}}_j, T_j)$, conditional on all other jurisdictions' policy choices, $\boldsymbol{\theta}_{\sim j} = \{\bar{\mathbf{e}}_t, T_t\}_{t \neq j}^J$, and the competitive equilibrium with market clearing across J virtual markets. The conditionally optimal lump-sump transfer is again identified by the Samuelson rule for public goods. The conditionally optimal cap on j's emissions delivered to t, $\bar{e}_j^t(\boldsymbol{\theta}_{\sim j})$ selected by each jurisdiction one-to-one corresponds to the Lagrange multiplier on the t emissions constraint in (18) which equals:

$$\delta_j^t(\boldsymbol{\theta}_{\sim j}) = \delta_j^{t,E,O} + \delta_j^{t,E,S} + \delta_j^{t,E,X} + \delta_j^{t,T,F} + \delta_j^{t,T,Z}, \tag{20}$$

where:

$$\delta_j^{t,E,O} = \left(\gamma_j^j \phi_{je}\beta_j^j + \beta \sum_{k\neq j}^{J} \gamma_{kj}\phi_{ke}\beta_j^k\right)\left(\frac{d\alpha_j f_j^{\delta_j^t}}{d\bar{e}_j^t}\right),$$

$$\delta_j^{t,E,S} = \sum_{l\neq j}^{J}\left[\left(\gamma_j^j \phi_{je}\beta_l^j + \sum_{k\neq j}^{J} \gamma_{kj}\phi_{ke}\beta_l^k\right)\left(\frac{d\alpha_l f_l^{\delta_j^t}}{d\bar{e}_j^t}\right)\right],$$

$$\delta_j^{t,E,X} = \left[\sum_{l\neq j}^{J} \gamma_{lj}\phi_{le}\left(1-\beta_l^l\right)\frac{d\alpha_l f_l^{\delta_j^t}}{d\bar{e}_j^t}\right],$$

$$\delta_j^{t,T,F} = \left(f_j^{\delta_j^t} - F_j^{\delta_j^t}\right)\left(\frac{dw^{\delta_j^t}}{d\bar{e}_j^t}\right),$$

$$\delta_j^{t,T,Z} = \sum_{l=1}^{J}\left(\beta_j^l \alpha_j f_j^{\delta_j^t} - \bar{e}_j^l\right)\left(\frac{dz_l^{\delta_j^t}}{d\bar{e}_j^t}\right),$$

and given: $\frac{d\alpha_l f_l^{\delta_j^t}}{d\bar{e}_j^t} = \alpha_l f_{lw}^{\delta_j^t}\left[\left(\frac{dw^{\delta_j^t}}{d\bar{e}_j^t}\right) + \alpha_l \sum_{k=1}^{J}\left(\beta_l^k \frac{dz_k^{\delta_j^t}}{d\bar{e}_j^t}\right)\right]$. For a specific choice of J, closed form solutions for $\frac{dw^{\delta_j^t}}{d\bar{e}_j^t}$ and $\frac{dz_l^{\delta_j^t}}{d\bar{e}_j^t}$ for all $l, t = 1, ..., J$ can be obtained as the solution to a system of linear equations as shown in the Appendix. Fortunately, it can be shown that $\frac{dw^{\delta_j^t}}{d\bar{e}_j^t} = \frac{dw^{\boldsymbol{\delta}}}{d\bar{e}^t}$ and $\frac{dz_l^{\delta_j^t}}{d\bar{e}_j^t} = \frac{dz_l^{\boldsymbol{\delta}}}{d\bar{e}^t}$ for all $j = 1, ..., J$, and this is all that is needed to proceed.

(20) provides an expression that is similar to (15), except the own jurisdiction *permit market terms of trade effect* now reflects the sum of permit market terms of trade effects across all $l = 1, ..., J$ virtual markets. Moreover, the *other jurisdictions' emissions Pigouvian correction*, $\delta_j^{t,E,X}$, always equals zero since $\beta_l^l = 1$ for all $l = 1, ..., J$. The spillover-damage misalignment distortion is no longer present because the deployment of J virtual markets ensures full alignment between spillovers and damages within each virtual market on delivered emissions.

As shown in the Appendix, total differentiation of all $k = 1, ..., J$ permit markets with respect to $\bar{e}_j^t$, implies that the sum of the *own emissions Pigouvian correction*, $\delta_j^{t,E,O}$, and the *spillback emissions Pigouvian correction*, $\delta_j^{t,E,S}$, equal $\gamma_j^t \phi_{te}$. When all jurisdictions internalize global damages this equals ϕ_{te}. Since $\delta_j^t = z_t$ for all $j = 1, ..., J$ given (19), then summing (20) across all j, implies, after imposing market clearing in the market for fossil fuels and market clearing in all permit markets, that $z_t = \phi_{te}$ for all $t = 1, ..., J$. As illustrated in (17), this coincides with $\delta_j^{t,T,F} = -\delta_j^{t,T,Z}$. Finally, as shown in the Appendix substitution of these permit prices into (19), together with the resulting competitive equilibrium identify the Pareto optimal allocation. Therefore, when all jurisdictions internalize global damages and emissions spillovers are not uniform, decentralized cap competition with permit trading across J permit markets, clearly identifies both the Pareto efficient *level and distribution* of emissions across all jurisdictions. This occurs because of: the construction of J virtual markets on delivered emissions which align with the boundaries by which damages are assessed, the fact that permits can be freely traded within each virtual market, and the assumption that all jurisdictions internalize global damages. Only a single permit market was necessary when spillovers were uniform and global, since one unit of emissions released by any jurisdiction had the same impact on global damages as a unit of emissions released by any other jurisdiction.

In addition to the assumption that jurisdictions internalize global damages, the results reviewed in this section have also relied upon the non-trivial assumption that each jurisdiction is willing to set caps on the emissions they deliver to other jurisdictions. The latter assumption implied a personalized virtual market in the emissions which locally cause damages to each jurisdiction. This has an intuitive correspondence with the suggestion, originally by Lindahl (1958), that personalized markets for public goods may also provide a means to achieve the Pareto optimal allocation.[36] When spillovers are non-uniform both personalized markets and the global internalization of damages by all jurisdictions are required for the allocation to be Pareto optimal. However, as shown in the Appendix, in the case of a global pollutant such as GHG emissions (and hence, a global public bad), the requirement that jurisdictions compete in caps across J personalized virtual markets in delivered emissions can actually serve as a substitute for the assumption that all jurisdictions internalize global damages. This is not possible in the case of a non-uniform pollutant or a public good which imposes non-uniform spillovers because the J personalized markets are necessary to identify the efficient distribution of emissions across jurisdictions.

Public Goods

Our central result hinges upon the fact that the market for emissions is missing (i.e., emissions are exogenous to consumers' utility maximization and producers' profit

[36]There has been some disagreement as to what Lindahl (1958) actually shows. See van den Nouweland (2015) for a useful historical review of the Lindahl equilibrium concept as it has evolved in the public goods literature.

maximization problems) *and* that all jurisdictions internalize global damages. If the market for emissions were not missing and all consumers (not jurisdictions) internalized global damages then they could address the inefficient provision of the externality themselves without the need for intervention by decentralized governments.

In the Appendix,[37] I amend the model to instead consider the voluntary provision of a 'global' public good which is non-rival in consumption across all jurisdictions. If consumers internalize the benefits from this global public good to all consumers, the decentralized decisions of all consumers and all producers, under the assumptions of perfectly competitive markets and the resulting competitive market equilibrium, yield the Pareto optimal allocation. The Pareto efficient public good subsidy is thus the null vector across all jurisdictions. Decentralized governments that also internalize global benefits may intervene by choosing taxes/subsidies, a la Section III, but doing so introduces effects analogous to the terms of trade and spillback distortions considered above. If governments instead did not internalize global benefits, then an additional benefit internalization distortion would emerge analogous to the damage internalization distortion considered above. If decentralized governments choose mandates on local consumption of the global public good (instead of emissions caps) but allow free trade in public good provision permits (instead of emissions permits) across all jurisdictions, then a virtual market for the global public good can be constructed in parallel to the real one. Decentralized competition in public good mandates with free trade in public good provision permits ensures Pareto efficiency so long as all jurisdictions internalize global benefits. This is of course duplicitous when consumers internalize global benefits themselves since the real market for the public good is itself all that is needed for efficiency.[38]

If instead consumers do not internalize global benefits, then the competitive equilibrium will again result in the well-established under-provision of the global public good. Decentralized competition in public good mandates with free trade in public good

[37] This model could be extended further to consider non-spatially uniform public goods analogous to what has been reviewed in the preceding section.

[38] The results of this extension thus complement the Lindahl equilibrium (Lindahl, 1958) of personalized markets for public goods. While decentralized mandate competition with free trade in public good provision allowances creates a virtual market by which all decentralized jurisdictions can potentially express their preferences for the Pareto efficient allocation, this is only possible if jurisdictions' governments' preferences are for the Pareto efficient allocation; that is, if all jurisdictions' governments internalize the global benefits of the public good. That consumers can also achieve this independent of government intervention when they can directly determine their consumption of the global public good, reflects the fact that when consumers internalize global benefits they explicitly address the consumption externality from the global public good.

provision permits will achieve the Pareto efficient allocation of the global public good so long as all jurisdictions' governments internalize global benefits even when consumers themselves do not.

The core insight of this and the previous model extension, however, is that our central theoretical results can be applied very generally to consider the allocation of many kinds of public goods and externalities. In contexts in which a central authority can adjudicate the mechanisms by which decentralized governments make decisions, it suggests that the central authority should establish the criterion by which decentralized governments should be required to make policy decisions (i.e., internalize global benefits/damages), should restrict decentralized governments to certain instruments (i.e., public good mandates or emissions caps), and should require that decentralized governments permit free trade in public good provisions or emissions allowances across all jurisdictions. In contexts, such as the Paris Agreement, in which universal agreement has not been obtained in any of these three areas, it suggests a space by which future negotiations can possibly enhance the efficiency of mitigation mechanism established under the Paris Agreement. However, given the Westphalian system of national sovereignty established after the Thirty Year's War and enshrined in the United Nations system, there is no global government which can impose such a system directly.

Multiple Traded Commodities, Private Adaptation, and End-of-Pipe Abatement

The Appendix presents a generalization of the model developed here in the presence of two mobile factors, capital and fossil fuels, both of which are elastically supplied. Our central results regarding the efficiency of decentralized cap competition under the prior set of assumptions is unaffected, although some of the total derivatives become more difficult to sign when the elasticity of substitution between fossil fuels and capital demanded in the production of the private good, and, when, fossil fuels and capital are not separable in the utility function, the elasticity of substitution between fossil fuels and capital supplied, is not zero. When these elasticities are zero across all jurisdictions then the analysis is similar to that considered above, with the exception that marginal policies in (13), (14), and (15) now include a capital market terms of trade effect. In this case, decentralized cap competition with permit trading, ensures that the permit market terms of trade effect exactly offsets the sum of the fossil fuel and capital market terms of trade effects. Without loss of generality, the model can be expanded to consider any number of mobile factors with elastic or inelastic supplies, and our central findings are unaffected.

In the Appendix, we also extend the model to allow for end-of-pipe abatement, building on the two mobile factor model reviewed in the previous paragraph. For simplicity, we assume end-of-pipe abatement is produced exclusively from capital. Unlike Fell and Kaffine (2014), our central results regarding the efficiency of decentralized cap competition under our prior assumptions are unaffected by this extension, which simply alters the capital market terms of trade effect.

Finally, the Appendix also considers an extension to the above model when the representative consumer in each jurisdiction also selects the amount of private adaptation to invest in to mitigate local damages from the emissions realized in their district. Adaptation in a district reflects investment in a private good for which markets are already present to reduce damages from a global externality, which prior to decentralized cap competition with permit trading, for which a market did not exist. This modification introduces an additional adaptation market term of trade effect, but the sum of the term of trade effects again equal zero under decentralized cap competition with permit trading. If all jurisdictions internalize global damages, then the model again achieves the Pareto optimal allocation.

V. Conclusion

Cap and trade systems to address externalities were originally conceived of as constructing an 'artificial market' through which price contestation among many permit buyers and sellers decentrally identifies the market clearing price of permits, without the need for centralized coordination except to set the cap itself and enforce property rights (Crocker, 1966; Dales, 1968; Baumol and Oates, 1971; Montgomery, 1972). Economic agents observing the price of permits as well as other prices, their own emissions, and their own production technologies or preferences select the optimal amount of abatement to produce for themselves, such that the marginal costs of abatement are equalized across all agents and the total sum of permits equals the cap determined by the central authority. The resulting distribution of abatement across economic agents will be cost-effective under certain standard assumptions, which have been maintained throughout our analysis (Hahn and Stavins, 2011). Importantly, under the same assumptions, for a given cap level, the distribution of permits across agents will have no bearing on the cost-effectiveness of the cap and trade system although certain permit allocations may be preferred for equity reasons. That the allocation of permits has no bearing on the cost-effectiveness of a particular cap is known as the independence property of cap and trade systems (Hahn and Stavins, 2011).

Our analysis identifies a second benefit from artificial markets. So long as all governments agree to allow free trade of permits when they make their uncoordinated, decentralized cap choices, then governments can observe the equilibrium permit price, the ratio of price changes, and their own fossil fuel and permit market trade flows in order to select their caps given the last lines of (16) and (17). When spillovers are global and jurisdictions internalize global damages, then decentralized governments will select a vector of caps that achieves the Pareto efficient level of emissions reductions. The resulting cumulative cap on emissions will be realized without centralized coordination, and the individual caps selected by each jurisdiction will reflect the allocation of permits assigned to each jurisdiction. Thus, from the vantage point of the decentralized cap equilibrium, the allocation of permits across jurisdictions *does* affect the efficiency of the cumulative sum of caps, since allocations that do not satisfy (17) can introduce terms of trade distortions and thus possibly encourage some jurisdictions to select caps such that the cumulative sum of caps no longer equals the Pareto efficient emissions level. The fact that these terms of trade distortions can be eliminated from the introduction of virtual markets is important as it suggests that decentralized governments through policy competition linked to the general equilibrium price system can make markets both complete and efficient.

Although environmental economists pioneered the development of cap and trade systems for the regulation of pollution, our analysis suggests that virtual markets may provide strategic benefits in many other public good and externality contexts in which decentralization posits significant challenges to efficient provision. Our findings suggests that virtual markets present a remarkable mechanism by which decentralized governments through their own atomistic choices and without coordination can address the missing and incomplete market problem, even when the distortions caused by those market failures impose trans-boundary spillovers across governments' borders. In particular, it suggests the minimum criteria that a centralized government would need to impose in order for decentralized governments to address a wide range of public good and externality problems.

Although my central result is very robust, in the context of global efforts to reduce GHG emissions, it is self-evident that the supposition that decentralized governments can achieve the Pareto efficient allocation rests on a knife's edge. The fundamental incentives to free-ride are still pervasive, and, under the Westphalian system of national sovereignty, individual nation-states are limited in their capacity to coerce other nation-states to contribute to mitigation. Under the Paris Agreement, nation-states currently are not restricted in their choice of policy instrument. When policy instrument choice is itself endogenous it is highly likely that some governments may prefer not using caps to reduce emissions at all or to restrict the free trade of emissions permits. Even if all governments could commit to mitigate using caps and allow free trade in emissions permits, purely self-interested governments will likely benefit from internalizing only their own damages when making their cap choices.[39] Domestic distributional concerns may constrain the ability for governments to commit to comprehensive free trade in emissions permits. Finally, a mechanism for monitoring, reporting, and verifying emissions reductions would itself be difficult to establish in the absence of a global government.

My analysis does suggest that previous failed attempts to reach an *ex ante* agreement on a global cap on GHG emissions may not be the only path forward for addressing anthropogenic climate change. The Paris Agreement provides a novel, largely voluntary approach which, if modified along the lines suggested above (restricting governments to propose emissions caps and to accept free trade in emissions permits, and internalize a global estimate of the social cost of carbon), could achieve the Pareto efficient level of global emissions without being directly negotiated. As with most challenges that transcend national borders, the penultimate ability of the human race to resolve such challenges as climate change depends in the final analysis on the capacity of individuals to contemplate the implications of their choices on others outside their own nation's borders. However, my model abstracts from the historical release of GHG emissions, a stock pollutant, by nation-states. Therefore, the resulting Nash equilibrium of caps identified by the model is agnostic with respect to equity considerations which have proven important obstacles in past negotiations to address this important global challenge.

[39]Importantly, if governments were able to agree to restrict themselves to caps and allow the free trade of permits, but were unable to achieve agreement regarding using a global social cost of carbon, the resulting Nash equilibrium may yield a smaller cumulative reduction in global emissions that than what could be achieved using other instruments or by restricting free trade in emissions permits. The extent that this is likely is the focus of on-going empirical work by the author.

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Appendix

Think Globally, Cap Locally, and Trade Often: The Critical Importance of Virtual Markets for Efficient Decentralized Policymaking in the Presence of Spillovers

Joel R. Landry

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Section I provides the intermediate steps in the derivation of the analytic expressions reported in the paper. Section II provides additional details on model extensions.

I. Analytical Derivations

A. *Decentralized Competition in Taxes*

Under tax competition each j government solves, after imposing the j government's balanced budget constraint:

$$\max_{g_j \geq 0, \tau_j} W_j\left(\bar{F}_j - F_j\right) + I_j + \tau_j \alpha_j f_j - g_j + V_j\left(g_j\right) - \phi\left(\boldsymbol{\gamma}_j \mathbf{e}\right), \tag{A.1}$$

where $I_j = \pi_j + wF_j$ is non-tax income.

The first order conditions to (A.1) are given by:

$$\begin{aligned} V_{jg} &= 1, \text{ and} \\ \alpha_j \tau_j &= \sum_{t=1}^{J} \gamma_j^t \phi_{te}\left(\frac{de_t}{df_j}\right) + \left(f_j - F_j\right)\left(\frac{dw}{df_j}\right), \end{aligned} \tag{A.2}$$

given:

$$\begin{aligned} \frac{dI_j}{df_j} &= \frac{d\pi_j}{df_j} + w\frac{dF_j}{df_j} + F_j\frac{dw}{df_j}, \\ \frac{d\pi_j}{df_j} &= -f_j\left(\frac{dw}{df_j} + \alpha_j\frac{d\tau_j}{df_j}\right), \end{aligned} \tag{A.3}$$

(5), and the first-order conditions to the utility maximization problem for representative consumer j (in particular, $w - W_{jF} = 0$).

Total differentiation of final good supply and fossil fuel demanded for the production of the final good for $t \neq j$:

$$\begin{aligned} \frac{dX_t}{df_j} &= X_{tw}\frac{dw}{df_j}, \text{ and} \\ \frac{df_t}{df_j} &= f_{tw}\frac{dw}{df_j}, \end{aligned} \tag{A.4}$$

where,

$$
\begin{aligned}
X_{tw} &= \left(\frac{h_{tf}}{h_{tff}}\right), \text{ and} \\
f_{tw} &= \left(\frac{1}{h_{tff}}\right). \qquad \text{(A.5)}
\end{aligned}
$$

Note that $X_{tw} \leq 0$ and $f_{tw} \leq 0$.

Total differentiation of final good supply for the production of the final good for j:

$$
\frac{dX_j}{df_j} = X_{jw} = h_{jf}. \qquad \text{(A.6)}
$$

Total differentiation of fossil fuel supply provides:

$$
\frac{dF_t}{df_j} = F_{tw}\frac{dw}{df_j}, \qquad \text{(A.7)}
$$

where:

$$
F_{tw} = \left(-\frac{1}{W_{tFF}}\right). \qquad \text{(A.8)}
$$

Note that $F_{tw} \geq 0$.

Total differentiation of (10) provides:

$$
\begin{aligned}
\sum_{t\neq j}^{J} \frac{df_t}{df_j} + 1 &= \sum_{t=1}^{J} \frac{dF_t}{df_j}, \text{ and} \\
\sum_{t=1}^{J} \frac{dx_t}{df_j} &= \sum_{t=1}^{J} \frac{dX_t}{df_j}. \qquad \text{(A.9)}
\end{aligned}
$$

Substituting fossil fuel demanded and supplied from (A.4) and (A.7) into the first line of (A.9) provides:

$$
\begin{aligned}
&\sum_{t\neq j}^{J} f_{tw}\frac{dw}{df_j} + 1 = \sum_{t=1}^{J} F_{tw}\frac{dw}{df_j} \Leftrightarrow \\
&\frac{dw}{df_j} = \bar{w}_j, \qquad \text{(A.10)}
\end{aligned}
$$

where:

$$
\bar{w}_j = \frac{1}{\sum_{t=1}^{J} F_{tw} - \sum_{t\neq j}^{J} f_{tw}}. \qquad \text{(A.11)}
$$

Note that $\bar{w}_j \geq 0$.

Substituting $\bar{w}_j$ into (A.4) provides:

$$\frac{df_t}{df_j} = f_{tw}\bar{w}_j = \bar{f}_t \tag{A.12}$$

Total differentiation of (1) provides for $t \neq j$:

$$\frac{de_t}{df_j} = \alpha_t \left(1-\beta\right) \frac{df_t}{df_j} + \beta \sum_{l\neq j}^{J} \alpha_l \frac{df_l}{df_j} + \beta\alpha_j, \tag{A.13}$$

and, for j:

$$\frac{de_j}{df_j} = \alpha_j + \beta \sum_{l\neq j}^{J} \alpha_l \frac{df_l}{df_j}. \tag{A.14}$$

Substituting (A.12) into (A.13) and (A.14) provides:

$$\frac{de_t}{df_j} = \alpha_t \left(1-\beta\right) \bar{f}_t + \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l + \beta\alpha_j,$$

$$\frac{de_j}{df_j} = \alpha_j + \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l. \tag{A.15}$$

Upon observing that $\sum_{t=1}^{J} \gamma_j^t \phi_{te} \left(\frac{de_t}{df_j}\right) = \gamma_j^j \phi_{je} \frac{de_j}{df_j} + \sum_{t\neq j}^{J} \gamma_j^t \phi_{te} \frac{de_t}{df_j} = \left[\alpha_j \left(\gamma_j^j \phi_{je} + \beta \sum_{t\neq j}^{J} \gamma_j^t \phi_{te}\right)\right] + \left[\gamma_j^j \phi_{je} \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l\right] + \left[\sum_{t\neq j}^{J} \gamma_j^t \phi_{te} \left(\alpha_t \left(1-\beta\right) \bar{f}_t + \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l\right)\right]$, it's evident that the second line of (A.2) can be re-written as:

$$\alpha_j \tau_j \left(\boldsymbol{\theta}_{\sim j}\right) = \alpha_j \tau_j^{E,O} + \alpha_j \tau_j^{E,S} + \alpha_j \tau_j^{E,X} + \alpha_j \tau_j^{T},$$

where:

$$\alpha_j \tau_j^{E,O} = \alpha_j \left(\gamma_j^j \phi_{je} + \beta \sum_{t\neq j}^{J} \gamma_j^t \phi_{te}\right),$$

$$\alpha_j \tau_j^{E,S} = \gamma_j^j \phi_{je} \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l,$$

$$\alpha_j \tau_j^{E,X} = \sum_{t\neq j}^{J} \gamma_j^t \phi_{te} \left(\alpha_t \left(1-\beta\right) \bar{f}_t + \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l\right), \text{ and}$$

$$\alpha_j \tau_j^{T} = \left(f_j - F_j\right) \bar{w}_j. \tag{A.16}$$

Consider the case of symmetric jurisdictions when $\phi_e > 0$. In that case (A.16) implies:

$$\alpha\tau = \alpha\phi_e\left[\gamma_j + \gamma_t\beta\left(J-1\right)\right] + \alpha\phi_e\left[\gamma_t\left(1+\beta\left(J-1\right)\right) + \beta\left(\gamma_j-\gamma_t\right)\right]\left(\delta-1\right), \tag{A.17}$$

where $(\delta - 1) = (J-1)\frac{df_t}{df_j} \Leftrightarrow \delta = (J-1)\frac{df_t}{df_j} + 1$. Therefore, δ equals the total change in fossil fuels from a one unit change in f_j induced by j's tax choice. Since governments hope to achieve a unit-for-unit reduction in emissions and therefore fossil fuels from their tax case (given the symmetry assumption), $\delta - 1$, reflects the extent to which a government's tax choice causes fossil fuels to leak across borders. Depending on β, this generates the spillover-damage misalignment $\left(\alpha\phi_e\beta\left(\gamma_j-\gamma_t\right)\left(\delta-1\right)\right)$ and spillback $\left(\alpha\phi_e\gamma_t\left(1+\beta\left(J-1\right)\right)\left(\delta-1\right)\right)$ distortions in the last term in (A.17). After some manipulation δ equals:

$$\delta = \frac{JF_{tw}}{JF_{tw} - (J-1)\,f_{tw}} = \frac{J\eta_{Fw}}{\left(J\eta_{Fw} - (J-1)\,\eta_{fw}\right)} \geq 0. \tag{A.18}$$

B. Decentralized Competition in Caps Without Permit Trading

Under cap competition each j government solves, after imposing the j government's balanced budget constraint:

$$\max_{g_j\geq 0,\bar{e}_j} W_j\left(\bar{F}_j - F_j\right) + I_j - g_j + V_j\left(g_j\right) - \phi\left(\boldsymbol{\gamma}_j\mathbf{e}\right), \tag{A.19}$$

where $I_j = \pi_j + wF_j$ is non-tax income.

The first order conditions to (A.19) are given by:

$$\begin{aligned} &V_{jg} = 1, \text{ and} \\ &\alpha_j\lambda_j = \sum_{t=1}^{J}\gamma_j^t\phi_{te}\left(\frac{de_t}{df_j}\right) + \left(f_j - F_j\right)\left(\frac{dw}{df_j}\right), \end{aligned} \tag{A.20}$$

given:

$$\begin{aligned} \frac{dI_j}{df_j} &= \frac{d\pi_j}{df_j} + w\frac{dF_j}{df_j} + F_j\frac{dw}{df_j}, \\ \frac{d\pi_j}{df_j} &= \alpha_j\lambda_j - f_j\frac{dw}{df_j}, \end{aligned} \tag{A.21}$$

(7), and the first-order conditions to the utility maximization problem for representative consumer j (in particular, $w - W_{jF} = 0$).

Total differentiation of final good supply and fossil fuel demanded for the production of the final good for $t \neq j$:

$$\begin{aligned} \frac{dX_t}{df_j} &= X_{tw}i\left(\lambda_t = 0\right)\frac{dw}{df_j}, \text{ and} \\ \frac{df_t}{df_j} &= f_{tw}i\left(\lambda_t = 0\right)\frac{dw}{df_j}, \end{aligned} \tag{A.22}$$

where,

$$X_{tw} = \left(\frac{h_{tf}}{h_{tff}}\right), \text{ and}$$

$$f_{tw} = \left(\frac{1}{h_{tff}}\right). \tag{A.23}$$

Note that $X_{tw} \leq 0$ and $f_{tw} \leq 0$.

Total differentiation of final good supply for the production of the final good for j (assuming $\lambda_j > 0$):

$$\frac{dX_j}{df_j} = h_{jf} = X_{j0}. \tag{A.24}$$

Total differentiation of fossil fuel supply provides:

$$\frac{dF_t}{df_j} = F_{tw}\frac{dw}{df_j}, \tag{A.25}$$

where:

$$F_{tw} = \left(-\frac{1}{W_{tFF}}\right). \tag{A.26}$$

Note that $F_{tw} \geq 0$.

Total differentiation of (10) provides:

$$\sum_{t\neq j}^{J}\frac{df_t}{df_j} + 1 = \sum_{t=1}^{J}\frac{dF_t}{df_j}, \text{ and}$$

$$\sum_{t=1}^{J}\frac{dx_t}{df_j} = \sum_{t=1}^{J}\frac{dX_t}{df_j}. \tag{A.27}$$

Substituting fossil fuel demanded and supplied from (A.22) and (A.25) into the first line of (A.27) provides:

$$\sum_{t\neq j}^{J} f_{tw} i\left(\lambda_t = 0\right)\frac{dw}{df_j} + 1 = \sum_{t=1}^{J} F_{tw}\frac{dw}{df_j} \Leftrightarrow$$

$$\frac{dw}{df_j} = \bar{w}_j, \tag{A.28}$$

where:

$$\bar{w}_j = \frac{1}{\sum_{t=1}^{J} F_{tw} - \sum_{t\neq j}^{J} f_{tw} i\left(\lambda_t = 0\right)}. \tag{A.29}$$

Note that $\bar{w}_j \geq 0$.

Substituting $\bar{w}_j$ into (A.22) provides:

$$\frac{df_t}{df_j} = f_{tw} i\left(\lambda_t = 0\right)\bar{w}_j = \bar{f}_t. \tag{A.30}$$

Total differentiation of (1) provides for $t \neq j$:

$$\frac{de_t}{df_j} = \alpha_t\left(1-\beta\right)\frac{df_t}{df_j} + \beta\sum_{l\neq j}^{J}\alpha_l\frac{df_l}{df_j} + \beta\alpha_j, \tag{A.31}$$

and, for j:

$$\frac{de_j}{df_j} = \alpha_j + \beta\sum_{l\neq j}^{J}\alpha_l\frac{df_l}{df_j}. \tag{A.32}$$

Substituting (A.30) into (A.31) and (A.32) provides:

$$\begin{aligned}\frac{de_t}{df_j} &= \alpha_t\left(1-\beta\right)\bar{f}_t + \beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l + \beta\alpha_j,\\ \frac{de_j}{df_j} &= \alpha_j + \beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l.\end{aligned} \tag{A.33}$$

Upon observing that $\sum_{t=1}^{J}\gamma_j^t\phi_{te}\left(\frac{de_t}{df_j}\right) = \gamma_j^j\phi_{je}\frac{de_j}{df_j} + \sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\frac{de_t}{df_j} = \left[\alpha_j\left(\gamma_j^j\phi_{je} + \beta\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\right)\right] + \left[\gamma_j^j\phi_{je}\beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l\right] + \left[\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\left(\alpha_t\left(1-\beta\right)\bar{f}_t + \beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l\right)\right]$, it's evident that the second line of (A.20) can be re-written as:

$$\alpha_j\lambda_j\left(\boldsymbol{\theta}_{\sim j}\right) = \alpha_j\lambda_j^{E,O} + \alpha_j\lambda_j^{E,S} + \alpha_j\lambda_j^{E,X} + \alpha_j\lambda_j^{T},$$

where:

$$\begin{aligned}\alpha_j\lambda_j^{E,O} &= \alpha_j\left(\gamma_j^j\phi_{je} + \beta\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\right),\\ \alpha_j\lambda_j^{E,S} &= \gamma_j^j\phi_{je}\beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l,\\ \alpha_j\lambda_j^{E,X} &= \sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\left(\alpha_t\left(1-\beta\right)\bar{f}_t + \beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l\right), \text{ and}\\ \alpha_j\lambda_j^{T} &= \left(f_j - F_j\right)\bar{w}_j.\end{aligned} \tag{A.34}$$

Consider the case of symmetric jurisdictions when $\phi_e > 0$. In that case (A.34) implies (again assuming that $\lambda > 0$ and thus all jurisdictions select binding caps):

$$\alpha\lambda = \alpha\phi_e \left[\gamma_j + \gamma_t \beta \left(J - 1\right)\right]. \tag{A.35}$$

Since all jurisdictions select binding caps under symmetry, there is no spillback distortion. If some jurisdictions did not select binding caps then an analogous expression to (A.18) holds, except where δ is replaced with $\delta^{\bar{e}} \left(J - 1\right) \frac{df_i}{df_j} + 1$. After some manipulation this yields:

$$\delta^{\bar{e}} = \frac{JF_{tw}}{JF_{tw} - \left(J - 1\right) f_{tw} i \left(\lambda = 0\right)} = \frac{\eta_{Fw}}{\left(J\eta_{Fw} - \left(J - 1\right) \eta_{fw} i \left(\lambda = 0\right)\right)}. \tag{A.36}$$

C. *Decentralized Competition in Caps With Permit Trading*

Under cap competition with permit trading each j government solves, after imposing the j government's balanced budget constraint:

$$\max_{g_j \geq 0, \bar{e}_j} W_j \left(\bar{F}_j - F_j\right) + I_j - g_j + V_j \left(g_j\right) - \phi\left(\boldsymbol{\gamma}_j \mathbf{e}\right), \tag{A.37}$$

where $I_j = \pi_j + wF_j$ is non-tax income.

The first order conditions to (A.37) are given by:

$$V_{jg} = 1, \text{ and}$$

$$\alpha_j \lambda_j = \alpha_j \sum_{t=1}^{J} \gamma_j^t \phi_{te} \left(\frac{de_t}{d\bar{e}_j}\right) + \alpha_j \left(f_j - F_j\right) \left(\frac{dw}{d\bar{e}_j}\right) + \left(\alpha_j f_j - \bar{e}_j\right) \alpha_j \left(\frac{dz}{d\bar{e}_j}\right), \tag{A.38}$$

given:

$$\frac{dI_j}{d\bar{e}_j} = \frac{d\pi_j}{d\bar{e}_j} + w\frac{dF_j}{d\bar{e}_j} + F_j \frac{dw}{d\bar{e}_j},$$

$$\frac{d\pi_j}{d\bar{e}_j} = z - f_j \frac{dw}{d\bar{e}_j} + \left(\bar{e}_j - \alpha_j f_j\right) \left(\frac{dz}{d\bar{e}_j}\right), \tag{A.39}$$

(9), and the first-order conditions to the utility maximization problem for representative consumer j (in particular, $w - W_{jF} = 0$).

Total differentiation of final good supply and capital and fossil fuel demanded for the production of the final good for $t \neq j$:

$$\frac{dX_t}{d\bar{e}_j} = X_{tw} \frac{dw}{d\bar{e}_j} + X_{tz} \frac{dz}{d\bar{e}_j}, \text{ and}$$

$$\frac{df_t}{d\bar{e}_j} = f_{tw} \frac{dw}{d\bar{e}_j} + f_{tz} \frac{dz}{d\bar{e}_j}, \tag{A.40}$$

where,

$$
\begin{aligned}
X_{tw} &= \left(\frac{h_{tf}}{h_{tff}}\right), \\
X_{tz} &= \left(\frac{\alpha_t h_{tf}}{h_{tff}}\right), \\
f_{tw} &= \left(\frac{1}{h_{tff}}\right), \text{ and} \\
f_{tz} &= \left(\frac{\alpha_t}{h_{tff}}\right).
\end{aligned}
\tag{A.41}
$$

Note that $X_{tw} \leq 0$, $X_{tz} \leq 0$, $f_{tw} \leq 0$, and $f_{tz} \leq 0$.

Total differentiation of the allowances supplied/demanded provides:

$$
\begin{aligned}
\frac{dy_t}{d\bar{e}_j} &= \alpha_t \frac{df_t}{d\bar{e}_j}, \text{ for } t \neq j, \text{ and} \\
\frac{dy_j}{d\bar{e}_j} &= \alpha_j \frac{df_j}{d\bar{e}_j} - 1.
\end{aligned}
\tag{A.42}
$$

Total differentiation of fossil fuel supply provides:

$$
\frac{dF_t}{d\bar{e}_j} = F_{tw} \frac{dw}{d\bar{e}_j},
\tag{A.43}
$$

where:

$$
F_{tw} = \left(-\frac{1}{W_{tFF}}\right).
\tag{A.44}
$$

Note that $F_{tw} \geq 0$.

Total differentiation of (10) provides:

$$
\begin{aligned}
\sum_{t=1}^{J} \frac{df_t}{d\bar{e}_j} &= \sum_{t=1}^{J} \frac{dF_t}{d\bar{e}_j}, \\
\sum_{t=1}^{J} \frac{dx_t}{d\bar{e}_j} &= \sum_{t=1}^{J} \frac{dX_t}{d\bar{e}_j}, \text{ and} \\
\sum_{t=1}^{J} \frac{dy_t}{d\bar{e}_j} &= 0.
\end{aligned}
\tag{A.45}
$$

Substituting fossil fuel demanded and supplied from (A.40) and (A.43) into the first line of (A.45) provides:

$$\begin{aligned}
&\sum_{t=1}^{J} f_{tw}\frac{dw}{d\bar{e}_j} + \sum_{t=1}^{J} f_{tz}\frac{dz}{d\bar{e}_j} = \sum_{t=1}^{J} F_{tw}\frac{dw}{d\bar{e}_j} \Leftrightarrow \\
&\frac{dw}{d\bar{e}_j} = w_{jz}\frac{dz}{d\bar{e}_j},
\end{aligned} \tag{A.46}$$

where:

$$w_{jz} = \frac{\sum_{t=1}^{J} f_{tz}}{\sum_{t=1}^{J}(F_{tw} - f_{tw})}. \tag{A.47}$$

Note that $w_{jz} \leq 0$.

Substituting allowances demanded/supplied from (A.42) given (A.40) into the last line of (A.45) provides:

$$\begin{aligned}
&\sum_{t=1}^{J}\left(\alpha_t f_{tw}\frac{dw}{d\bar{e}_j} + \alpha_t f_{tz}\frac{dz}{d\bar{e}_j}\right) - 1 = 0 \Leftrightarrow \\
&\frac{dz}{d\bar{e}_j} = z_{jw}\frac{dw}{d\bar{e}_j} + z_{j,0},
\end{aligned} \tag{A.48}$$

where:

$$\begin{aligned}
z_{jw} &= -\frac{\sum_{t=1}^{J}\alpha_t f_{tw}}{\sum_{t=1}^{J}\alpha_t f_{tz}}, \text{ and} \\
z_{j,0} &= \frac{1}{\sum_{t=1}^{J}\alpha_t f_{tz}}.
\end{aligned} \tag{A.49}$$

Note that: $z_{jw} \leq 0$ and $z_{j,0} \leq 0$.

Substituting (A.46) into (A.48) provides:

$$\frac{dz}{d\bar{e}_j} = \frac{z_{j,0}}{1 - z_{jw}w_{jz}} = \bar{z}_j. \tag{A.50}$$

Substituting (A.50) into (A.46) provides:

$$\frac{dw}{d\bar{e}_j} = w_{jz}\bar{z}_j = \bar{w}_j. \tag{A.51}$$

Substituting $\bar{w}_j$,and $\bar{z}_j$ into (A.40) provides:

$$\frac{df_t}{d\bar{e}_j} = f_{tw}\bar{w}_j + f_{tz}\bar{z}_j = \bar{f}_t \tag{A.52}$$

Total differentiation of (1) provides for all $t = 1, ..., J$:

$$\frac{de_t}{d\bar{e}_j} = \alpha_t (1-\beta) \frac{df_t}{d\bar{e}_j} + \beta \sum_{l=1}^{J} \alpha_l \frac{df_l}{d\bar{e}_j}. \tag{A.53}$$

Substituting (A.52) into (A.53) provides:

$$\frac{de_t}{d\bar{e}_j} = \alpha_t (1-\beta) \bar{f}_t + \beta \sum_{l=1}^{J} \alpha_l \bar{f}_l. \tag{A.54}$$

Upon observing that $\alpha_j \sum_{t=1}^{J} \gamma_j^t \phi_{te} \left(\frac{de_t}{d\bar{e}_j}\right) = \alpha_j \gamma_j^j \phi_{je} \frac{de_j}{d\bar{e}_j} + \alpha_j \sum_{t\neq j}^{J} \gamma_j^t \phi_{te} \frac{de_t}{d\bar{e}_j} = \alpha_j \left[\left(\gamma_j^j \phi_{je} + \beta \sum_{t\neq j}^{J} \gamma_j^t \phi_{te}\right)\right] \left(\alpha_j \bar{f}_j\right) + \alpha_j \left[\gamma_j^j \phi_{je} \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l\right] + \alpha_j \left[\sum_{t\neq j}^{J} \gamma_j^t \phi_{te} \left((1-\beta) \alpha_t \bar{f}_t + \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l\right)\right]$, it's evident that the second line of (A.38) can be re-written as:

$$\alpha_j \mu_j (\boldsymbol{\theta}_{\sim j}) = \alpha_j \mu_j^{E,O} + \alpha_j \mu_j^{E,S} + \alpha_j \mu_j^{E,X} + \alpha_j \mu_j^{T,F} + \alpha_j \mu_j^{tz},$$

where:

$$\begin{aligned}
\alpha_j \mu_j^{E,O} &= \alpha_j \left(\gamma_j^j \phi_{je} + \beta \sum_{t\neq j}^{J} \gamma_j^t \phi_{te}\right) \alpha_j \bar{f}_j, \\
\alpha_j \mu_j^{E,S} &= \alpha_j \gamma_j^j \phi_{je} \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l, \\
\alpha_j \mu_j^{E,X} &= \alpha_j \sum_{t\neq j}^{J} \gamma_j^t \phi_{te} \left((1-\beta) \alpha_t \bar{f}_t + \beta \sum_{l\neq j}^{J} \alpha_l \bar{f}_l\right), \\
\alpha_j \mu_j^{T,F} &= (f_j - F_j) \alpha_j \bar{w}_j, \text{ and} \\
\alpha_j \mu_j^{tz} &= (\alpha_j f_j - \bar{e}_j) \alpha_j \bar{z}_j.
\end{aligned} \tag{A.55}$$

D. Pareto Optimal Allocation

Given the assumption of quasi-linear utility, the solution to the Pareto optimality problem is exactly equivalent to the solution when a utilitarian social welfare function is maximized. The latter problem is given by:

$$
\begin{aligned}
&\max_{\{f_j,F_j\}_{j=1}^J} \quad \sum_{j=1}^J W_j\left(\bar{F}_j - F_j\right) + \sum_{j=1}^J h_j\left(f_j\right) \\
&\qquad\qquad - \sum_{j=1}^J \phi_j \left(\alpha_j f_j + \beta \sum_{l\neq j}^J \alpha_l f_l\right) \\
&\qquad\qquad \text{subject to:} \\
&\qquad\qquad \sum_{j=1}^J f_j \leq \sum_{j=1}^J F_j \ \left(\omega\right). \qquad \text{(A.56)}
\end{aligned}
$$

The first-order conditions to (A.56) are given by:

$$
\begin{aligned}
h_{jf} - \alpha_j \left(\phi_{je} + \beta \sum_{l\neq j}^J \phi_{le}\right) &= \omega, \ \text{ for all } j = 1, ..., J, \\
\omega &= W_{jF}, \ \text{ for all } j = 1, ..., J. \qquad \text{(A.57)}
\end{aligned}
$$

It is also possible to identify the vector of taxes that maximize the Utilitarian social welfare function conditional on the resulting economic equilibrium. This problem is given by:

$$
\begin{aligned}
&\max_{\{\tau_j\}_{j=1}^J, w} \quad \sum_{j=1}^J W_j\left(\bar{F}_j - F_j\left(w\right)\right) \\
&\qquad + \sum_{j=1}^J h_j\left(f_j\left(w,\tau_j\right)\right) \\
&\qquad - \sum_{j=1}^J \phi_j \left(\alpha_j f_j\left(w,\tau_j\right) + \beta \sum_{l\neq j}^J \alpha_l f_l\left(w,\tau_l\right)\right) \\
&\qquad \text{subject to:} \\
&\qquad \sum_{j=1}^J f_j\left(w,\tau_j\right) \leq \sum_{j=1}^J F_j\left(w\right) \ \left(\delta\right), \qquad \text{(A.58)}
\end{aligned}
$$

and given the maximization of (2) subject to the consumer's private budget constraint and (4).

The first-order conditions to (A.58) are given by:

$$\sum_{j=1}^{J}\left[F_{jw}\left(\delta - W_{jF}\right) + f_{jw}\left(h_{jf} - \delta\right) - \alpha_j f_{jw}\left(\phi_{je} + \beta\sum_{l\neq j}^{J}\phi_{le}\right)\right] = 0, \text{ and}$$

$$\text{(A.59)} \qquad \left(h_{jf} - \delta\right) f_{j\tau} - \alpha_j f_{j\tau}\left(\phi_{je} + \beta\sum_{l\neq j}^{J}\phi_{le}\right) = 0, \text{ for all } j = 1, ..., J.$$

Given $W_{jF} = w$, (5) which implies that $\alpha_j \tau_j = h_{jf} - w$, and observing that $\delta = w$, the last line of (A.59) provides the vector of first-best Pigouvian taxes:

$$\text{(A.60)} \qquad \tau_j^{PO} = (1-\beta)\,\phi_{je} + \beta\sum_{l=1}^{J}\phi_{le}, \text{ for all } j = 1, ..., J.$$

Note that substituting in (A.60) into (A.59) returns (A.57), given $\omega = w$.

II. Extensions

A. Non-Uniform Spillovers

Next, I consider the case of non-uniform spillovers and cap competition across J virtual markets. Total emissions delivered to j from j and all other jurisdictions are, $e_j = \sum_{t=1}\beta_t^j \alpha_t f_t$, where we assume $\beta_j^j = 1$ for all $j = 1, ..., J$.

Under cap competition with free trade in permits across all J virtual markets each j government solves, after imposing the j government's balanced budget constraint:

$$\text{(A.61)} \qquad \max_{g_j \geq 0, \bar{\mathbf{e}}_j} W_j\left(\bar{F}_j - F_j\right) + I_j - g_j + V_j\left(g_j\right) - \phi\left(\boldsymbol{\gamma}_j \mathbf{e}\right),$$

where $I_j = \pi_j + wF_j$ is non-tax income.

The first order conditions to (A.61) are given by:

$$V_{jg} = 1, \text{ and}$$

(A.62)

$$\alpha_j\delta_j = \alpha_j\sum_{t=1}^{J}\gamma_j^t\phi_{te}\left(\frac{de_t}{d\bar{e}_j^t}\right) + \alpha_j\left(f_j - F_j\right)\left(\frac{dw}{d\bar{e}_j^t}\right) + \sum_{l=1}^{J}\left(\beta_j^l\alpha_j f_j^{\delta_j^t} - \bar{e}_j^l\right)\left(\frac{dz_l}{d\bar{e}_j^t}\right),$$

given:

$$\frac{dI_j}{d\bar{e}_j^t} = \frac{d\pi_j}{d\bar{e}_j^t} + w\frac{dF_j}{d\bar{e}_j^t} + F_j\frac{dw}{d\bar{e}_j^t},$$

$$\text{(A.63)} \qquad \frac{d\pi_j}{d\bar{e}_j^t} = z_t - f_j\frac{dw}{d\bar{e}_j^t} + \sum_{l=1}^{J}\left(\bar{e}_j^l - \beta_j^l\alpha_j f_j\right)\left(\frac{dz_l}{d\bar{e}_j^t}\right),$$

(19), and the first-order conditions to the utility maximization problem for representative consumer j (in particular, $w - W_{jF} = 0$).

Total differentiation of final good supply and capital and fossil fuel demanded for the production of the final good for $t \neq j$:

$$\frac{dX_t}{d\bar{e}_j^t} = X_{tw}\frac{dw}{d\bar{e}_j^t} + \sum_{l=1}^{J} X_{tz_l}\frac{dz_l}{d\bar{e}_j^t}, \text{ and}$$

$$\frac{df_t}{d\bar{e}_j^t} = f_{tw}\frac{dw}{d\bar{e}_j^t} + \sum_{l=1}^{J} f_{tz_l}\frac{dz_l}{d\bar{e}_j}, \tag{A.64}$$

where,

$$X_{tw} = \left(\frac{h_{tf}}{h_{tff}}\right),$$

$$X_{tz_l} = \left(\frac{\alpha_t \beta_t^l h_{tf}}{h_{tff}}\right) \text{ for } l = 1, ..., J,$$

$$f_{tw} = \left(\frac{1}{h_{tff}}\right), \text{ and}$$

$$f_{tz_l} = \left(\frac{\alpha_t \beta_t^l}{h_{tff}}\right) \text{ for } l = 1, ..., J. \tag{A.65}$$

Note that $X_{tw} \leq 0$, $X_{tz_l} \leq 0$, $f_{tw} \leq 0$, and $f_{tz_l} \leq 0$.

Total differentiation of the allowances supplied/demanded provides:

$$\frac{dy_t^k}{d\bar{e}_j^t} = \beta_t^k \alpha_t \frac{df_t}{d\bar{e}_j^t} \text{ for } t \neq j \text{ and all } k = 1, ..., J,$$

$$\frac{dy_j^t}{d\bar{e}_j^t} = \beta_j^t \alpha_j \frac{df_j}{d\bar{e}_j^t} - 1, \text{ and}$$

$$\frac{dy_j^l}{d\bar{e}_j^t} = \beta_j^l \alpha_j \frac{df_j}{d\bar{e}_j^t} \text{ for } l \neq t. \tag{A.66}$$

Total differentiation of fossil fuel supply provides:

$$\frac{dF_t}{d\bar{e}_j^t} = F_{tw}\frac{dw}{d\bar{e}_j^t}, \tag{A.67}$$

where:

$$F_{tw} = \left(-\frac{1}{W_{tFF}}\right). \tag{A.68}$$

Note that $F_{tw} \geq 0$.

Total differentiation of the market clearing conditions provides:

$$\begin{aligned}
\sum_{l=1}^{J} \frac{df_l}{d\bar{e}_j^t} &= \sum_{l=1}^{J} \frac{dF_l}{d\bar{e}_j^t}, \\
\sum_{l=1}^{J} \frac{dx_l}{d\bar{e}_j^t} &= \sum_{l=1}^{J} \frac{dX_l}{d\bar{e}_j^t}, \text{ and} \\
\sum_{l=1}^{J} \frac{dy_l^k}{d\bar{e}_j^t} &= 0 \text{ for all } k = 1, ..., J.
\end{aligned} \tag{A.69}$$

Substituting fossil fuel demanded and supplied from (A.64) and (A.67) into the first line of (A.69) provides:

$$\begin{aligned}
\sum_{l=1}^{J} f_{lw} \left(\frac{dw}{d\bar{e}_j^t}\right) + \sum_{m=1}^{J} \left(\sum_{l=1}^{J} f_{lz_m}\right) \left(\frac{dz_m}{d\bar{e}_j^t}\right) = \sum_{l=1}^{J} F_{lw} \left(\frac{dw}{d\bar{e}_j^t}\right) \Leftrightarrow \\
\left[\sum_{l=1}^{J} (f_{lw} - F_{lw})\right] \left(\frac{dw}{d\bar{e}_j^t}\right) + \sum_{m=1}^{J} \left(\sum_{l=1}^{J} f_{lz_m}\right) \left(\frac{dz_m}{d\bar{e}_j^t}\right) = 0.
\end{aligned} \tag{A.70}$$

Substituting allowances demanded/supplied from (A.66) into the last line of (A.69)provides:

$$\begin{aligned}
\frac{de_k}{d\bar{e}_j^t} &= \left(\sum_{l=1}^{J} \beta_l^k \alpha_l f_{lw}\right) \left(\frac{dw}{d\bar{e}_j^t}\right) + \sum_{m=1}^{J} \left(\sum_{l=1}^{J} \beta_l^k \alpha_l f_{lz_m}\right) \left(\frac{dz_m}{d\bar{e}_j^t}\right) = 0 \text{ for } k \neq t \\
\frac{de_t}{d\bar{e}_j^t} &= \left(\sum_{l=1}^{J} \beta_l^t \alpha_l f_{lw}\right) \left(\frac{dw}{d\bar{e}_j^t}\right) + \sum_{m=1}^{J} \left(\sum_{l=1}^{J} \beta_l^t \alpha_l f_{lz_m}\right) \left(\frac{dz_m}{d\bar{e}_j^t}\right) = 1.
\end{aligned} \tag{A.71}$$

(A.70) and (A.71) provide the following analytic expression for the $J+1$ vector of total derivatives, $\bar{\mathbf{z}}_j^t = \left\{\frac{dw}{d\bar{e}_j^t}, \frac{dz_1}{d\bar{e}_j^t}, ..., \frac{dz_J}{d\bar{e}_j^t}\right\}$:

$$\bar{\mathbf{z}}_j^t = \mathbf{A}^{-1}\mathbf{b}_{t+1}, \tag{A.72}$$

where the $J+1$ row vector of coefficients $\mathbf{A}_1 = \left\{\left[\sum_{l=1}^{J} (f_{lw} - F_{lw})\right], \left(\sum_{l=1}^{J} f_{lz_1}\right), ..., \left(\sum_{l=1}^{J} f_{lz_J}\right)\right\}$, the $J+1$ row vector of coefficients $\mathbf{A}_{k+1} = \left\{\left(\sum_{l=1}^{J} \beta_l^k \alpha_l f_{lw}\right), \left(\sum_{l=1}^{J} \beta_l^k \alpha_l f_{lz_1}\right), ..., \left(\sum_{l=1}^{J} \beta_l^k \alpha_l f_{lz_J}\right)\right\}$ for all $k = 1, ..., J$, the $J+1$ by $J+1$ matrix of coefficients $\mathbf{A} = \{\mathbf{A}_l\}_{l=1}^{J+1}$, and $\mathbf{b}_{t+1}$ is the $t+1$ column basis vector of length $J+1$. $\mathbf{A}$ can be inverted so long as $\mathbf{A}$ has full rank. Functionally this requires that the J by J matrix of spillover coefficients $\boldsymbol{\beta}$ has full rank. For the case of a pollutant which generates uniform spillovers when $\beta_j^t = \beta$ for all $j, t = 1, ..., J$, this is not the case and the solution can be obtained as before in Section I.C; J markets can concurrently exist even in the case of a uniform pollutant, but functionally they are all identical and the caps chosen on delivered emissions to t for a particular jurisdiction j in the resulting Nash equilibrium are the same for all $t = 1, ..., J$. Note also that for the particular case of a uniform global pollutant, when $\beta_j^t = 1$ for all $j, t = 1, ..., J$, that (A.71) instead implies that $\frac{de_k}{d\bar{e}_j^t} = 1$ for all $k = 1, ..., J$ since $\frac{de_k}{d\bar{e}_j^t} = \frac{de}{d\bar{e}_j^t}$ for all $k = 1, ..., J$. For the general case when $\boldsymbol{\beta}$ has full rank, the solution to (A.72) implies that $\bar{\mathbf{z}}_j^t = \bar{\mathbf{z}}^t$ for all $j = 1, ..., J$, reflecting the fact the markets for fossil fuels and the J permit markets are global. For a particular J, (A.72) can also be expressed in terms of elasticities as before.

Substituting (A.72) into (A.64) provides:

$$\frac{df_l}{d\bar{e}_j^t} = \bar{f}_l^t \tag{A.73}$$

Substituting (A.73) into (A.71) and given (A.71), provides, for all $l = 1, ..., J$:

$$\begin{aligned} \frac{de_k}{d\bar{e}_j^t} &= \sum_{l=1}^{J} \beta_l^k \alpha_l \bar{f}_l^t = 0 \text{ for } k \neq t \\ \frac{de_t}{d\bar{e}_j^t} &= \sum_{l=1}^{J} \beta_l^t \alpha_l \bar{f}_l^t = 1. \end{aligned} \tag{A.74}$$

After some manipulation the last line of (A.62) can be re-written as (20). For the general case when $\boldsymbol{\beta}$ has full rank (and therefore $\beta_j^t \neq \beta$ for all $t, j = 1, ..., J$), when all jurisdictions internalize global damages, summing (20) across all $j = 1, ..., J$, implies, after imposing market clearing in the market for fossil fuels and market clearing in all permit markets, that $z_t = \phi_{te}$ for all $t = 1, ..., J$.

Next, we return to the case of a global pollutant when $\beta_j^t = 1$ for all $j, t = 1, ..., J$. Given that $\frac{de_k}{d\bar{e}_j^t} = 1$ for all $k = 1, ..., J$, *when all jurisdictions internalize own damages*, summing (20) across all $j = 1, ..., J$, implies, after imposing market clearing in the market for fossil fuels and market clearing in all permit markets, that $z_t = \left(\frac{1}{J}\right)\left(\sum_{l=1}^{J} \phi_{le}\right)$ for all $t = 1, ..., J$.

In either of these two cases, substitution of the appropriate z_t for all $t = 1, ..., J$ into (19), together with the remaining first-order conditions and the market clearing conditions provides an allocation which supports the Pareto optimal allocation identified by (A.76), since $\sum_{t=1}^{J} z_t = \left(\sum_{l=1}^{J} \phi_{le}\right)$.

Pareto Optimal Allocation

Again given that $\boldsymbol{\gamma}_j$ equals the basis vector for all j, the Pareto optimal allocation solves:

$$
\begin{aligned}
&\max_{g_j, F_j, f_j, x_j \geq 0} \quad W_1\left(\bar{F}_1 - F_1\right) + x_1 + V_1\left(g_1\right) - \phi\left(\boldsymbol{\gamma}_1 \mathbf{e}\right) \\
&\text{subject to:} \\
&\quad W_j\left(\bar{F}_j - F_j\right) + x_j + V_j\left(g_j\right) - \phi\left(\boldsymbol{\gamma}_j \mathbf{e}\right) \geq \bar{u}_j \ \left(\lambda_j\right), \text{ for all } j > 1, \\
&\sum_{t=1}^{J}\left(x_t + g_t\right) = \sum_{t=1}^{J} h_t\left(f_t\right) \ (\delta), \\
&\sum_{t=1}^{J} f_t = \sum_{t=1}^{J} F_t \ (\omega), \\
&e_j = \sum_{t=1} \beta_t^j \alpha_t f_t, \text{ for all } j.
\end{aligned} \tag{A.75}
$$

The first-order conditions to (A.75) imply, after some manipulation:

$$
\begin{aligned}
&W_{jF} = h_{jf} - \alpha_j\left(\sum_{t=1}^{J} \beta_j^t \phi_{te}\right) = \omega, \text{ for all } j = 1, ..., J, \text{ and} \\
&V_{jg} = 1, \text{ for all } j = 1, ..., J.
\end{aligned} \tag{A.76}
$$

The vector of taxes that support the Pareto optimal allocation are now given by:

$$
\tau_j^{PO} = \left(\sum_{t=1}^{J} \beta_j^t \phi_{te}\right), \text{ for all } j = 1, ..., J. \tag{A.77}
$$

Lindahl Personalized Markets for a Global Pollutant

An alternative method to address the under-provision of public good/over-provision of public bads problem was proposed by **?**. **?** proposed establishing personalized markets in the public good/bad. We focus on the (global) public bads case in light of the model we have so far considered. In this case the private good producer is now required to pay for damages accrued to each jurisdiction for each unit of emissions that they produce. Thus, they now solve:

$$
\max_{f_j \geq 0} \quad h_j\left(f_j\right) - \left(w + \alpha_j \sum_{t=1}^{J} z_t\right) f_j. \tag{A.78}
$$

The first-order condition to (A.78) is given by:

$$
h_{jf} = w + \alpha_j \sum_{t=1}^{J} z_t. \tag{A.79}
$$

This generates the supply of the private good in jurisdiction j, $X_j(\mathbf{z}, w)$, the demand for fossil fuels in j, $f_j(\mathbf{z}, w)$, and the supply of produced emissions in j, $\hat{e}_j(\mathbf{z}, w) = \alpha_j f_j(\mathbf{z}, w)$. Focusing on the case of global emissions, realized emissions in j are still as given in (1), i.e., $e_j = \sum_{t=1}^{J} \alpha_t f_t$.

Consumers now need a mechanism for expressing their private preferences for the public bad. One method for achieving this, as noted by **?**, is to establish a personalized market for each consumer in the public bad. We modify consumer's preferences in (3) to accommodate this case. Specifically, we replace $\phi\left(\boldsymbol{\gamma}_j \mathbf{e}\right)$ with $\phi_j(e_j)$. In effect, this amounts to imposing $\gamma_j^j = 1$ and $\gamma_j^t = 0$ for all $j = 1, ..., J$ and $t \neq j$ in (3). We now introduce a new, personalized market for each consumer in a virtual good, m_j. Although e_j is non-excludable, m_j is assumed to be. As such the consumer in j now solves:

$$\max_{F_j \geq 0, m_j \geq 0} \quad W_j\left(\bar{F}_j - F_j\right) + wF_j + I_j + z_j m_j + V_j(g_j) - \phi_j(m_j). \tag{A.80}$$

The first-order conditions to (A.80) are given by:

$$\begin{aligned} w &= W_{jF}, \\ z_j &= \phi_{je}(m_j). \end{aligned} \tag{A.81}$$

The last first-order condition in (A.81) is very important as it states that the price for emissions in j's personalized market should be set equal to their marginal willingness to pay for a unit of avoided emissions realized in their jurisdiction.

Market clearing is as before (see (10)), except that we also require that $e_j = m_j$ for each $j = 1, ..., J$, which reflects the excludability assumptions imposed on each of our new J virtual markets. Given a global pollutant, this can also be written as: $e = m_j$ since $e_j = e$ for all $j = 1, .., J$. It's clear from (A.79), (A.81), and the new market clearing conditions, for all $j = 1, ..., J$, that expanding the number of markets from one to J virtual markets renders the equilibrium allocation Pareto optimal, even if jurisdictions do not internalize global damages. To be precise, the last line of (A.81) first identifies the prices on emissions in each jurisdiction realized in the Lindahl equilibrium as being equal to marginal damages in each jurisdiction, given that emissions realized everywhere must be the same in the case of a (global) public bad, i.e., $e = m_j$ for all $j = 1, .., J$. Given this, (A.79) then identifies the Pareto optimal level of emissions.

The Lindahl equilibrium concept has been regarded as impractical for two reasons. First, solving (A.80) requires the ability to construct a personal, excludable market for each consumer for a public bad that has historically been considered non-excludable. Second, if each consumer has a private market in the public bad, they may wish to exercise market power. Similarly, consumers would have no incentive to truthfully report their marginal willingness to pay to avoid emissions since misreporting provides another opportunity for them to free-ride off of others.

Our analysis shows that while emissions can be made excludable it is more difficult to make damages from emissions excludable. A cap and trade system with appropriate enforcement and monitoring can render a public bad privately excludable and thus capable of being virtually commoditized and transacted in markets. There is little reason to suspect that many public goods cannot also be appropriately monitored and enforced, and thus mandate and trade systems could also allow for excludability. For example, if a park is constructed, a new road is built, or more bombers purchased it is not typically difficult to ascertain who supplied these goods and thus who should be awarded a valuable permit for their provision.

As shown in the text, cap competition by governments with free trade in permits within a single virtual market eliminates the spillback and terms of trade distortions arising from decentralized efforts to constrain the public bad.[1] This is possible because of the excludability provided by the introduction of the single new virtual market on global emissions when trading in permits is allowed across jurisdictions. This single market cannot alone address the reality that damages from emissions are non-rival in consumption across jurisdictions and so the assumption of global internalization of damages is also necessary. J permit markets, however, can provide an alternative mechanism for addressing non-rivalry in emissions damages. When emissions are global, the Nash equilibrium of caps that results are identical across the J markets. The limitations of the Lindahl solution of personalized markets for the public bad also bear repeating for this alternative method by which decentralized governments establish J virtual markets in the public bad. For instance, a government contemplating introducing a new virtual market in the presence of some pre-existing virtual market(s) is unlikely to be willing to do so given that they can free-ride off of the damage reductions provided by the pre-existing virtual market(s). Therefore, when market entry is itself within governments' choice sets, it is unlikely that decentralized governments will be willing to establish sufficient virtual markets necessary to support the Pareto optimal allocation if they do not internalize global damages. Of course, if governments do internalize global damages then a single virtual market will likely be superior to J virtual markets since it will entail lower transaction and enforcement costs which have heretofore been abstracted from in the current analysis.

B. *Multiple Mobile Factors and Abatement*

Consumer Demand

Now suppose the representative consumer in each jurisdiction has preferences given by:

$$u_j = U_j\left(\bar{K}_j - K_j\right) + W_j\left(\bar{F}_j - F_j\right) + x_j + v_j\left(g_j, \mathbf{e}\right), \tag{A.82}$$

where $U_{iK} > 0$, $U_{iKK} < 0$, $W_{iF} > 0$, and $W_{iFF} < 0$. Now the consumer's private budget constraint is given by: $x_j = \pi_j + rK_j + wF_j - T_j$, where r is the rate of return to capital. The consumer maximizes (A.82) subject to this constraint, taking the amount of local public good provided and emissions as exogenous. This provides the Walrasian demand for the private good, $x_j\left(w, r, T_j\right)$, the amount of capital supplied, $K_j\left(r\right)$, and the amount of fossil fuels supplied, $F_j\left(w\right)$, where $K_{jr} = \frac{-1}{U_{jKK}} > 0$ and $F_{jw} = \frac{-1}{W_{jFF}} > 0$.

Final Good Production

A representative final good producer located in each jurisdiction produces a final good using clean (capital) and dirty (fossil fuels) mobile factors and labor as inputs according to the following production function: $X_j = h_j\left(f_j, k_j, l_j\right)$. Since labor is fixed in each jurisdiction, we can impose market clearing in the labor market, $l_j = \bar{L}_j$, to obtain $X_j = h_j\left(f_j, k_j\right)$, which is analogous to specifying a decreasing returns to scale production

[1] Recall, that in the case of a global pollutant the spillover-damage internalization is also zero.

function in capital and fossil fuels where $h_{jf} > 0$, $h_{jk} > 0$, $h_{jff} < 0$, $h_{jkk} < 0$, and $h_{jff}h_{jkk} - h_{jfk}h_{jkf} > 0$.

When a government imposes an emissions cap and also allows permits to be used for compliance, y_j, the representative producer maximizes profits by solving:

$$\max_{f_j \geq 0, y_j} \quad h_j\left(f_j, k_j\right) - wf_j - rk_j - zy_j$$

$$\text{subject to:}$$

$$\alpha_j f_j - y_j \leq \bar{e}_j \ \left(\mu_j\right). \tag{A.83}$$

The first order conditions to (A.83) are given by:

$$\begin{aligned} h_{jf} &= w + \alpha_j \mu_j, \\ h_{jk} &= r, \\ \mu_j &= z, \\ \left(\bar{e}_j - \alpha_j f_j + y_j\right)\mu_j &= 0, \alpha_j f_j - y_j \leq \bar{e}_j, \mu_j \geq 0. \end{aligned} \tag{A.84}$$

(A.84) generates the unconditional demands for fossil fuels, $f_j\left(w, r, z\right)$, and capital, $k_j\left(w, r, z\right)$. We also obtain the supply of the final good, $X_j\left(w, r, z\right)$, the amount of permits supplied/demanded, $y_j\left(w, r, z, \bar{e}_j\right) = \alpha_j f_j\left(w, r, z\right) - \bar{e}_j$, profits, $\pi_j\left(w, r, z, \bar{e}_j\right) = h_j\left(f_j\left(w, r, z\right), k_j\left(w, r, z\right)\right) - \left(w + \alpha_j z\right) f_j\left(w, r, z\right) - rk_j\left(w, r, z\right) + z\bar{e}_j$, and source emissions, $\alpha_j f_j\left(w, r, z\right)$.

A *competitive equilibrium* with a permit market is the vector of prices and government expenditures on local public goods, $\left(w, r, z, \{g_i\}_{i=1}^{J}\right)$, and resulting quantities that solve all consumers' utility maximization problems and all firms' profit maximization problems, such that all markets clear:

$$\begin{aligned} \sum_{j=1}^{J} f_j\left(w, r, z, \theta_j\right) &\leq \sum_{j=1}^{J} F_j\left(w\right), \\ \sum_{j=1}^{J} k_j\left(w, r, z, \theta_j\right) &\leq \sum_{j=1}^{J} K_j\left(r\right), \\ \sum_{j=1}^{J} y_j\left(w, r, z, \theta_j\right) &\leq 0, \\ \sum_{j=1}^{J} x_j\left(w, r, z, \theta_j\right) + \sum_{j=1}^{J} g_j\left(\theta_j\right) &\leq \sum_{j=1}^{J} X_j\left(w, r, z, \theta_j\right), \end{aligned} \tag{A.85}$$

and the governments budget constraint is balanced for all j such that: $T_j = g_j$.

Under cap competition with permit trading each j government solves, after imposing the j government's balanced budget constraint:

$$\max_{g_j \geq 0, \bar{e}_j} U_j\left(\bar{K}_j - K_j\right) + W_j\left(\bar{F}_j - F_j\right) + I_j - g_j + V_j\left(g_j\right) - \sum_{t=1}^{J} \gamma_j^t \phi_t\left(e_t\right), \tag{A.86}$$

where $I_j = \pi_j + rK_j + wF_j$ is total non-tax income.

The first order conditions to (A.86) are given by:

$$\begin{aligned} & V_{jg} = 1, \text{ and} \\ & \alpha_j \lambda_j = \alpha_j \sum_{t=1}^{J} \gamma_j^t \phi_{te}\left(\frac{de_t}{d\bar{e}_j}\right) + \alpha_j\left(f_j - F_j\right)\left(\frac{dw}{d\bar{e}_j}\right) + \alpha_j\left(k_j - K_j\right)\left(\frac{dr}{d\bar{e}_j}\right) \\ & \qquad + \left(\alpha_j f_j - \bar{e}_j\right)\alpha_j\left(\frac{dz}{d\bar{e}_j}\right), \end{aligned} \tag{A.87}$$

given:

$$\begin{aligned} \frac{dI_j}{d\bar{e}_j} &= \frac{d\pi_j}{d\bar{e}_j} + r\frac{dK_j}{d\bar{e}_j} + K_j\frac{dr}{d\bar{e}_j} + w\frac{dF_j}{d\bar{e}_j} + F_j\frac{dw}{d\bar{e}_j}, \\ \frac{d\pi_j}{d\bar{e}_j} &= z - f_j\frac{dw}{d\bar{e}_j} - k_j\frac{dr}{d\bar{e}_j} + \left(\bar{e}_j - \alpha_j f_j\right)\left(\frac{dz}{d\bar{e}_j}\right), \end{aligned} \tag{A.88}$$

(A.84), and the first-order conditions to the utility maximization problem for representative consumer j (in particular, $r - U_{jK} = 0$ and $w - W_{jF} = 0$).

Total differentiation of final good supply and capital and fossil fuel demanded for the production of the final good for $t \neq j$:

$$\begin{aligned} \frac{dX_t}{d\bar{e}_j} &= X_{tr}\frac{dr}{d\bar{e}_j} + X_{tw}\frac{dw}{d\bar{e}_j} + X_{tz}\frac{dz}{d\bar{e}_j}, \\ \frac{dk_t}{d\bar{e}_j} &= k_{tr}\frac{dr}{d\bar{e}_j} + k_{tw}\frac{dw}{d\bar{e}_j} + k_{tz}\frac{dz}{d\bar{e}_j}, \text{ and} \\ \frac{df_t}{d\bar{e}_j} &= f_{tr}\frac{dr}{d\bar{e}_j} + f_{tw}\frac{dw}{d\bar{e}_j} + f_{tz}\frac{dz}{d\bar{e}_j}, \end{aligned} \tag{A.89}$$

where,

$$X_{tr} = \left(\frac{h_{tf}h_{tfk} - h_{tk}h_{tff}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$X_{tw} = \left(\frac{h_{tk}h_{tkf} - h_{tf}h_{tkk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$X_{tz} = \left(\frac{\alpha_t \left(h_{tk}h_{tkf} - h_{tf}h_{tkk} \right)}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$k_{tr} = \left(\frac{-h_{tff}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$k_{tw} = \left(\frac{h_{tkf}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$k_{tz} = \left(\frac{\alpha_t h_{tkf}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$f_{tr} = \left(\frac{h_{tfk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right),$$
$$f_{tw} = \left(\frac{-h_{tkk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right), \text{ and}$$
$$f_{tz} = \left(\frac{-\alpha_t h_{tkk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}} \right). \tag{A.90}$$

Note that when capital and fossil fuels are technical complements with respect to the production of the final good (e.g., $h_{tkf} \geq 0$) that $X_{tw} \leq 0$, $X_{tr} \leq 0$, $X_{tz} \leq 0$, $k_{tw} \leq 0$, $k_{tr} \leq 0$, $k_{tz} \leq 0$, $f_{tw} \leq 0$, $f_{tr} \leq 0$, and $f_{tz} \leq 0$.

Total differentiation of the allowances supplied/demanded provides:

$$\frac{dy_t}{d\bar{e}_j} = \alpha_t \frac{df_t}{d\bar{e}_j}, \text{ for } t \neq j, \text{ and}$$
$$\frac{dy_j}{d\bar{e}_j} = \alpha_j \frac{df_j}{d\bar{e}_j} - 1. \tag{A.91}$$

Total differentiation of all remaining supplies provides:

$$\frac{dK_t}{d\bar{e}_j} = K_{tr} \frac{dr}{d\bar{e}_j}, \text{ and}$$
$$\frac{dF_t}{d\bar{e}_j} = F_{tw} \frac{dw}{d\bar{e}_j}, \tag{A.92}$$

where:

$$K_{tr} = \left(-\frac{1}{U_{tKK}}\right), \text{ and}$$

$$F_{tw} = \left(-\frac{1}{W_{tFF}}\right). \tag{A.93}$$

Note that $K_{tr} \geq 0$ and $F_{tw} \geq 0$.

Total differentiation of (A.85) provides:

$$\sum_{t=1}^{J} \frac{df_t}{d\bar{e}_j} = \sum_{t=1}^{J} \frac{dF_t}{d\bar{e}_j},$$

$$\sum_{t=1}^{J} \frac{dk_t}{d\bar{e}_j} = \sum_{t=1}^{J} \frac{dK_t}{d\bar{e}_j},$$

$$\sum_{t=1}^{J} \frac{dx_t}{d\bar{e}_j} = \sum_{t=1}^{J} \frac{dX_t}{d\bar{e}_j}, \text{ and}$$

$$\sum_{t=1}^{J} \frac{dy_t}{d\bar{e}_j} = 0. \tag{A.94}$$

Substituting fossil fuel demanded and supplied from (A.89) and (A.92) into the first line of (A.94) provides:

$$\sum_{t=1}^{J} f_{tw} \frac{dw}{d\bar{e}_j} + \sum_{t=1}^{J} f_{t,r} \frac{dr}{d\bar{e}_j} + \sum_{t=1}^{J} f_{tz} \frac{dz}{d\bar{e}_j} = \sum_{t=1}^{J} F_{tw} \frac{dw}{d\bar{e}_j} \Leftrightarrow$$

$$\frac{dw}{d\bar{e}_j} = w_{j,r} \frac{dr}{d\bar{e}_j} + w_{jz} \frac{dz}{d\bar{e}_j}, \tag{A.95}$$

where:

$$w_{j,r} = \frac{\sum_{t=1}^{J} f_{t,r}}{\sum_{t=1}^{J} (F_{tw} - f_{tw})}, \text{ and}$$

$$w_{jz} = \frac{\sum_{t=1}^{J} f_{tz}}{\sum_{t=1}^{J} (F_{tw} - f_{tw})}. \tag{A.96}$$

Again if $h_{tkf} \geq 0$, then $w_{j,r} \leq 0$ and $w_{jz} \leq 0$.

Substituting allowances demanded/supplied from (A.91) given (A.89) into the last line of (A.94) provides:

$$\sum_{t=1}^{J}\left(\alpha_t f_{t,r}\frac{dr}{d\bar{e}_j}+\alpha_t f_{tw}\frac{dw}{d\bar{e}_j}+\alpha_t f_{tz}\frac{dz}{d\bar{e}_j}\right)-1=0 \Leftrightarrow$$

$$\frac{dz}{d\bar{e}_j}=z_{j,r}\frac{dr}{d\bar{e}_j}+z_{jw}\frac{dw}{d\bar{e}_j}+z_{j,0}, \tag{A.97}$$

where:

$$z_{j,r}=-\frac{\sum_{t=1}^{J}\alpha_t f_{t,r}}{\sum_{t=1}^{J}\alpha_t f_{tz}},$$

$$z_{jw}=-\frac{\sum_{t=1}^{J}\alpha_t f_{tw}}{\sum_{t=1}^{J}\alpha_t f_{tz}}, \text{ and}$$

$$z_{j,0}=\frac{1}{\sum_{t=1}^{J}\alpha_t f_{tz}}. \tag{A.98}$$

Again if $h_{tkf}\geq 0$, then $z_{j,r}\leq 0$, $z_{jw}\leq 0$, and $z_{j,0}\leq 0$.

Substituting capital demanded and supplied from (A.89) and (A.92) into the second line of (A.94) provides:

$$\sum_{t=1}^{J}k_{tr}\frac{dr}{d\bar{e}_j}+\sum_{t=1}^{J}k_{tw}\frac{dw}{d\bar{e}_j}+\sum_{t=1}^{J}k_{tz}\frac{dz}{d\bar{e}_j}=\sum_{t=1}^{J}K_{tr}\frac{dr}{d\bar{e}_j} \Leftrightarrow$$

$$\frac{dr}{d\bar{e}_j}=r_{jw}\frac{dw}{d\bar{e}_j}+r_{jz}\frac{dz}{d\bar{e}_j}, \tag{A.99}$$

where:

$$r_{jw}=\frac{\sum_{t=1}^{J}k_{tw}}{\sum_{t=1}^{J}\left(K_{t,r}-k_{t,r}\right)}, \text{ and}$$

$$r_{jz}=\frac{\sum_{t=1}^{J}k_{tz}}{\sum_{t=1}^{J}\left(K_{t,r}-k_{t,r}\right)}. \tag{A.100}$$

Again if $h_{tkf}\geq 0$, then $r_{jw}\leq 0$ and $r_{jz}\leq 0$.

Substituting (A.95) into (A.97) provides:

$$\frac{dz}{d\bar{e}_j}=\hat{z}_{j,r}\frac{dr}{d\bar{e}_j}+\hat{z}_{j,0}, \tag{A.101}$$

where:

$$\hat{z}_{j,r}=\frac{z_{j,r}+z_{jw}w_{j,r}}{1-z_{jw}w_{jz}}, \text{ and}$$

$$\hat{z}_{j,0}=\frac{z_{j,0}}{1-z_{jw}w_{jz}}. \tag{A.102}$$

Substituting (A.101) into (A.95) provides:

$$\frac{dw}{d\bar{e}_j} = \hat{w}_{j,r}\frac{dr}{d\bar{e}_j} + \hat{w}_{j,0}, \tag{A.103}$$

where:

$$\begin{aligned} \hat{w}_{j,r} &= \frac{w_{j,r} + z_{j,r}w_{jz}}{1 - z_{jw}w_{jz}}, \text{ and} \\ \hat{w}_{j,0} &= \frac{w_{jz}z_{j,0}}{1 - z_{jw}w_{jz}}. \end{aligned} \tag{A.104}$$

Substituting (A.101) and (A.103) into (A.99) provides:

$$\frac{dr}{d\bar{e}_j} = \frac{r_{jw}\hat{w}_{j,0} + r_{jz}\hat{z}_{j,0}}{1 - r_{jw}\hat{w}_{j,r} - r_{jz}\hat{z}_{j,r}} = \bar{r}_j. \tag{A.105}$$

Substituting (A.105) into (A.103) provides:

$$\frac{dw}{d\bar{e}_j} = \hat{w}_{j,r}\bar{r}_j + \hat{w}_{j,0} = \bar{w}_j. \tag{A.106}$$

Substituting (A.105) into (A.101) provides:

$$\frac{dz}{d\bar{e}_j} = \hat{z}_{j,r}\bar{r}_j + \hat{z}_{j,0} = \bar{z}_j. \tag{A.107}$$

Substituting $\bar{r}_j$, $\bar{w}_j$, and $\bar{z}_j$ into (A.89) provides:

$$\frac{df_t}{d\bar{e}_j} = f_{t,r}\bar{r}_j + f_{tw}\bar{w}_j + f_{tz}\bar{z}_j = \bar{f}_t \tag{A.108}$$

Total differentiation of (1) provides for all $t = 1, ..., J$:

$$\frac{de_t}{d\bar{e}_j} = \alpha_t(1-\beta)\frac{df_t}{d\bar{e}_j} + \beta\sum_{l=1}^{J}\alpha_l\frac{df_l}{d\bar{e}_j}. \tag{A.109}$$

Substituting (A.108) into (A.109) provides:

$$\frac{de_t}{d\bar{e}_j} = \alpha_t(1-\beta)\bar{f}_t + \beta\sum_{l=1}^{J}\alpha_l\bar{f}_l. \tag{A.110}$$

Upon observing that $\alpha_j\sum_{t=1}^{J}\gamma_j^t\phi_{te}\left(\frac{de_t}{d\bar{e}_j}\right) = \alpha_j\gamma_j^j\phi_{je}\frac{de_j}{d\bar{e}_j} + \alpha_j\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\frac{de_t}{d\bar{e}_j} = \alpha_j\left[\left(\gamma_j^j\phi_{je} + \beta\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\right)\right](\alpha_j\bar{f}_j) + \alpha_j\left[\gamma_j^j\phi_{je}\beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l\right] + \alpha_j\left[\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\left((1-\beta)\alpha_t\bar{f}_t + \beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l\right)\right]$, it's evident that the second line of (A.87) can be re-written as:

$$\alpha_j\mu_j\left(\boldsymbol{\theta}_{\sim j}\right) = \alpha_j\mu_j^{E,O} + \alpha_j\mu_j^{E,S} + \alpha_j\mu_j^{E,X} + \alpha_j\mu_j^{T,F} + \alpha_j\mu_j^{T,K} + \alpha_j\mu_j^{tz},$$

where:

$$\begin{aligned}
\alpha_j\mu_j^{E,O} &= \alpha_j\left(\gamma_j^j\phi_{je} + \beta\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\right)\alpha_j\bar{f}_j,\\
\alpha_j\mu_j^{E,S} &= \alpha_j\gamma_j^j\phi_{je}\beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l,\\
\alpha_j\mu_j^{E,X} &= \alpha_j\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\left((1-\beta)\,\alpha_t\bar{f}_t + \beta\sum_{l\neq j}^{J}\alpha_l\bar{f}_l\right),\\
\alpha_j\mu_j^{T,F} &= \left(f_j - F_j\right)\alpha_j\bar{w}_j,\\
\alpha_j\mu_j^{T,K} &= \left(k_j - K_j\right)\alpha_j\bar{r}_j, \text{ and}\\
\alpha_j\mu_j^{tz} &= \left(\alpha_j f_j - \bar{e}_j\right)\alpha_j\bar{z}_j.
\end{aligned} \tag{A.111}$$

WITH TECHNOLOGICAL ABATEMENT

When technological abatement is included in the model, end-of-pipe abatement can be supplied according to $a_j = m_j\left(k_j^A\right)$, where $0 = m_j(0)$, $\frac{\partial m_j}{\partial k_j^A} > 0$, and $\frac{\partial^2 m_j}{\left(\partial k_j^A\right)^2} < 0$ and given that k_j^A is the amount of capital used for technological abatement and k_j^X is the amount of capital used directly in the production of the final good. With technological abatement the firm's profit maximization problem under cap competition with permit trading is given by:

$$\max_{f_j\geq 0, k_j^X\geq 0, k_j^A\geq 0, y_j} \quad h_j\left(f_j, k_j^X\right) - wf_j - r\left(k_j^X + k_j^A\right) - zy_j$$

$$\begin{aligned}
&\text{subject to:}\\
&m_j\left(k_j^A\right) \leq \alpha_j f_j \;\; (\lambda_j)\\
&\alpha_j f_j - m_j\left(k_j^A\right) - y_j \leq \bar{e}_j \;\; (\mu_j)\,.
\end{aligned} \tag{A.112}$$

The first order conditions to (A.83) are given by:

$$\begin{aligned}
h_{jf} &= w + \alpha_j\mu_j - \alpha_j\lambda_j,\\
h_{jk} &= r,\\
\mu_j m_{jk} &\leq r + m_{jk}\lambda_j \;(= \text{ when } k_j^A > 0),\\
\mu_j &= z, \text{ and}\\
\left(\bar{e}_j - \left(\alpha_j f_j - m_j\left(k_j^A\right) - y_j\right)\right)\mu_j &= 0, \alpha_j f_j - m_j\left(k_j^A\right) - y_j \leq \bar{e}_j, \mu_j \geq 0.
\end{aligned} \tag{A.113}$$

(A.113) directly implies the unconditional demand for fossil fuels, capital, and abatement, or, $f_j(w,r,z)$, $k_j(w,r,z) = k_j^X(w,r,z) + k_j^A(r,z)$, and $a_j(r,z) = m_j\left(k_j^A(r,z)\right)$, respectively. We also obtain the supply of the final good, $X_j(w,r,z)$, the amount of permits supplied/demanded, $y_j(w,r,z,\bar{e}_j) = \alpha_j f_j(w,r,z) - a_j(r,z) - \bar{e}_j$, and profits, $\pi_j(w,r,z,\bar{e}_j) = h_j\left(f_j(w,r,z), k_j^X(w,r,z)\right) - wf_j(w,r,z) - z(\alpha_j f_j(w,r,z) - a_j(r,z)) - rk_j(w,r,z) + z\bar{e}_j$. Finally, the emissions released by the jurisdiction are given by $\alpha_j f_j(w,r,z) - a_j(r,z)$.

Emissions realized in each district are now given by:

$$\begin{aligned} e_j &= \alpha_j(f_j - a_j) + \beta \sum_{t \neq j}^{J} \alpha_t(f_t - a_t) \\ &= (1-\beta)\alpha_j(f_j - a_j) + \beta \sum_{t=1}^{J} \alpha_t(f_t - a_t). \end{aligned} \tag{A.114}$$

Now, the first order conditions to (A.86) are given by:

$$\begin{aligned} &V_{jg} = 1, \text{ and} \\ &\alpha_j \lambda_j = \alpha_j \sum_{t=1}^{J} \gamma_j^t \phi_{te}\left(\frac{de_t}{d\bar{e}_j}\right) + \alpha_j(f_j - F_j)\left(\frac{dw}{d\bar{e}_j}\right) + \alpha_j(k_j - K_j)\left(\frac{dr}{d\bar{e}_j}\right) \\ &\qquad + (\alpha_j f_j - a_j - \bar{e}_j)\alpha_j\left(\frac{dz}{d\bar{e}_j}\right), \end{aligned} \tag{A.115}$$

given:

$$\begin{aligned} \frac{dI_j}{d\bar{e}_j} &= \frac{d\pi_j}{d\bar{e}_j} + r\frac{dK_j}{d\bar{e}_j} + K_j\frac{dr}{d\bar{e}_j} + w\frac{dF_j}{d\bar{e}_j} + F_j\frac{dw}{d\bar{e}_j}, \\ \frac{d\pi_j}{d\bar{e}_j} &= z - f_j\frac{dw}{d\bar{e}_j} - k_j\frac{dr}{d\bar{e}_j} + (\bar{e}_j + a_j - \alpha_j f_j)\left(\frac{dz}{d\bar{e}_j}\right), \end{aligned} \tag{A.116}$$

(A.113), and the first-order conditions to the utility maximization problem for representative consumer j (in particular, $r - U_{jK} = 0$ and $w - W_{jF} = 0$).

Assuming, $\lambda_t = 0$, total differentiation of final good supply and capital and fossil fuel demanded for the production of the final good for all t is now:

$$\begin{aligned} \frac{dk_t}{d\bar{e}_j} &= k_{tr}\frac{dr}{d\bar{e}_j} + k_{tw}\frac{dw}{d\bar{e}_j} + k_{tz}\frac{dz}{d\bar{e}_j}, \text{ and} \\ \frac{df_t}{d\bar{e}_j} &= f_{tr}\frac{dr}{d\bar{e}_j} + f_{tw}\frac{dw}{d\bar{e}_j} + f_{tz}\frac{dz}{d\bar{e}_j}, \end{aligned} \tag{A.117}$$

where,

$$
\begin{aligned}
k_{tr} &= \left(\frac{-h_{tff}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}}\right) + \left(\frac{1}{zm_{tkk}}\right),\\
k_{tw} &= \left(\frac{h_{tkf}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}}\right),\\
k_{tz} &= \left(\frac{\alpha_t h_{tkf}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}}\right) - \left(\frac{m_{tk}}{zm_{tkk}}\right),\\
f_{tr} &= \left(\frac{h_{tfk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}}\right),\\
f_{tw} &= \left(\frac{-h_{tkk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}}\right), \text{ and}\\
f_{tz} &= \left(\frac{-\alpha_t h_{tkk}}{h_{tkf}h_{tfk} - h_{tkk}h_{tff}}\right). \qquad \text{(A.118)}
\end{aligned}
$$

In addition, total differentiation of abatement for all t, provides:

$$\frac{da_t}{d\bar{e}_j} = a_{tr}\frac{dr}{d\bar{e}_j} + a_{tz}\frac{dz}{d\bar{e}_j}, \qquad \text{(A.119)}$$

where,

$$
\begin{aligned}
a_{tr} &= \left(\frac{m_{tk}}{zm_{tkk}}\right), \text{ and}\\
a_{tz} &= -\left(\frac{m_{tk}^2}{zm_{tkk}}\right). \qquad \text{(A.120)}
\end{aligned}
$$

Total differentiation of the allowances supplied/demanded provides:

$$
\begin{aligned}
\frac{dy_t}{d\bar{e}_j} &= \alpha_t\frac{df_t}{d\bar{e}_j} - \frac{da_t}{d\bar{e}_j}, \text{ for } t \neq j, \text{ and}\\
\frac{dy_j}{d\bar{e}_j} &= \alpha_j\frac{df_j}{d\bar{e}_j} - \frac{da_j}{d\bar{e}_j} - 1. \qquad \text{(A.121)}
\end{aligned}
$$

Substituting allowances demanded/supplied from (A.121) given (A.117) and (A.119) into the last line of (A.94) provides:

$$
\begin{aligned}
\sum_{t=1}^{J}\left(\left(\alpha_t f_{t,r} - a_{tr}\right)\frac{dr}{d\bar{e}_j} + \alpha_t f_{tw}\frac{dw}{d\bar{e}_j} + \left(\alpha_t f_{tz} - a_{tz}\right)\frac{dz}{d\bar{e}_j}\right) - 1 = 0 \Leftrightarrow&\\
\frac{dz}{d\bar{e}_j} = z_{j,r}\frac{dr}{d\bar{e}_j} + z_{jw}\frac{dw}{d\bar{e}_j} + z_{j,0},& \qquad \text{(A.122)}
\end{aligned}
$$

where:

$$
\begin{aligned}
z_{j,r} &= -\frac{\sum_{t=1}^{J}\left(\alpha_t f_{t,r} - a_{tr}\right)}{\sum_{t=1}^{J}\left(\alpha_t f_{tz} - a_{tz}\right)},\\
z_{jw} &= -\frac{\sum_{t=1}^{J}\alpha_t f_{tw}}{\sum_{t=1}^{J}\left(\alpha_t f_{tz} - a_{tz}\right)}, \text{ and}\\
z_{j,0} &= \frac{1}{\sum_{t=1}^{J}\left(\alpha_t f_{tz} - a_{tz}\right)}.
\end{aligned} \tag{A.123}
$$

We proceed as before to obtain $\bar{r}_j$, $\bar{w}_j$, and $\bar{z}_j$. Substituting $\bar{r}_j$, $\bar{w}_j$, and $\bar{z}_j$ into (A.119) provides:

$$
\frac{da_t}{d\bar{e}_j} = a_{tr}\bar{r}_j + a_{tz}\bar{z}_j = \bar{a}_t. \tag{A.124}
$$

Total differentiation of (A.114) provides for all t:

$$
\frac{de_t}{d\bar{e}_j} = (1-\beta)\left(\alpha_t\frac{df_t}{d\bar{e}_j} - \frac{da_t}{d\bar{e}_j}\right) + \beta\sum_{l=1}^{J}\left(\alpha_l\frac{df_l}{d\bar{e}_j} - \frac{da_l}{d\bar{e}_j}\right). \tag{A.125}
$$

Substituting (A.124) into (A.125) provides:

$$
\frac{de_t}{d\bar{e}_j} = (1-\beta)\left(\alpha_t\bar{f}_t - \bar{a}_t\right) + \beta\sum_{l=1}^{J}\left(\alpha_l\bar{f}_l - \bar{a}_l\right). \tag{A.126}
$$

Finally, the LaGrange multiplier on the conditionally optimal cap can be written as:

$$
\alpha_j\mu_j\left(\boldsymbol{\theta}_{\sim j}\right) = \alpha_j\tau_j^{E,O} + \alpha_j\tau_j^{E,S} + \alpha_j\tau_j^{E,X} + \alpha_j\mu_j^{T,F} + \alpha_j\mu_j^{T,K} + \alpha_j\mu_j^{tz},
$$

where:

$$
\begin{aligned}
\alpha_j\mu_j^{E,O} &= \alpha_j\left(\gamma_j^j\phi_{je} + \beta\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\right)\left(\alpha_j\bar{f}_j - \bar{a}_j\right),\\
\alpha_j\mu_j^{E,S} &= \alpha_j\gamma_j^j\phi_{je}\beta\sum_{l\neq j}^{J}\left(\alpha_l\bar{f}_l - \bar{a}_l\right),\\
\alpha_j\mu_j^{E,X} &= \alpha_j\sum_{t\neq j}^{J}\gamma_j^t\phi_{te}\left((1-\beta)\left(\alpha_t\bar{f}_t - \bar{a}_t\right) + \beta\sum_{l\neq j}^{J}\left(\alpha_l\bar{f}_l - \bar{a}_l\right)\right),\\
\alpha_j\mu_j^{T,F} &= \left(f_j - F_j\right)\alpha_j\bar{w}_j,\\
\alpha_j\mu_j^{T,K} &= \left(k_j - K_j\right)\alpha_j\bar{r}_j, \text{ and}\\
\alpha_j\mu_j^{tz} &= \left(\alpha_j f_j - a_j - \bar{e}_j\right)\alpha_j\bar{z}_j.
\end{aligned} \tag{A.127}
$$

Pareto Optimal Allocation

We next characterize for the Pareto optimal allocation for the case of two mobile factors and end-of-pipe abatement. Maximization of the utilitarian social welfare function is now given by:

$$\begin{aligned}
\max_{\left\{f_j, k_j^X, k_j^A, F_j, K_j\right\}_{j=1}^J} \quad & \sum_{j=1}^J U_j\left(\bar{K}_j - K_j\right) + \sum_{j=1}^J W_j\left(\bar{F}_j - F_j\right) + \sum_{j=1}^J h_j\left(k_j^X, f_j\right) \\
& - \sum_{j=1}^J \phi_j\left(\left(\alpha_j f_j - m_j\left(k_j^A\right)\right) + \beta \sum_{l \neq j}^J \left(\alpha_l f_l - m_l\left(k_l^A\right)\right)\right) \\
& \text{subject to:} \\
& \sum_{j=1}^J f_j \leq \sum_{j=1}^J F_j \ \left(\mu_F\right), \text{ and} \\
& \sum_{j=1}^J \left(k_j^X + k_j^A\right) \leq \sum_{j=1}^J K_j \ \left(\mu_K\right).
\end{aligned} \tag{A.128}$$

The first-order conditions to (A.128) are given by:

$$\begin{aligned}
h_{jf} - \alpha_j\left(\phi_{je} + \beta \sum_{l \neq j}^J \phi_{le}\right) &= \mu_F, \text{ for all } j = 1, ..., J, \\
h_{jk} &= \mu_K, \text{ for all } j = 1, ..., J, \\
\left(\phi_{je} + \beta \sum_{l \neq j}^J \phi_{le}\right) & m_{jk}\mu_K, \text{ for all } j = 1, ..., J, \\
\mu_F &= W_j', \text{ for all } j = 1, ..., J, \text{ and} \\
\mu_K &= U_j', \text{ for all } j = 1, ..., J.
\end{aligned} \tag{A.129}$$

The firm's profit maximization problem with two mobile factors, end-of-pipe abatement and an emissions tax is given by:

$$\begin{aligned}
\max_{f_j \geq 0, k_j^X \geq 0, k_j^A \geq 0} \quad & h_j\left(f_j, k_j^X\right) - w f_j - \tau_j\left(\alpha_j f_j - m_j\left(k_j^A\right)\right) - r\left(k_j^X + k_j^A\right) \\
& \text{subject to:} \\
& m_j\left(k_j^A\right) \leq \alpha_j f_j \ \left(\mu_j\right).
\end{aligned} \tag{A.130}$$

The first order conditions to (A.130) are given by:

$$\begin{aligned}
h_{jf} &= w + \alpha_j \tau_j - \alpha_j \mu_j, \\
h_{jk} &= r, \text{ and} \\
\tau_j m_{jk} &\leq r + m_{jk}\mu_j \ (= \text{ when } k_j^A > 0).
\end{aligned} \tag{A.131}$$

The unconditional demand for fossil fuels, capital, abatement are given by $f_j(w,r,\tau_j)$, $k_j(w,r,\tau_j) = k_j^X(w,r,\tau_j) + k_j^A(r,\tau_j)$, and $a_j(r,\tau_j) = m_j\left(k_j^A(r,\tau_j)\right)$, respectively. We also obtain the supply of the final good, $X_j(w,r,\tau_j)$, and profits, $\pi_j(w,r,\tau_j) = h_j\left(f_j(w,r,\tau_j), k_j^X(w,r,\tau_j)\right) - wf_j(w,r,\tau_j) - \tau_j(\alpha_j f_j(w,r,\tau_j) - a_j(r,\tau_j)) - rk_j(w,r,\tau_j)$. Finally, the emissions released by the jurisdiction now equal: $\alpha_j f_j(w,r,\tau_j) - a_j(r,\tau_j)$.

Again, it is possible to identify the vector of taxes that maximize the Utilitarian social welfare function conditional on the resulting economic equilibrium. This problem is given by:

$$\begin{aligned}
\max_{\{\tau_j\}_{j=1}^J, w, r} \quad & \sum_{j=1}^J U_j\left(\bar{K}_j - K_j(r)\right) + \sum_{j=1}^J W_j\left(\bar{F}_j - F_j(w)\right) \\
& + \sum_{j=1}^J h_j\left(f_j(w,r,\tau_j), k_j^X(w,r,\tau_j)\right) \\
& - \sum_{j=1}^J \phi_j\left(\alpha_j f_j(w,r,\tau_j) - a_j(r,\tau_j)\right) \\
& - \sum_{j=1}^J \phi_j\left(\beta \sum_{l\neq j}^J \left(\alpha_l f_l(w,r,\tau_l) - a_l(r,\tau_l)\right)\right) \\
& \text{subject to:} \\
& \sum_{j=1}^J f_j(w,r,\tau_j) \leq \sum_{j=1}^J F_j(w) \ \ (\delta_F), \text{ and} \\
& \sum_{j=1}^J \left(k_j^X(w,r,\tau_j) + k_j^A(r,\tau_j)\right) \leq \sum_{j=1}^J K_j(r) \ \ (\delta_K),
\end{aligned} \tag{A.132}$$

and given the maximization of (A.82) subject to the consumer's private budget constraint and (A.130).

The first-order conditions to (A.132) are given by:

$$\sum_{j=1}^{J}\left[F_{jw}\left(\delta_F - W_j'\right) + f_{jw}\left(h_{jf}-\delta_F\right) + k_{jw}^{X}\left(h_{jk}-\delta_K\right) - k_{jw}^{A}\delta_K\right],$$

$$-\sum_{j=1}^{J}\left[\left(\alpha_j f_{jw} - m_{jk}k_{jw}^{A}\right)\left(\phi_{je} + \beta\sum_{l\neq j}^{J}\phi_{le}\right)\right] = 0,$$

$$\sum_{j=1}^{J}\left[K_{jr}\left(\delta_K - U_j'\right) + f_{jr}\left(h_{jf}-\delta_F\right) + k_{jr}^{X}\left(h_{jk}-\delta_K\right) - k_{jr}^{A}\delta_K\right],$$

$$-\sum_{j=1}^{J}\left[\left(\alpha_j f_{jr} - m_{jk}k_{jr}^{A}\right)\left(\phi_{je} + \beta\sum_{l\neq j}^{J}\phi_{le}\right)\right] = 0, \text{ and}$$

$$\left(h_{jf}-\delta_F\right)f_{j\tau} + \left(h_{jk}-\delta_K\right)k_{j\tau}^{X} - \delta_K k_{j\tau}^{A}$$

$$-\left(\alpha_j f_{j\tau} - m_{jk}k_{j\tau}^{A}\right)\left(\phi_{je} + \beta\sum_{l\neq j}^{J}\phi_{le}\right) = 0, \text{ for all } j = 1,...,J. \tag{A.133}$$

As shown above, the vector of first best Pigouvian taxes is again (A.60).

C. Private Adaptation

Next, I extend the model to account for private adaptation. To do so, I modify the emissions damage function. Emissions damages were given by: $\phi\left(\boldsymbol{\gamma}_j\mathbf{e}\right) = \phi\left(\gamma_{1,j}e_1,...,\gamma_{J,j}e_J\right)$. Emissions damages are now instead: $\phi_j\left(\boldsymbol{\gamma}_j\mathbf{e},\psi_j\left(a_j\right)\right) = \phi\left(\gamma_{1,j}e_1,...,\gamma_{j-1,j}e_{j-1},\gamma_j^j e_j - \psi_j\left(a_j\right),\gamma_{j+1,j}e_{j+1}...,\gamma_{J,j}e_J\right)$, where $\psi_{ja}>0$ and $\psi_{jaa}<0$ and a_j is the amount of private adaptation chosen by the consumer in j. Local marginal damages are still ϕ_{te} and global marginal damages with respect to emissions are still $\sum_{t=1}^{J}\phi_{te}$. We also still have jurisdiction j's weighted global marginal damages, $\sum_{t=1}^{J}\gamma_j^t\phi_{te}$. Now, the private marginal benefit to jurisdiction j from adaptation is $\phi_{te}\psi_{ja}$.

A representative consumer in each district has preferences as in (2) and (3), with emissions damages as modified above. We assume that the consumer only internalizes local damages (from their perspective $\gamma_j^t = 0$ for all $t \neq j$). The consumer solves, taking g_j and e_j as fixed:

$$\max_{F_j\geq 0, a_j\geq 0} \quad W_j\left(\bar{F}_j - F_j\right) + I_j + wF_j - pa_j + V_j\left(g_j\right) - \phi_j\left(e_j - \psi_j\left(a_j\right)\right). \tag{A.134}$$

where p is the price of adaptation, I_j is the consumer's non-fossil fuel income, given that the consumer's private budget constraint is: $x_j = I_j + wF_j$.

The first-order conditions to (A.134) are given by:

$$w = W_{jF}, \text{ and}$$

$$p = \phi_{te}\psi_{ja}. \tag{A.135}$$

which provides the supply function for fossil fuels, $F_j\left(w\right)$, the amount of the private final good demanded, $x_j(p,w)$, and the amount of the private adaptation demanded by jurisdiction j, $a_j(p)$.

The final good is again produced following (A.154). In the case of a cap with permit trading, the firm instead solves (8) relabeling f_j with f_j^X. Again suppose that the local public good is produced linearly from the numeraire.

A private adaptation supplier in each jurisdiction maximizes profits according to:

$$\max_{f_j^A \geq 0} \quad pb_j\left(f_j^A\right) - wf_j^A. \tag{A.136}$$

The first-order condition to (A.136) is given by:

$$pb_{jf} = w, \tag{A.137}$$

the solution to which yields the demand for fossil fuels for the global public good, $f_j^A(p,w)$, the supply of adaptation by the firm in j, $A_j(p,w)$, and the profits to the firm in j, $\pi_j^A(p,w)$. A *competitive equilibrium before cap competition* is the search for the price vector (p,w) and a vector of government expenditures on local public goods, $\{g_i\}_{i=1}^J$, such that: $\sum_{t=1}^J h_t\left(f_t^X\right) = \sum_{t=1}^J (x_t + g_t)$, $\sum_{t=1}^J b_t\left(f_t^A\right) = \sum_{t=1}^J a_t$, and $\sum_{t=1} F_t = \sum_{t=1}^J \left(f_t^X + f_t^A\right)$, given that $I_j = \pi_j^X + \pi_j^A - T_j$ and $g_j = T_j$ for all $j = 1,...,J$, and the resulting quantities that solve (A.154), (A.134), and (A.136). A *competitive equilibrium under cap competition with trading* is the search for the price vector (p,w,z) and a vector of government expenditures on local public goods, $\{g_i\}_{i=1}^J$, such that: $\sum_{t=1}^J h_t\left(f_t^X\right) = \sum_{t=1}^J (x_t + g_t)$, $\sum_{t=1}^J b_t\left(f_t^A\right) = \sum_{t=1}^J a_t$, $\sum_{t=1} F_t = \sum_{t=1}^J \left(f_t^X + f_t^A\right)$, and $\sum_{t=1}^J y_t = 0$, given that $I_j = \pi_j^X + \pi_j^A - T_j$ and $g_j = T_j$ for all $j = 1,...,J$, and the resulting quantities that solve (8), (A.134), and (A.136).

Under cap competition with permit trading each j government solves, after imposing the j government's balanced budget constraint, $g_j = T_j$:

$$\max_{g_j \geq 0, \bar{e}_j} W_j\left(\bar{F}_j - F_j\right) + I_j - pa_j - g_j + V_j(g_j) - \phi_j\left(\boldsymbol{\gamma}_j \mathbf{e}, \psi_j(a_j)\right), \tag{A.138}$$

where $I_j = \pi_j^X + \pi_j^A + wF_j$ is now non-tax income.

The first order conditions to (A.138) are given by:

$$\begin{aligned} & V_{jg} = 1, \text{ and} \\ & \mu_j = \sum_{t=1}^J \gamma_j^t \phi_{te} \frac{de_t^{\mu_j}}{d\bar{e}_j} + \left(f_j^X + f_j^A - F_j\right)\left(\frac{dw^{\mu_j}}{d\bar{e}_j}\right) + \left(\alpha_j f_j^X - \bar{e}_j\right)\left(\frac{dz^{\mu_j}}{d\bar{e}_j}\right) \\ & \quad + (a_j - A_j)\left(\frac{dp^{\mu_j}}{d\bar{e}_j}\right), \end{aligned} \tag{A.139}$$

given:

$$\frac{dI_j}{d\bar{e}_j} = \frac{d\pi_j^{X,\mu_j}}{d\bar{e}_j} + \frac{d\pi_j^{A,\mu_j}}{d\bar{e}_j} + w\frac{dF_j^{\mu_j}}{d\bar{e}_j} + F_j\frac{dw^{\mu_j}}{d\bar{e}_j},$$

$$\frac{d\pi_j^{X,\mu_j}}{d\bar{e}_j} = z - f_j^X\frac{dw^{\mu_j}}{d\bar{e}_j} + \left(\bar{e}_j - \alpha_j f_j^X\right)\frac{dz^{\mu_j}}{d\bar{e}_j},$$

$$\frac{d\pi_j^{A,\mu_j}}{d\bar{e}_j} = A_j\frac{dp^{\mu_j}}{d\bar{e}_j} - f_j^A\frac{dw^{\mu_j}}{d\bar{e}_j}, \tag{A.140}$$

(9), (A.135), and (A.137). Again, one can obtain closed form solutions for $\frac{dw^{\mu_j}}{d\bar{e}_j}$, $\frac{dz^{\mu_j}}{d\bar{e}_j}$, and $\frac{dp^{\mu_j}}{d\bar{e}_j}$ by totally differentiating the first-order conditions and the appropriate market clearing conditions. Since w^{μ_j}, z^{μ_j}, and p^{μ_j} are determined by national markets, it is again the case that the total derivatives do not vary across all j. Moreover, total differentiation of the permit market clearing condition, implies $\frac{de_t^{\mu_j}}{d\bar{e}_j} = 1$ for all t when $\beta = 1$. Summing (A.139) across all j, and imposing the market clearing conditions for fossil fuels, permits, and private adaptation and the second first order condition from (9), we have $\mu_j = z = \left(\frac{1}{J}\right)\sum_{j=1}^{J}\sum_{t=1}^{J}\gamma_j^t\phi_{te}$ for all j. Together with the first-order conditions this supports the Pareto optimal allocation that solves (A.141) when $\beta = 1$ and all jurisdictions internalize global damages.

Pareto Optimal Allocation

Again given that $\boldsymbol{\gamma}_j$ equals the basis vector for all j, the Pareto optimal allocation solves:

$$\max_{a_j,g_j,F_j,f_j^X,f_j^A,x_j\geq 0} \quad W_1\left(\bar{F}_1 - F_1\right) + x_1 + V_1\left(g_1\right) - \phi_1\left(e_1 - \psi_1\left(a_1\right)\right)$$

subject to:

$$W_j\left(\bar{F}_j - F_j\right) + x_j + V_j\left(g_j\right) - \phi_j\left(e_j - \psi_j\left(a_j\right)\right) \geq \bar{u}_j \ \left(\lambda_j\right), \text{ for all } j > 1,$$

$$\sum_{t=1}^{J} a_t = \sum_{t=1}^{J} b_t\left(f_t^A\right) \ (\rho),$$

$$\sum_{t=1}^{J}\left(x_t + g_t\right) = \sum_{t=1}^{J} h_t\left(f_t^X\right) \ (\delta),$$

$$\sum_{t=1}^{J}\left(f_t^X + f_t^A\right) = \sum_{t=1}^{J} F_t \ (\omega),$$

$$e_j = \alpha_j f_j^X + \beta\sum_{l\neq j}\alpha_l f_l^X, \text{ for all } j. \tag{A.141}$$

The first-order conditions to (A.141) imply, after some manipulation:

$$W_{jF} = h_{jf} - \alpha_j \left(\phi_{je} + \beta \left(\sum_{t \neq j}^{J} \phi_{te} \right) \right)$$
$$= \phi_{je} \psi_{ja} b_{jf} = \omega, \text{ for all } j = 1, ..., J, \text{ and}$$
$$V_{jg} = 1, \text{ for all } j = 1, ..., J. \tag{A.142}$$

D. Preferences

Suppose instead that preferences are given by $u_j \left(\bar{F}_j - F_j, x_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right)$ with $u_{jF} > 0$, $u_{jFF} < 0$, $u_{jx} > 0$, $u_{jg} > 0$, and $u_{jgg} < 0$. Under cap competition with permit trading the consumer now solves, taking g_j, $\mathbf{e}$, and I_j as fixed:

$$\max_{F_j \geq 0} \quad u_j \left(\bar{F}_j - F_j, I_j + wF_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right). \tag{A.143}$$

given that the consumer's private budget constraint is: $x_j = I_j + wF_j$ and where $I_j = \pi_j - T_j$ is the consumer's non fossil fuel income.

The first-order condition to (A.143) is given by:

$$w = \left(\frac{u_{jF}}{u_{jx}} \right), \tag{A.144}$$

which provides the supply function for fossil fuels, $F_j \left(w, I_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right)$ and the amount of the private final good demanded, $x_j \left(w, I_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right)$. Moreover, the consumer's indirect utility function can be written as: $U_j \left(w, I_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right) = u_j \left(\bar{F}_j - F_j \left(w, I_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right), x_j \left(w, I_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right), g_j, \boldsymbol{\gamma}_j \mathbf{e} \right)$, where $U_{jF} = -u_{jF}$, $U_{jx} = u_{jx}$, $U_{jg} = u_{jg}$, and $U_{je_t} = \gamma_j^t u_{je_t}$ for all $t = 1, ..., J$.

Under cap competition with permit trading the numeraire is still produced following (8), which yields profits equal to $\pi_j = \pi_j \left(w, z, \bar{e}_j \right)$. Moreover, the government's balanced budget constraint is given by $g_j = T_j$. Thus, non-fossil fuel income equals $I_j = I_j \left(w, z, \bar{e}_j, g_j \right) = \pi_j \left(w, z, \bar{e}_j \right) - g_j$.

Under cap competition with permit trading each j government solves:

$$\max_{g_j \geq 0, \bar{e}_j} U_j \left(w, I_j, g_j, \boldsymbol{\gamma}_j \mathbf{e} \right), \tag{A.145}$$

given $I_j = I_j \left(w, z, \bar{e}_j, g_j \right)$ and given (10) with permit trading.

The first order conditions to (A.145) are given by:

$$\left(\frac{u_{jg}}{u_{jx}} \right) = 1, \text{ and}$$
$$\mu_j = - \sum_{t=1}^{J} \gamma_j^t \left(\frac{u_{je_t}}{u_{jx}} \right) \left(\frac{de_t^{\mu_j}}{d\bar{e}_j} \right) + \left(f_j - F_j \right) \left(\frac{dw^{\mu_j}}{d\bar{e}_j} \right) + \left(\alpha_j f_j - \bar{e}_j \right) \left(\frac{dz^{\mu_j}}{d\bar{e}_j} \right), \tag{A.146}$$

given:

$$\frac{dI_j}{dg_j} = -1,$$
$$\frac{dI_j}{d\bar{e}_j} = \frac{d\pi_j^{\mu_j}}{d\bar{e}_j} + w\frac{dF_j^{\mu_j}}{d\bar{e}_j} + F_j\frac{dw^{\mu_j}}{d\bar{e}_j},$$
$$\frac{d\pi_j^{\mu_j}}{d\bar{e}_j} = z - f_j\frac{dw^{\mu_j}}{d\bar{e}_j} + (\bar{e}_j - \alpha_j f_j)\frac{dz^{\mu_j}}{d\bar{e}_j}, \tag{A.147}$$

(9), and (A.144). Again, one can obtain closed form solutions for $\frac{dw^{\mu_j}}{d\bar{e}_j}$ and $\frac{dz^{\mu_j}}{d\bar{e}_j}$, by totally differentiating the first-order conditions and the appropriate market clearing conditions. Since both w^{μ_j} and z^{μ_j} are determined by national markets, it is again the case that the total derivatives do not vary across all j. Moreover, total differentiation of the permit market clearing condition when $\beta = 1$, implies $\frac{de_t^{\mu_j}}{d\bar{e}_j} = 1$ for all t. Summing (A.146) across all j, and imposing the market clearing conditions for fossil fuels and permits and the second first order condition from (9), we have $\mu_j = z = -\left(\frac{1}{J}\right)\sum_{j=1}^{J}\sum_{t=1}^{J}\gamma_j^t\left(\frac{u_{je_t}}{u_{jx}}\right)$ for all j. We say that jurisdictions exhibit *consistency in their assessment of marginal damages*, when: $\left(\frac{u_{te_t}}{u_{tx}}\right) = \left(\frac{1}{J}\right)\sum_{l=1}^{J}\left(\frac{u_{le_t}}{u_{lx}}\right)$ for all $t = 1, ..., J$. Consistency requires that jurisdiction t's marginal damages from a unit of emissions realized in their jurisdiction should equal the average of all jurisdictions marginal damage assessments for a unit of emissions realized in jurisdiction t. Under this assumption and the assumption that all jurisdictions internalize global damages, this expression together with the first-order conditions supports the Pareto optimal allocation that solves (A.148) when $\beta = 1$.

Pareto Optimal Allocation

Again given that $\boldsymbol{\gamma}_j$ equals the basis vector for all j, the Pareto optimal allocation solves:

$$\max_{a_j, g_j, F_j, f_j^X, f_j^A, x_j \geq 0} \quad u_1\left(\bar{F}_1 - F_1, x_1, g_1, e_1\right)$$

subject to:

$$u_j\left(\bar{F}_j - F_j, x_j, g_j, e_j\right) \geq \bar{u}_j \ (\lambda_j), \text{ for all } j > 1,$$
$$\sum_{t=1}^{J}(x_t + g_t) = \sum_{t=1}^{J} h_t(f_t) \ (\delta),$$
$$\sum_{t=1}^{J} f_t = \sum_{t=1}^{J} F_t \ (\omega),$$
$$e_j = \alpha_j f_j + \beta\sum_{l \neq j}\alpha_l f_l, \text{ for all } j. \tag{A.148}$$

The first-order conditions to (A.148) imply, after some manipulation:

$$\left(\frac{u_{1F}}{u_{1x}}\right) = h_{jf} + \alpha_j \left(\left(\frac{u_{je_j}}{u_{jx}}\right) + \beta \left(\sum_{t \neq j}^{J} \left(\frac{u_{te_j}}{u_{tx}}\right) \right) \right), \text{ for all } j = 1, ..., J, \text{ and}$$

$$\left(\frac{u_{jg}}{u_{jx}}\right) = 1, \text{ for all } j = 1, ..., J. \tag{A.149}$$

The vector of taxes that supports this Pareto optimal allocation is now: $\tau_j^{PO} = \left(\frac{u_{je_j}}{u_{jx}}\right) + \beta \left(\sum_{t \neq j}^{J} \left(\frac{u_{te_j}}{u_{tx}}\right)\right)$ for all $j = 1, ..., J$.

E. Global Public Goods

Instead of an externality with spillovers, consider instead an alternative version of the model in which a public good is present that is non-rival and non-excludable across all jurisdictions, $e = \sum_{t=1}^{J} e_t$, where e_t is the amount of this 'global' public good demanded in jurisdiction t. Now define each consumer's preferences as:

$$u_j = W_j \left(\bar{F}_j - F_j\right) + x_j + v_j \left(g_j, e\right), \tag{A.150}$$

where $v_{jg} > 0$, $v_{je} > 0$, $W_{iF} > 0$, and $W_{iFF} < 0$. We assume the following preferences for the local public good and the global public good:

$$v_j \left(g_j, e\right) = V_j \left(g_j\right) + \sum_{t=1}^{J} \gamma_j^t \phi_t \left(e\right), \tag{A.151}$$

where $V_{jg} > 0$, $V_{jgg} < 0$, $\phi_{te} > 0$ and $\phi_{tee} < 0$.

Note that we still retain the possibility for a local public good, g_j, as before. Define I_j as the consumer's non-fossil fuel income in j, such that the consumer's private budget constraint is: $x_j = I_j + wF_j$.

Consider first the case in which the consumer does not select either the local or global public goods. The consumer in j solves the following utility maximization problem, taking g_j and e as fixed:

$$\max_{F_j \geq 0} \quad W_j \left(\bar{F}_j - F_j\right) + I_j + wF_j + V_j \left(g_j\right) + \sum_{t=1}^{J} \gamma_j^t \phi_t \left(e\right). \tag{A.152}$$

The first-order condition to (A.152) is given by:

$$w = W_{jF}, \tag{A.153}$$

which provides the supply function for fossil fuels, $F_j \left(w\right)$, and the amount of the private final good demanded by jurisdiction j, $x_j \left(w\right)$.

The final good is produced by a representative producer who maximizes profits, taking all prices as exogenous, according to:

$$\max_{f_j^X \geq 0} \quad h_j\left(f_j^X\right) - w f_j^X. \tag{A.154}$$

The first-order condition to (A.154) is given by:

$$h_{jf} = w. \tag{A.155}$$

(A.155) yields the unconditional demand for fossil fuels $f_j^X(w)$. We also obtain the supply of the final good, $X_j(w)$, and profits, $\pi_j(w)$. Again suppose that the local public good is produced linearly from the numeraire.

Now allow the global public good to be produced according to $E_j = b_j\left(f_j^E\right)$, where $b_{jf} > 0$ and $b_{jff} < 0$. In the absence of any policy, since consumers are restricted from demanding any of the global public good, there is no market for the global public good and so the amount of public good supplied by all jurisdictions is zero, as is f_j^E. A *competitive equilibrium prior to global public good policies* is the search for the price of fossil fuels w and a vector of government expenditures on local public goods, $\{g_i\}_{i=1}^J$, such that: $\sum_{t=1}^J h_t\left(f_t^X\right) = \sum_{t=1}^J (x_t + g_t)$ and $\sum_{t=1} F_t = \sum_{t=1}^J \left(f_t^X + f_t^E\right)$, given that $I_j = \pi_j^X - T_j$ and $g_j = T_j$ for all $j = 1, ..., J$, and the resulting quantities that solve (A.152) and (A.154).

In the case of a tax for the global public good, τ_j,[2] a global public good producer in each j jurisdiction solves:

$$\max_{f_j^E \geq 0} \quad \tau_j b_j\left(f_j^E\right) - w f_j^E. \tag{A.156}$$

The first-order condition to (A.156) is given by:

$$\tau_j b_{jf} = w, \tag{A.157}$$

the solution to which yields the demand for fossil fuels for the global public good, $f_j^E(w, \tau_j)$, the supply of the global public good by the firm in j, $E_j(w, \tau_j)$, and the profits to the firm in j, $\pi_j^E(w, \tau_j)$.

In the case of a public good mandate, $\bar{E}_j$, with permit trading a global public good producer in each j jurisdiction instead solves:

$$\max_{f_j^E \geq 0, y_j} \quad - w f_j^E - z y_j$$

$$\text{subject to:}$$

$$b_j\left(f_j^E\right) + y_j \geq \bar{E}_j \ (\mu_j), \tag{A.158}$$

where z is the equilibrium price of permits.

[2] As for the externality case, when $\tau_j < 0$ the tax is in fact a subsidy.

The first order conditions to (A.158) are given by:

$$\begin{aligned}
\mu_j b_{jf} &= w, \\
\mu_j &= z, \\
\left(b_j\left(f_j^E\right) + y_j - \bar{E}_j\right)\mu_j &= 0, b_j\left(f_j^E\right) + y_j \geq \bar{E}_j, \mu_j \geq 0,
\end{aligned} \tag{A.159}$$

the solution to which yields the demand for fossil fuels for the global public good, $f_j^E(w,z)$, the supply of the global public good by the firm in j, $E_j(w,z)$, the demand/supply of public good permits, $y_j\left(w,z,\bar{E}_j\right) = \bar{E}_j - E_j(w,z)$, and the profits to the firm in j, $\pi_j^E\left(w,z,\bar{E}_j\right)$.

Under tax competition each j government solves, after imposing the j government's balanced budget constraint in this instance, $g_j = T_j + \tau_j E_j$, and given that $e = \sum_{t=1}^J E_t$:

$$\max_{g_j \geq 0, \tau_j} W_j\left(\bar{F}_j - F_j\right) + I_j + \tau_j E_j - g_j + V_j(g_j) + \sum_{t=1}^J \gamma_j^t \phi_t(e), \tag{A.160}$$

where $I_j = \pi_j^X + \pi_j^E + wF_j$ is now non-tax income.

The first order conditions to (A.160) are given by:

$$\begin{aligned}
&V_{jg} = 1, \text{ and} \\
&\tau_j = -\sum_{t=1}^J \gamma_j^t \phi_{te} - \sum_{t=1}^J \gamma_j^t \phi_{te}\left(\sum_{l \neq j} \frac{dE_l^{\tau_j}}{dE_j}\right) + \left(f_j^X + f_j^E - F_j\right)\left(\frac{dw^{\tau_j}}{dE_j}\right),
\end{aligned} \tag{A.161}$$

given:

$$\begin{aligned}
\frac{dI_j}{dE_j} &= \frac{d\pi_j^{X,\tau_j}}{dE_j} + \frac{d\pi_j^{E,\tau_j}}{dE_j} + w\frac{dF_j^{\tau_j}}{dE_j} + F_j\frac{dw^{\tau_j}}{dE_j}, \\
\frac{d\pi_j^X}{dE_j} &= -f_j^X\frac{dw^{\tau_j}}{dE_j}, \\
\frac{d\pi_j^E}{dE_j} &= -\frac{d\tau_j}{dE_j}E_j - f_j^E\frac{dw^{\tau_j}}{dE_j},
\end{aligned} \tag{A.162}$$

(A.153), (A.155), and (A.157). As before, one can obtain closed form solutions for $\frac{dE_l^{\tau_j}}{dE_j}$ and $\frac{dw^{\tau_j}}{dE_j}$ by totally differentiating the first-order conditions and the market clearing conditions.

The term $\tau_j^{E,O} = -\sum_{t=1}^J \gamma_j^t \phi_{te}$ is analogous to the own emissions Pigouvian correction. When $\gamma_j^t = 1$ for all $t, j = 1, ..., J$, then, if $\tau_j = \tau_j^{E,O}$ for all j, substitution into (A.157) for all j, implies $\sum_{t=1}^J \phi_{te} = \left(\frac{w}{b_{jf}}\right)$, which together with the remaining first-order conditions and the market clearing conditions would support the Pareto optimal allocation given in (A.181). If instead $\gamma_j^j = 1$ for all $j = 1, ..., J$ and $\gamma_j^t = 0$ for all $t \neq j$, then a benefit internalization distortion will be present. If $\tau_j = \tau_j^{E,O}$ for all j then this is analogous to the classic under-provision of public goods result. A second term also emerges that is analogous to the sum of the spillback and other jurisdictions' emissions Pigouvian corrections, $\tau_j^{E,S} + \tau_j^{E,X} = -\sum_{t=1}^{J} \gamma_j^t \phi_{te} \left(\sum_{l \neq j} \frac{dE_l^{\tau_j}}{dE_j} \right)$. Finally, a fossil fuel market terms of trade effect is also present, $\tau_j^T = \left(f_j^X + f_j^E - F_j \right) \left(\frac{dw^{\tau_j}}{dE_j} \right)$.

Under mandate competition with permit trading each j government solves, after imposing the j government's balanced budget constraint in this instance, $g_j = T_j$, market clearing in the global public good, $\sum_{t=1}^{J} \bar{E}_t = \sum_{t=1}^{J} E_t$, and given that $e = \sum_{t=1}^{J} E_t$:

$$\max_{g_j \geq 0, \bar{E}_j} W_j \left(\bar{F}_j - F_j \right) + I_j - g_j + V_j \left(g_j \right) + \sum_{t=1}^{J} \gamma_j^t \phi_t \left(\bar{E}_j + \sum_{l \neq j} \bar{E}_l \right), \tag{A.163}$$

where $I_j = \pi_j^X + \pi_j^E + wF_j$ is now non-tax income.

The first order conditions to (A.163) are given by:

$$\begin{aligned} &V_{jg} = 1, \text{ and} \\ &-\mu_j = -\sum_{t=1}^{J} \gamma_j^t \phi_{te} + \left(f_j^X + f_j^E - F_j \right) \left(\frac{dw^{\mu_j}}{d\bar{E}_j} \right) + \left(\bar{E}_j - E_j \right) \left(\frac{dz^{\mu_j}}{d\bar{E}_j} \right), \end{aligned} \tag{A.164}$$

given:

$$\begin{aligned} \frac{dI_j}{d\bar{E}_j} &= \frac{d\pi_j^{X,\mu_j}}{d\bar{E}_j} + \frac{d\pi_j^{E,\mu_j}}{d\bar{E}_j} + w\frac{dF_j^{\mu_j}}{d\bar{E}_j} + F_j \frac{dw^{\mu_j}}{d\bar{E}_j}, \\ \frac{d\pi_j^X}{d\bar{E}_j} &= -f_j^X \frac{dw^{\mu_j}}{d\bar{E}_j}, \\ \frac{d\pi_j^E}{d\bar{E}_j} &= -z + \left(E_j - \bar{E}_j \right) \frac{dz^{\mu_j}}{d\bar{E}_j} - f_j^E \frac{dw^{\mu_j}}{d\bar{E}_j}, \end{aligned} \tag{A.165}$$

(A.153), (A.155), and (A.158). Again, one can obtain closed form solutions for $\frac{dw^{\mu_j}}{d\bar{E}_j}$ and $\frac{dz^{\mu_j}}{d\bar{E}_j}$ by totally differentiating the first-order conditions and the appropriate market clearing conditions. Since both w^{μ_j} and z^{μ_j} are determined by national markets, it is again the case that the total derivatives do not vary across all j. Therefore summing (A.165) across all j, and imposing the market clearing conditions for fossil fuels and permits, and the second first order condition from (A.159), we have $\mu_j = z = \sum_{t=1}^{J} \gamma_j^t \phi_{te}$ for all j. Therefore, when $\gamma_j^t = 1$ for all $t, j = 1, ..., J$, then substitution into (A.159) for all j, implies $\sum_{t=1}^{J} \phi_{te} = \left(\frac{w}{b_{jf}} \right)$ for all j, which together with the remaining first-order conditions and the market clearing conditions support the Pareto optimal allocation given in (A.181).

The term $\mu_j^{E,O} = -\sum_{t=1}^{J} \gamma_j^t \phi_{te}$ is again analogous to the own emissions Pigouvian correction. The analogous expressions for the spillback and other jurisdictions' emissions Pigouvian corrections are both equal to zero. A fossil fuel market terms of trade effect is again present, $\mu_j^{T,F} = \left(f_j^X + f_j^E - F_j\right)\left(\frac{dw^{\mu_j}}{d\bar{E}_j}\right)$, as is a permit market terms of trade effect, $\mu_j^{tz} = \left(\bar{E}_j - E_j\right)\left(\frac{dz^{\mu_j}}{d\bar{E}_j}\right)$. As for the emissions cap case, the sum of these two terms of trade effects again equals zero under decentralized mandate competition with permit trading so long as all jurisdictions internalize global benefits.

With Local Provision of the Global Public Good

Now, suppose instead that a representative consumer in each jurisdiction also voluntarily selects some level of global public good to consume, e_j. The consumer solves, taking g_j, e_l for all $l \neq j$, and $\bar{E}_l$ for all $l = 1, ..., J$, as fixed:

(A.166)

$$\max_{F_j \geq 0, e_j \geq 0} \quad W_j\left(\bar{F}_j - F_j\right) + I_j + wF_j - pe_j + V_j\left(g_j\right) + \sum_{t=1}^{J} \hat{\gamma}_j^t \phi_t \left(e_j + \sum_{l \neq j} e_l + \sum_{l=1}^{J} \bar{E}_l\right).$$

where p is the private market price of the global public good and $\hat{\gamma}_j^t$ for all $t = 1, ..., J$ are the consumer located in j's benefit internalization weights, which need not equal those used by the government in k, γ_j^t.

The first-order conditions to (A.166) are given by:

$$w = W_{jF}, \text{ and}$$

$$p \geq \sum_{t=1}^{J} \hat{\gamma}_j^t \phi_{te}, = \text{ if } e_j > 0. \tag{A.167}$$

which provides the supply function for fossil fuels, $F_j(w)$, the amount of the private final good demanded, $x_j(p, w)$, and the amount of the global public good demanded by jurisdiction j, $e_j(p)$.

Given that individuals can free-ride off of the consumption choices of other consumers, $\sum_{l \neq j} e_l$, as well as the amount consumed by governments, $\sum_{l=1}^{J} \bar{E}_l$, it will be the case that for some k jurisdiction that $\sum_{t=1}^{J} \hat{\gamma}_k^t \phi_{te} = \text{argmax}\left\{\sum_{t=1}^{J} \hat{\gamma}_j^t \phi_{te}\right\}_{j=1}^{J}$. For this k jurisdiction, define $\hat{e}_k(p)$ as the solution to the equation $p = \sum_{t=1}^{J} \hat{\gamma}_k^t \phi_{te}\left(\hat{e}_k\right)$, where we assume that $\hat{e}_k(0)$ is appropriately bounded. If $\hat{e}_k(p) > \sum_{l=1}^{J} \bar{E}_l$, then the k jurisdiction also demands a positive amount of the public good, $e_k\left(p, \left\{\bar{E}_l\right\}_{l=1}^{J}\right) = \hat{e}_k(p) - \sum_{l=1}^{J} \bar{E}_l > 0$, and otherwise the k jurisdiction demands $e_k(p) = 0$ since the k consumer can free-ride off of the public good demanded by governments. Because all other $j \neq k$ consumers free-ride off the public good demanded by k or demanded by governments, $e_j(p) = 0$ for all $j \neq k$.

Observe that when $e_k(p) > 0$ and $\bar{E}_l = 0$ for all $l = 1, ..., J$, we are likely to observe an instance of an incomplete private market for the public good, unless $\hat{\gamma}_k^t = 1$ for all $t = 1, ..., J$, in which case the k consumer selects an $e_k(p) > 0$ that supports the Pareto optimal allocation in (A.181).

When decentralized governments select non-zero public good mandates there is the possibility that their mandate choices when permits are freely traded can replace an incomplete private market for the public good with a virtual market that is complete. A complete virtual market will be achieved when the vector of mandates selected by governments, $\left\{\bar{E}_l\right\}_{l=1}^{J}$, with free trade in permits supports the Pareto optimal allocation in (A.181), in which case $z > 0$, where z is the permit price relaized in the equilibrium. Governments may also select caps $\sum_{l=1}^{J} \bar{E}_l$ that exceed $\hat{e}_k(p)$, but which fall short of the Pareto optimal allocation. In this case, governments will replace an incomplete private market for the public good with an incomplete (although less so) virtual market for the public good. In either of these two cases, consumers will directly demand no quantity of the public good themselves since they can free-ride off of the consumption mandated by governments, and, therefore $p = 0$ and $z > 0$.

However, governments may also create a virtual market that is more incomplete than that which consumers would be willing to establish if left to their own devices. In this case, $\hat{e}_k(p)$ exceeds $\sum_{l=1}^{J} \bar{E}_l$. The consumer k will then create a second, possibly incomplete real market for the public good, which fully accounts for the public good demanded by the virtual market, hence, $e_k\left(p, \left\{\bar{E}_l\right\}_{l=1}^{J}\right) = \hat{e}_k(p) - \sum_{l=1}^{J} \bar{E}_l > 0$, in this instance. Furthermore, $p > 0$ and $z > 0$.

Prior to any policies and only a real market for the public good exists, a global public good supplier maximizes profits according to:

$$\max_{f_j^E \geq 0} \quad pb_j\left(f_j^E\right) - wf_j^E. \tag{A.168}$$

The first-order condition to (A.168) is given by:

$$pb_{jf} = w, \tag{A.169}$$

the solution to which yields the demand for fossil fuels for the global public good, $f_j^E(p, w)$, the supply of the global public good by the firm in j, $E_j(p, w)$, and the profits to the firm in j, $\pi_j^E(p, w)$. A *competitive equilibrium prior to global public good policies* is the search for the price vector (p, w) and a vector of government expenditures on local public goods, $\{g_i\}_{i=1}^{J}$, such that: $\sum_{t=1}^{J} h_t\left(f_t^X\right) = \sum_{t=1}^{J}(x_t + g_t)$, $\sum_{t=1}^{J} b_t\left(f_t^E\right) = \sum_{t=1}^{J} e_t$, and $\sum_{t=1} F_t = \sum_{t=1}^{J}\left(f_t^X + f_t^E\right)$, given that $I_j = \pi_j^X + \pi_j^E - T_j$ and $g_j = T_j$ for all $j = 1, ..., J$, and the resulting quantities that solve (A.154), (A.166), and (A.168).

In the case of a tax for the global public good, τ_j, a global public good producer in each j jurisdiction solves:

$$\max_{f_j^E \geq 0} \quad (p - \tau_j)\, b_j\left(f_j^E\right) - wf_j^E. \tag{A.170}$$

The first-order condition to (A.170) is given by:

$$(p - \tau_j)\, b_{jf} = w, \tag{A.171}$$

the solution to which yields the demand for fossil fuels for the global public good, $f_j^E(p, w, \tau_j)$, the supply of the global public good by the firm in j, $E_j(p, w, \tau_j)$, and the profits to the firm in j, $\pi_j^E(p, w, \tau_j)$.

In the case of a public good mandate, $\bar{E}_j$, with permit trading we have:

$$\max_{f_j^E \geq 0, y_j} \quad pb_j\left(f_j^E\right) - wf_j^E - zy_j$$

$$\text{subject to:}$$

$$b_j\left(f_j^E\right) + y_j \geq \bar{E}_j \ (\mu_j)\,, \tag{A.172}$$

where z is the equilibrium price of permits.

The first order conditions to (A.172) are given by:

$$\begin{aligned}
(p+\mu_j)\, b_{jf} &= w,\\
\mu_j &= z,\\
\left(b_j\left(f_j^E\right) + y_j - \bar{E}_j\right)\mu_j &= 0, b_j\left(f_j^E\right) + y_j \geq \bar{E}_j, \mu_j \geq 0,
\end{aligned} \tag{A.173}$$

the solution to which yields the demand for fossil fuels for the global public good, $f_j^E\,(p,w,z)$, the supply of the global public good by the firm in j, $E_j\,(p,w,z)$, the demand/supply of public good permits, $y_j\left(p,w,z,\bar{E}_j\right) = \bar{E}_j - E_j\,(p,w,z)$, and the profits to the firm in j, $\pi_j^E\left(p,w,z,\bar{E}_j\right)$.

Under tax competition each j government solves, after imposing the j government's balanced budget constraint in this instance, $g_j = T_j + \tau_j E_j$:

$$\max_{g_j \geq 0, \tau_j} W_j\left(\bar{F}_j - F_j\right) + I_j - pe_j + \tau_j E_j - g_j + V_j\,(g_j) + \sum_{t=1}^{J} \gamma_j^t \phi_t\,(e)\,, \tag{A.174}$$

where $I_j = \pi_j^X + \pi_j^E + wF_j$ is now non-tax income and $e \equiv \sum_{t=1}^J e_t = \sum_{t=1}^J E_t$.

The first order conditions to (A.174) are given by:

$$\begin{aligned}
&V_{jg} = 1, \text{ and}\\
&\tau_j = -\sum_{t=1}^{J} \gamma_j^t \phi_{te}\left(\frac{de^{\tau_j}}{dE_j}\right) + \left(\sum_{t=1}^{J} \hat{\gamma}_k^t \phi_{te}\right)\left(\frac{de_j^{\tau_j}}{dE_j}\right)\\
&\quad + \left(f_j^X + f_j^E - F_j\right)\left(\frac{dw^{\tau_j}}{dE_j}\right) + (e_j - E_j)\left(\frac{dp^{\tau_j}}{dE_j}\right),
\end{aligned} \tag{A.175}$$

given:

$$\begin{aligned}
\frac{dI_j}{dE_j} &= \frac{d\pi_j^{X,\tau_j}}{dE_j} + \frac{d\pi_j^{E,\tau_j}}{dE_j} + w\frac{dF_j^{\tau_j}}{dE_j} + F_j\frac{dw^{\tau_j}}{dE_j},\\
\frac{d\pi_j^X}{dE_j} &= -f_j^X \frac{dw^{\tau_j}}{dE_j},\\
\frac{d\pi_j^E}{dE_j} &= \left(\frac{dp}{dE_j} - \frac{d\tau_j}{dE_j}\right) E_j - f_j^E \frac{dw^{\tau_j}}{dE_j},
\end{aligned} \tag{A.176}$$

(A.155), (A.167), and (A.171). As before, one can obtain closed form solutions for $\frac{dE_l^{\tau_j}}{dE_j}$ and $\frac{dw^{\tau_j}}{dE_j}$ by totally differentiating the first-order conditions and the market clearing conditions.

Under mandate competition with permit trading each j government solves, after imposing the j government's balanced budget constraint in this instance, $g_j = T_j$, and the permit market clearing condition with private provision of the global public good, $\sum_{t=1}^{J} y_t + \sum_{t=1}^{J} e_t = 0$ (and the resulting equilibrium price of permits (z):

$$\max_{g_j \geq 0, \bar{E}_j} W_j\left(\bar{F}_j - F_j\right) + I_j - pe_j - g_j + V_j\left(g_j\right) + \sum_{t=1}^{J} \gamma_j^t \phi_t\left(e\right), \tag{A.177}$$

where $I_j = \pi_j^X + \pi_j^E + wF_j$ is now non-tax income, and given that $\sum_{t=1}^{J} y_t + \sum_{t=1}^{J} e_t = 0 \Leftrightarrow e \equiv \sum_{t=1}^{J} e_t + \sum_{t=1}^{J} \bar{E}_t = \sum_{t=1}^{J} E_t$.

The first order conditions to (A.177) are given by:

$$\begin{aligned}
&V_{jg} = 1, \text{ and} \\
&-\mu_j = -\sum_{t=1}^{J} \gamma_j^t \phi_{te} + \left(\sum_{t=1}^{J} \hat{\gamma}_k^t \phi_{te}\right)\left(\frac{de_j^{\mu_j}}{d\bar{E}_j}\right) + \left(f_j^X + f_j^E - F_j\right)\left(\frac{dw^{\mu_j}}{d\bar{E}_j}\right) \\
&\quad + \left(\bar{E}_j - E_j\right)\left(\frac{dz^{\mu_j}}{d\bar{E}_j}\right) + \left(e_j - E_j\right)\left(\frac{dp^{\mu_j}}{d\bar{E}_j}\right) \text{ when } p^{\mu_j} > 0, \text{ or,} \\
&-\mu_j = -\sum_{t=1}^{J} \gamma_j^t \phi_{te} + \left(f_j^X + f_j^E - F_j\right)\left(\frac{dw^{\mu_j}}{d\bar{E}_j}\right) \\
&\quad + \left(\bar{E}_j - E_j\right)\left(\frac{dz^{\mu_j}}{d\bar{E}_j}\right) \text{ when } p^{\mu_j} = 0,
\end{aligned} \tag{A.178}$$

given:

$$\begin{aligned}
\frac{dI_j}{d\bar{E}_j} &= \frac{d\pi_j^{X,\mu_j}}{d\bar{E}_j} + \frac{d\pi_j^{E,\mu_j}}{d\bar{E}_j} + w\frac{dF_j^{\mu_j}}{d\bar{E}_j} + F_j\frac{dw^{\mu_j}}{d\bar{E}_j}, \\
\frac{d\pi_j^X}{d\bar{E}_j} &= -f_j^X\frac{dw^{\mu_j}}{d\bar{E}_j}, \\
\frac{d\pi_j^E}{d\bar{E}_j} &= -z + E_j\frac{dp^{\mu_j}}{d\bar{E}_j} + \left(E_j - \bar{E}_j\right)\frac{dz^{\mu_j}}{d\bar{E}_j} - f_j^E\frac{dw^{\mu_j}}{d\bar{E}_j} \text{ when } p^{\mu_j} > 0, \text{ or} \\
\frac{d\pi_j^E}{d\bar{E}_j} &= -z + \left(E_j - \bar{E}_j\right)\frac{dz^{\mu_j}}{d\bar{E}_j} - f_j^E\frac{dw^{\mu_j}}{d\bar{E}_j} \text{ when } p^{\mu_j} = 0,
\end{aligned} \tag{A.179}$$

(A.155), (A.167), and (A.172). Again, one can obtain closed form solutions for $\frac{dw^{\mu_j}}{d\bar{E}_j}$, $\frac{dz^{\mu_j}}{d\bar{E}_j}$, and $\frac{dp^{\mu_j}}{d\bar{E}_j}$ by totally differentiating the first-order conditions and the appropriate market clearing conditions. Since both w^{μ_j}, z^{μ_j}, and p^{μ_j} are are determined by national markets, it is again the case that the total derivatives do not vary across all j. Summing (A.179) across all j, and imposing the market clearing conditions for fossil fuels, the public good, and permits, and the second first order condition from (A.173), we have $\mu_j = z = \left(\frac{1}{J}\right)\left[\sum_{j=1}^{J}\sum_{t=1}^{J}\gamma_j^t\phi_{te} - \left(\sum_{t=1}^{J}\hat{\gamma}_k^t\phi_{te}\right)\left(\frac{de_k^{\mu_j}}{d\bar{E}_j}\right) + e_k\left(\frac{dz^{\mu_j}}{d\bar{E}_j}\right) + \left(\sum_{t=1}^{J}\bar{E}_t\right)\left(\frac{dp^{\mu_j}}{d\bar{E}_j}\right)\right]$ when $p^{\mu_j} > 0$, and $\mu_j = z = \left(\frac{1}{J}\right)\left[\sum_{j=1}^{J}\sum_{t=1}^{J}\gamma_j^t\phi_{te}\right]$ when $p^{\mu_j} = 0$, for all j.

The consumer in k is likely to under-provide the global public good relative to the Pareto optimal allocation in (A.181) and when this occurs the private market is likely to be incomplete. We are interested in the case when governments can secure the Pareto optimal allocation through their mandate choices and thus efficiently provide the global public good. When the private market is incomplete, and when all governments internalize global benefits, i.e., $\gamma_j^t = 1$ for all $t, j = 1, ..., J$, it will be the case that $\sum_{t=1}^{J}\bar{E}_t$ exceeds $\hat{e}_k^{\mu_j}(0)$, and thus $p^{\mu_j} = 0$. Therefore, when all governments internalize global benefits, $z = \sum_{t=1}^{J}\phi_{te}$. Substitution of this plus $p^{\mu_j} = 0$ into (A.173) for all j, implies $\sum_{t=1}^{J}\phi_{te} = \left(\frac{w}{b_{jf}}\right)$ for all j, which together with the remaining first-order conditions and the market clearing conditions support the Pareto optimal allocation given in (A.181).

Pareto Optimal Allocation

The Pareto optimal allocation solves:

$$\max_{e, g_j, F_j, f_j^X, f_j^E, x_j \geq 0} \quad W_1\left(\bar{F}_1 - F_1\right) + x_1 + V_1(g_1) + \phi_1(e)$$

subject to:

$$W_j\left(\bar{F}_j - F_j\right) + x_j + V_j(g_j) + \phi_j(e) \geq \bar{u}_j \ (\lambda_j), \text{ for all } j > 1,$$

$$e = \sum_{t=1}^{J} b_t\left(f_t^E\right) \ (\mu),$$

$$\sum_{t=1}^{J}(x_t + g_t) = \sum_{t=1}^{J} h_t\left(f_t^X\right) \ (\delta),$$

$$\sum_{t=1}^{J}\left(f_t^X + f_t^E\right) = \sum_{t=1}^{J} F_t \ (\beta). \quad \text{(A.180)}$$

The first-order conditions to (A.180) imply, after some manipulation:

$$W_{jF} = h_{jf} = \left(\sum_{t=1}^{J}\phi_{te}\right) b_{jf} = \beta, \text{ for all } j = 1, ..., J, \text{ and}$$

$$V_{jg} = 1, \text{ for all } j = 1, ..., J. \quad \text{(A.181)}$$