The Insurance Value of Progressive Taxation with Heterogenous Risk Aversion

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Abstract

Investment in human capital is lower when the returns to it are subject to uninsurable risk. Progressive income taxation offers a degree of insurance against such risk. Offsetting this effect are the two well-known distortions imposed by progressive taxation: lower expected net-of-tax returns to human-capital acquisition and distortion of the labor-supply decision. The net efficiency effect of progressive income taxation is therefore ambiguous, but there is a presumption that some degree of progressivity can be welfare-improving for risk-averse individuals. To derive the degree of progressivity that may be desirable on efficiency grounds, I construct a general-equilibrium model of an economy with two sectors, calibrated to approximate the U.S. labor market, that differ in terms of the productivity of human capital and the variability of lifetime earnings. Individuals, who differ only in terms of their risk aversion, sort themselves into the two sectors. The simple version of this model, which ignores the labor-leisure choice, suggests that a relatively high degree of income-tax progressivity maximizes

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aggregate welfare as measured by workers' willingness to pay for the insurance being provided. When each workers' supply of labor is allowed to vary in response to marginal tax rates, the efficient degree of progressivity is similar to that of the U.S. tax code.

Keywords: Optimal taxation, Insurance, Risk aversion heterogeneity, Human capital.

JEL Codes: H2, D5, D8, J2, J3.

1 Introduction

Sectors differ in the mean and variance of earnings. Because risk-averse workers dislike high earnings variance, a high mean earning must compensate for the high variance to attain a given value of expected utility. Workers will sort into sectors based on their preferences over the mean and variance of earnings such that less risk-averse workers will accept a higher variance of earnings in return for a higher mean earning. Progressivity in the average tax rate reduces the expected net-of-tax returns to human-capital acquisition, but it reduces the variance of lifetime earnings, thus providing insurance for risk-averse workers. An extension of the core model allows progressivity in the marginal tax rate to distort the labor supply decision. Once the labor-leisure distortion is taken into account, the efficient degree of progressivity is lower than that implied by the model without a distortion.

My core model considers two distinct channels through which progressivity affects the average tax rate. First, through the mean-consumption channel, progressive taxation reduces the expected net-of-tax returns to human-capital acquisition. Workers must accumulate human capital in order to gain access to higher average earnings in some sectors. The cost of obtaining human capital is certain in my model, but it does not generate refundable tax credits. Therefore, progressive income taxation reduces the expected net-of-tax returns to human-capital acquisition and consequently, mean consumption. Second, through the variance-of-consumption channel, a progressive tax offers insurance to workers in the sectors with a high volatility of lifetime earnings. Assume there is a distribution of lifetime earnings. When realized lifetime earnings are high (low), workers pay a higher (lower) tax rate; progressive income taxation shrinks the variance of consumption and provides insurance to risk-averse workers.[1] This insurance effect cannot be provided by the firm, which cannot

[1] For example, suppose a person wants to be a lawyer. He doesn't know whether he will be a successful lawyer or an unsuccessful lawyer. Before making the decision to become a lawyer, he understands the distribution of lifetime earnings. But he doesn't know whether his draw will be from high lifetime earnings or low lifetime earnings. The level of lifetime earnings will be revealed after he becomes a lawyer.

create a wage contract to insure against the variation in lifetime earnings. In addition, individual savings cannot insure against this risk due to the uncertainty of lifetime earnings. Therefore, tax policy is an approach the government can use to fill the missing insurance market. These two effects jointly determine whether there is an efficient degree of progressivity in the tax rates, in the aggregate economy, with workers that are heterogeneous in risk aversion.

Progressive income taxation also distorts the labor supply. A higher marginal income tax rate will give workers less incentive to work. In an extension of my core model, I introduce the labor supply decision, which lowers the efficient degree of progressivity. After calibrating my second model using Panel Study of Income Dynamics (PSID) data for the year 2000, I find that the efficient marginal tax rate in the low income bracket is 21.7%, and the efficient marginal tax rate in the high income bracket is 36.5%.

In this paper, the approach to welfare analysis follows Harberger (1971), who takes the individual willingness to pay as the measure. The idea is to compensate each worker with a certain amount of consumption in order to make the worker indifferent to a flat tax regime or a progressive tax regime (i.e., the expected utility of the worker in the flat tax regime and the progressive tax regime is the same). Since workers are heterogeneous in risk aversion, their compensation amounts will differ. Some workers will gain and some workers will lose under a progressive tax regime, compared to a flat tax regime. Therefore, each worker with a different level of risk aversion has an individual efficient degree of progressivity. In order to capture the aggregate level of welfare, I sum each worker's gain or loss, which follows the criteria of Kaldor-Hicks efficiency. Hence, the efficient degree of progressivity is found in the aggregate level.

This paper considers three effects in unison: the insurance effect, the reduction of expected net-of-tax returns to human-capital acquisition, and the labor supply distortion under uncertainty. This project also links heterogeneity in risk aversion among workers with income taxation topics.

The literature on how progressivity in the marginal rate distorts the number of hours worked is directly related to optimal income taxation (Mirrlees 1971; Saez 2001; Sachs et al. 2016). These models consider a world without uncertainty. Optimal marginal tax rates are interpreted in terms of redistribution, insurance, and incentive effects (Low and Maldoomb 2004; Boadway and Sato 2012; Heathcote et al. 2017). They consider the optimal income taxation under uncertainty, but do not consider sectoral choices. Brown and Rosen (1987) discuss how the market allows individuals to substitute the mean level of the wage for its variability across occupations and further predict how lowering the rate of taxation on earnings would impact an individual occupational choice, though optimal income taxation or the efficient degree of progressivity was not discussed in their paper.

Further tax literature explores how progressivity in the average rate reduces the expected net-of-tax returns to human-capital acquisition (Eaton and Rosen 1980; Guvenen et al. 2014). Stantcheva (2017) derives the optimal taxation and human capital policies in a life cycle model with risky human capital. These papers do not address how progressive taxation influences sectoral choice on an individual level. If an individual wants to sort into a high-skill sector, he needs to accumulate a high level of human capital. A progressive tax may make him less likely to choose a sector offering a high return, which therefore reduces the net-of-tax return to human-capital acquisition.

Though progressive taxation reduces the expected net-of-tax returns to human-capital acquisition and distorts the labor supply decision, it also provides insurance against risk of lifetime earnings, which reduces the variance of consumption (Varian 1980; Kniesner and Ziliak 2002). Heathcote et al. (2017) discusses how a progressive tax system can substitute for imperfect private insurance against idiosyncratic earnings risk, though it does not consider sectoral choices and heterogeneity in risk aversion.

One of the assumptions in my model is that workers are homogeneous in ability but heterogeneous in risk aversion when sorting into two sectors with different levels of lifetime earnings uncertainty. Rothschild and Scheuer (2013) consider optimal progressive taxation in a model where individuals can self-select into one of several possible sectors based on heterogeneity in a multidimensional skill vector. Cubas and Silos (2015) discuss progressive taxation and risky career choices. These general equilibrium models include heterogeneous abilities but do not consider heterogeneous risk aversion.

It is well documented in the labor literature that heterogeneous risk aversion is an important but unobserved factor that influences career choice. Guiso and Paiella (2005) use household survey data to construct a direct measure of absolute risk aversion based on the maximum price a consumer is willing to pay to buy a risky asset. They find that risk-averse consumers are less likely than the risk-prone to be self-employed and to be entrepreneurs and they are more likely to work in the public sector after controlling for the level of income, wealth, personal characteristics, educational attainment, and other attributes. When analyzing sectoral choice, considering heterogeneity in risk aversion is important because of the self-selection problem. If risk aversion is unobservable, estimates of the effect of labor income risk on sectoral choice will be inconsistent because the measure of income risk is correlated with the error term that contains an unobserved preference parameter. Moreover, Hagedorn et al. (2017) argue that observable worker and firm characteristics account for only 30% of the observed variation in wages. In recent studies, career choice and heterogeneity in risk aversion are jointly considered (Cozzi 2014; Barth et al. 2017). Lockwood et al. (2015) discuss heterogeneity in preferences and optimal redistribution in an optimal tax model. Gartner et al. (2017) argue the individual risk preferences and the demand for redistribution.

This paper explores the effect of progressive taxation on the equilibrium allocation of heterogeneously risk averse workers across two sectors. Furthermore, it presents an analysis of relative welfare change with the implementation of a progressive tax, as compared to a flax tax, under uncertainty. In the core model, two effects will be jointly considered in the welfare analysis of progressivity in the average rate: the insurance effect through the channel of reduction in the variance of consumption, and the reduction in expected net-of-tax returns to human-capital acquisition through the channel of mean consumption. All agents in the core model are full-time, full-year workers in the sector of their choice.[2] In the extension of the core model, I include the distortion of the labor supply decision caused by progressivity in the marginal income tax rate. After adding the labor-leisure choice, and calibrating the model using PSID data, I find the efficient degree of progressivity, which is less progressive than in the core model.

The structure of the paper is as follows: In section 2, I provide empirical evidence that individuals who are less risk averse will choose sectors with higher unobserved variance of wages. An income taxation model with a fixed labor supply will be shown in section 3. In section 4, a model with a variable labor supply will be analyzed. Section 5 concludes the paper.

2 Empirical Evidence of Heterogeneous Risk Aversion

One of the key features in my model is that workers are heterogeneous in risk aversion. In the 1996 PSID, the questionnaire includes questions related to hypothetical job choices. Based on the individual answers to the questions, Kimball, Sahm, and Shapiro (2009) estimate the average coefficient of relative risk aversion in each of six categories. Following their work, I further impute the individual coefficient of relative risk aversion. The detailed imputation method is given in Appendix A. Figure 1 shows that workers are indeed heterogeneous in risk aversion.

The model starts from the premise that individuals who are less risk averse will sort into

[2]Full-time, full-year workers are defined as working 35-plus hours per week and 40-plus weeks per year.

sectors with higher unobserved variance of wages. In addition, I assume there is a positive correlation between average wage and unobserved variance of wages across sectors. These are reasonable realistic assumptions based on the empirical evidence. The sample I choose to demonstrate the empirical evidence is full-time workers between 20 and 60 years old in the period 1971-2013. The real wage is equal to annual labor earnings divided by annual work hours, adjusted by the 1999 Consumer Price Index. Following Bonin et al. (2008), I discard observations of full-time employed workers whose wage information is extremely implausible, thus dropping observations of those in the top 1-percentile and bottom 1-percentile of the wage distribution. There are nine categories of industry. I calculate the mean and the unobserved variance of wages for each industry each year. The measure of unobserved variance is similar to that of Bonin et al. (2008) and Fouargea et al. (2014). The unobserved variance is obtained by the regression of the Mincer equation. I regress hourly mean wage for each industry each year on their education, gender, and experience, and include both a quadratic and cubic term for experience as well as industry fixed effects. The error is clustered at the industry level. Then I calculate the variance of the residual for each industry each year as the unobserved variance of wages. Figure 2 shows that there is a positive relationship between average wage and unobserved variance of wages across sectors each year. Figure 3 displays that, on average, workers who are more risk averse will sort into industries that have a smaller unobserved variance of wages. The bubble indicates the size of the industry.

3 Income Taxation and Welfare with Fixed Labor Supply

3.1 Setup

Suppose workers exhibit homogeneity in ability but heterogeneity in risk aversion. The labor market is divided into two sectors, each of which produces a homogeneous goods. The individual wage is endogenous in the general equilibrium framework, which is determined by the marginal product in the sector and by individual-specific labor productivity shocks. Exogenous policy changes (e.g., income taxation reform) can influence the allocation of workers between the two sectors and change the equilibrium wage in the labor market.

The production functions in the aggregate level are:

$$\begin{aligned} Y_1 = \alpha_1 L_1^{\beta_1} = \alpha_1 \Big[n(1-h(\theta_m)) \int_0^{+\infty} \eta_{i1} dF(\eta_{i1}) \Big]^{\beta_1}, \\ Y_2 = \alpha_2 L_2^{\beta_2} = \alpha_2 \Big[nh(\theta_m) \int_0^{+\infty} \eta_{i2} dF(\eta_{i2}) \Big]^{\beta_2}, \end{aligned} \tag{1}$$

where Y_1 and Y_2 are total output; L_1 and L_2 are total effective labor in sector 1 and sector 2. The total factor productivities are α_1 and α_2 in sector 1 and sector 2 and $\alpha_1 < \alpha_2$. The labor shares are β_1 and β_2. The proportion of labor working in sector 2 is $h(\theta_m)$, and θ_m is the coefficient of risk aversion for the marginal worker. The total population is n. I assume workers who are less risk averse (i.e., $\theta < \theta_m$) will choose to work in sector 2, while those who are more risk averse (i.e., $\theta > \theta_m$) will choose to work in sector 1. Therefore, the marginal worker's coefficient of risk aversion, θ_m, is endogenous in my model. It can be solved numerically and also depends on the tax regimes.

I assume that individual i's productivity shock in sector j follows a log-normal distribution $\eta_{ij} \sim lnN(\frac{-\sigma_j^2}{2}, \sigma_j^2)$, with $\sigma_2^2 > \sigma_1^2$, $E(\eta_{ij}) = 1$ and $Var(\eta_{i2}) > Var(\eta_{i1})$. Therefore, the mean wage for workers in sector 2 is higher than the mean wage for workers in sector 1 in order to compensate for the higher variance in sector 2. Each worker is paid a wage equal to his productivity of labor.

The timing of the model is as follows. First, workers must choose whether or not to obtain human capital before sectoral choices. In order to get into sector 2, which requires high human capital, workers need to pay the cost. Workers also know their risk aversion. Second, they choose their sector. Third, after that decision has been made, they learn their productivity shock and hence their wage. Under no tax scenario, income is equal to consumption. Appendix B shows that the consumption for an individual who chooses to work in sector 1 or sector 2 is [3]

$$\begin{aligned} c_{i1} &= \eta_{i1}\alpha_1\beta_1(n(1-h(\theta_m)))^{\beta_1-1}, \\ c_{i2} &= \eta_{i2}\alpha_2\beta_2(nh(\theta_m))^{\beta_2-1}-\gamma, \end{aligned} \tag{2}$$

where γ is the cost of accumulating human capital (i.e., the cost of higher productivity α_2) to work in sector 2.

On the worker side, the utility function exhibits constant relative risk aversion. Thus, the maximization problem is in the context of Von Neumann-Morgenstern expected utility. The utility function is given by

$$U(c) = \frac{c^{1-\theta}-1}{1-\theta}, \tag{3}$$

where c is consumption. The constant coefficient of relative risk aversion is $\theta \in [\underline{\theta}, \overline{\theta}]$, which is defined as $\theta = c\frac{-U''}{U'}$. A large value of θ implies that the worker is relatively more risk averse. I assume that θ follows a continuous distribution in the population, which will be calibrated from the data.

3.2 Equilibrium Conditions

Since workers care about after-tax earnings, that is, their consumption level, I examine the progressiveness of the average tax rate as a response to the progressiveness of the marginal tax rate in the tax function. Following Guner et al. (2012a,2012b) and Guner et al. (2014),

[3] This is a static model, which represents the steady state. Thus, there are no savings in this model. The after-tax wage is equal to consumption.

I use the log tax functional form[4]. The average tax rate and corresponding consumption is

$$\begin{aligned} t(w) &= a + \rho \cdot log(w), \\ c(w) &= (1-a) \cdot w - \rho \cdot w \cdot log(w) \end{aligned} \tag{4}$$

where c is the consumption and w is the wage. In this functional form, a is the scale parameter and ρ captures the curvature of the tax function. If ρ is equal to 0, then it is equivalent to the flat taxation case.

Therefore, the government has access to the following instruments: a progressive labor income tax with two free parameters that it can choose, and a proportional corporate income tax on profits which will be discussed later..

Based on the above functional form, the consumption for worker i in sector 1 or 2 is

$$\begin{aligned} c_{i1} &= (1-a)\big[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,P})))^{\beta_1-1}\big] - \rho\big[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,P})))^{\beta_1-1}\big]log\big[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,P})))^{\beta_1-1}\big], \\ c_{i2} &= (1-a)\big[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,P}))^{\beta_2-1}\big] - \rho\big[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,P}))^{\beta_2-1}\big]log\big[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,P}))^{\beta_2-1}\big] - \gamma, \end{aligned} \tag{5}$$

where $h(\theta_{m,P})$ is the proportion of workers choosing sector 2 and P indicates progressive tax regime.

The equilibrium condition in this model (for the marginal worker m) is $E[U_m(c_{m2})] =$

[4] I have tried two other functional forms. The first corresponds to the function used in Heathcote et al. (2016): $c(w) = aw^{\rho}$. In order to keep tax revenue constant, when I reduce the value of ρ to make the tax system more progressive, the value of a needs to be increased. Then I find that the marginal tax rate, $1 - a\rho w^{\rho-1}$, becomes negative if a is large enough. Thus, this functional form does not fit my model well. The second functional form corresponds to the function used in Guvenen et al. (2014). The function form of total tax paid is $T = aw^{\rho}$. When I increase the value of ρ to make the tax system more progressive, the value of a needs to decrease to hold the tax revenue constant. However, when the values of ρ and a are simultaneously changed, the degree of progressiveness only changes in a very small amount. That is, there is an upper bound for the degree of progressiveness when the values of ρ and a are changed simultaneously to make the tax system more progressive and, at the same time keep the tax revenue constant. Hence, the second tax function form is not ideal.

$E[U_m(c_{m1})]$, which is identical to

$$\begin{aligned}
&\int_0^{+\infty} \frac{\left\{(1-a)w_{m1} - \rho w_{m1} log(w_{m1})\right\}^{1-\theta_{m,P}} - 1}{1-\theta_{m,P}} f(\eta_{m1}) d\eta_{m1} \\
&= \int_0^{+\infty} \frac{\left\{(1-a)w_{m2} - \rho w_{m2} log(w_{m2}) - \gamma\right\}^{1-\theta_{m,P}} - 1}{1-\theta_{m,P}} f(\eta_{m2}) d\eta_{m2},
\end{aligned} \tag{6}$$

where $\eta_{mj} \sim lnN(\frac{-\sigma_j^2}{2}, \sigma_j^2)$. The above equation is nonlinear, so the solution $\theta_{m,P}$ will be given in the simulation section under certain parameter values.

In the flat tax regime F, the value of ρ is equal to zero. The value of a is equivalent to the flat tax rate τ. Based on the same logic, I find the marginal worker $\theta_{m,F}$, who is indifferent to choosing sector 1 or sector 2. The equilibrium allocation in sector 2 is equal to $h(\theta_{m,F})$.

3.3 Welfare Analysis

There are two levels of welfare analysis in this paper. The first is the individual level. Because individuals are heterogeneous in risk aversion, it is important to explore the difference in welfare change in the two tax regimes with respect to the degree of risk aversion. The second is the aggregate level. It can tell us whether there exists an efficient degree of progressivity based on the insurance effect through the variance-of-consumption channel, and on the reduction of expected net-of-tax returns to human-capital acquisition through the mean-consumption channel.

Since workers are heterogeneous in risk aversion, adding their utility together to measure the aggregate welfare is not appropriate.I follow Harberger (1971), who uses the individual's willingness to pay to measure welfare, and mitigates the aggregation problem. The detailed methodology follows Lucas (1987). The idea is to compensate with a certain amount of consumption to make worker i indifferent to either a progressive tax regime (P) or a flat tax regime (F). Individual i might remain in the same sector, or he might switch sectors between the two tax regimes. But the compensating consumption itself will not affect the worker's decision about which sector to enter. It's a hypothetical compensation after the endogenous sectoral choice is made.

The formulas for the welfare change for individual i if he does not change sector to work (i.e., both in sector j), or if he switches from sector j to sector k to work are given by

$$\begin{aligned} E[U_{ij,P}(c_{ij})] &= E[U_{ij,F}(c_{ij} + \Delta c_i)], \quad or \\ E[U_{ik,P}(c_{ik})] &= E[U_{ij,F}(c_{ij} + \Delta c_i)]. \end{aligned} \tag{7}$$

If there is a welfare gain for worker i in the progressive tax regime, then $\Delta c_i > 0$. If there is a welfare loss for worker i in the progressive tax regime, then $\Delta c_i < 0$. Workers with different levels of risk aversion, θ_i, will be compensated with different levels of consumption Δc_i. Therefore, after compensating by the amount of consumption Δc_i, the welfare level of worker i will be exactly the same in the progressive tax regime and the flat tax regime.

In the aggregate level, following the criterion of Kaldor-Hicks efficiency, the total welfare change for all workers involves aggregating the compensating level of consumption and checking how it changes when the tax system is more progressive:

$$\Delta c = \sum_i \Delta c_i. \tag{8}$$

In order to fully consider the change in welfare in the economy under the general equilibrium framework, I attribute the residual of the output to the capital owners. They are risk neutral and pay the flat corporate income tax. The change in welfare for the capital owners is equal to the difference of the residual output between a progressive tax regime and a flat tax regime after imposing the constant flat corporate income tax rate τ_c.

$$\Delta\Pi_1 = (1-\tau_c)\alpha_1(n(1-h(\theta_{m,P})))^{\beta_1}[1-\beta_1] - (1-\tau_c)\alpha_1(n(1-h(\theta_{m,F})))^{\beta_1}[1-\beta_1], \quad (9)$$

$$\Delta\Pi_2 = (1-\tau_c)\alpha_2(n(h(\theta_{m,P})))^{\beta_2}[1-\beta_2] - (1-\tau_c)\alpha_2(n(h(\theta_{m,F})))^{\beta_2}[1-\beta_2]. \quad (10)$$

The total welfare change for capital owners is

$$\Delta\Pi = \Delta\Pi_1 + \Delta\Pi_2. \quad (11)$$

The calculation of the welfare change in the aggregate economy is equal to the total welfare change for all workers plus the total welfare change for capital owners. This measurement is based on money instead of utility, and it varies with the degree of progressivity in income taxation.

$$\Delta TW = \Delta c + \Delta\Pi. \quad (12)$$

Welfare comparisons are based on the equal tax revenue condition. That is, the expected tax revenue collected from all workers in the flat tax regime and in the progressive tax regime is equal, and the degree of progressivity does not affect expected tax revenue. The tax revenue can be used as government spending, but it will not impact wages and it doesn't impact different individuals differently.[5] Therefore, under the parametric assumption of the tax function, when ρ is changed to make the tax system more progressive, the value of a needs to be changed too in the tax function to make the expected tax revenue stay the same. The relationship between a and ρ, given the constant tax revenue, is described in Appendix

[5] As Eaton and Rosen (1980) argue, "the assumption that the government is only concerned with the expected revenue need not imply that the government is risk neutral. If the shock associated with wage is independent across individuals and if the number of individuals is large enough,then the law of large numbers will guarantee the government a constant total revenue despite uncertainty on the individual level. The government is, in this case, simply a more efficient risk pooler than the individual."

C.

3.4 Calibration

In the data set, I focus on working-age individuals, aged 20-60 in the year 2000, whose population weight is nonzero. Each observation in the sample is weighted by its PSID supplied sample weight. I drop those individuals who are not full-time workers, because the core model assumes there is no labor-leisure choice (i.e., no adjustment of hours of work).[6] Wage is equal to the individual's labor earnings divided by the work hours of the individual. I further drop observations with reported hourly wages below the federal minimum wage rate of \$5.15/hour or above \$100/hour. The remaining size of the sample is 2,353 individuals.

Since η_{i1} and η_{i2} follow a lognormal distribution, there is some probability that the wage might be zero, Therefore, I use a shifted lognormal distribution instead to calibrate the parameters in the distribution.

I first calibrate h_m, which is the proportion of workers who attend at least some college. For the labor share β_1 and β_2, I impose values of 2/3.[7] For the cost of human capital γ, it is approximately 20% of the mean hourly wage for workers who attend at least some college. Given h^*, β_1, β_2, and γ, productivity parameters α_1 and α_2 are calibrated to match the target data moments: the pre-tax mean hourly wage in sector 1 and that in sector 2. The pre-tax mean hourly wage in sector 1 is equal to the pre-tax mean hourly wage for all workers who had obtained at most a high school diploma in the year 2000: \$16.41. The pre-tax mean hourly wage in sector 2 is equal to the pre-tax mean wage for all workers who had attended at least some college in the year 2000: \$24.84. I search for the parameters α_1 and α_2 that minimize the squared deviations between the model and data moments. Table 1 shows each

[6] In the Cross-national Equivalent File version of PSID, if the individual had positive wages and worked at least 1,820 hours last year (35 hours per week on average), then the individual was employed full-time.

[7] I am not aware of any paper that offers a reasonable but different β in different industries in the U.S. Thus, I adopt the labor share in the aggregate economy in the U.S, which is 2/3.

parameter in the model.

The remaining unknown variables are σ_1^2 and σ_2^2. Although they are unobserved, they can be solved for in the equilibrium with no tax case given the other calibrated parameters. Suppose $\sigma_2^2 = 3\sigma_1^2$; [8]then I am searching σ_1^2 to solve equation (6) if $a = 0$ and $\rho = 0$. Given the following equation, the ratio of $Var(w_2)$ and $Var(w_1)$ is approximately 2.66. Once σ_2^2 and σ_1^2 are calibrated, they are considered exogenous and will not be changed if the tax regime is changed.

$$\frac{Var(w_2)}{Var(w_1)} = \frac{[(e^{\sigma_2^2} - 1)(\alpha_2\beta_2(n(1-h_m))^{\beta_2-1})]^2}{[(e^{\sigma_1^2} - 1)(\alpha_1\beta_1(nh_m)^{\beta_1-1})]^2} \tag{13}$$

The estimated risk aversion distribution in the sample approximately follows a shifted lognormal distribution. Thus, I draw a three-parameter lognormal distribution (shape=0.30, scale=0.3, threshold=2.0), which approximates the targeted risk aversion distribution for the sample in the year of 2000. It is shown in Figure 4.

3.5 Results

I first simulate the model under the flat tax regime, where $a = 0.23$ and $\rho = 0$ (i.e. $\tau = 0.23$) in the year 2000. Given the values of a, ρ, and the above calibrated parameters shown in Table 1, the equilibrium allocation can be solved using equation (6). After the equilibrium allocations, I simulate two individual welfare changes by using equation (7), whose coefficient of risk aversion is the minimum and the maximum in the distribution of θ. Then I simulate the change of aggregate welfare in equation (8). These simulations will be continuously done as the tax system becomes more progressive, that is, when the value of ρ becomes bigger. All welfare comparisons are done under a constant tax revenue condition, meaning that as ρ

[8]The linear relationship between σ_2^2 and σ_1^2 is assumed in order to have one unknown parameter to be solved in one equation. More generally, other linear relationship could also be assumed.

becomes bigger a must correspondingly become smaller. Figure 5 shows how ρ and a must vary in order to keep tax revenue constant.

3.5.1 Individual Welfare Change with Respect to the Change in ρ

Equation (7) provides the formulas to calculate the individual welfare change. Since the worker with the lowest coefficient of risk aversion (i.e., the least risk averse worker) will always stay in sector 2, and the worker with the highest coefficient of risk aversion (i.e., the most risk averse worker) will always stay in sector 1, it is worthwhile to explore their welfare change. Figure 6 shows the welfare change for these workers as ρ increases and the tax becomes more progressive. For the least risk averse worker ($\theta = 2.51$), welfare increases initially because of the insurance effect. However, at higher levels of progressivity the human capital effect dominates and welfare decreases. For the most risk averse worker ($\theta = 5.88$), a more progressive tax provides more insurance. Unlike workers in sector 2, the most risk averse worker does not need to pay the cost to enter sector 1; thus the mean consumption does not change as much when the tax is more progressive. Based on these facts, welfare strictly increases for the most risk averse worker as the tax becomes more progressive.

3.5.2 Aggregate Welfare Change with respect to the Change of ρ

I next consider whether there is a progressive tax rate that maximizes aggregate welfare. Figure 7 shows that when the tax is more progressive, some workers move from sector 2 to sector 1. The economic intuition is that although a higher degree of progressiveness provides more insurance to workers in sector 2, the average tax rate in the progressive tax regime in sector 2 is higher than the average tax rate in sector 1, which decreases the mean after-tax wage in sector 2 more. When the cost of the lower mean after-tax wage is greater than the benefit of lower variance in sector 2, workers start to move from sector 2 to sector 1.

Giving the changing allocation of workers across sectors as the tax rate becomes more progressive, it is easy to understand the welfare change for capital owners shown in Figure 8. When the tax is more progressive, fewer workers are employed in sector 2. Output is reduced in sector 2, and the capital owner in sector 2 must pay higher wages because the remaining workers have higher marginal productivity. Thus, the capital owner in sector 2 is worse off (Figure 8(b)). Similar analysis is applied in the opposite direction for the capital owner in sector 1 (Figure 8(a)). Figure 8(c) shows the net change of welfare for both capital owners in the economy. The net effect shows the welfare of capital owners strictly decreases as the tax system becomes more progressive because total output is reduced.

Figure 9 presents the hump relationship between the change in welfare for the whole economy and ρ. In the aggregate level, when the income tax rate becomes more progressive, total welfare first increases and then decreases. The aggregate effect is composed of the insurance effect through the variance-of-consumption channel, the reduction of expected net-of-tax returns to human-capital acquisition through the mean-consumption channel, and the reduction of output. At low levels of progressivity, the insurance effect dominates the reduction in expected net-of-tax returns to human-capital acquisition and the output effect, which makes total welfare increase. However, when the level of progressivity is very high, the reduction in expected net-of-tax returns to human-capital acquisition is large, which dominates the insurance effect. More workers choose to work in the low productivity sector (i.e., sector 1) and total output is reduced. Thus, total welfare eventually decreases.

The maximal welfare gain (i.e., dTW) under progressive income tax system, as compared with flat income tax system, is the point where $\rho = 0.200$ and $a = -0.372$. I transfer the values of those two parameters into average tax rates, because the insurance effect and the reduction in return to human capital are based on the progressivity of average tax rates. In addition, the progressive average tax rates lead to an output reduction. The corresponding average tax rates ($ATRs$) are shown below. The marginal tax rate is the average tax rate plus $\rho = 0.200$.

$$
ATR_1 = \begin{cases} 8.7\% & w_{1,min} = 9.91 \\ 18.8\% & w_1 = 16.41 \\ 44.5\% & w_{1,max} = 59.49. \end{cases}
$$

$$
ATR_2 = \begin{cases} 23.9\% & w_{2,min} = 21.21 \\ 27.1\% & w_2 = 24.85 \\ 53.2\% & w_{2,max} = 91.92. \end{cases}
$$

4 Income Taxation and Welfare With Variable Labor Supply

The additional distortion of progressiveness is the labor supply decision. The following model will deal with the trade-off among the insurance effect, the reduction of expected net-of-tax returns to human-capital acquisition, and the labor supply distortion imposed by progressive income taxation. In order to make the model more tractable, I use a discrete distribution of realized wage and a piecewise linear tax function.[9]

4.1 Setup

There are two sectors in the model. In sector 1, wage is exogenously given as w_1; thus no uncertainty about earnings exists in sector 1. In sector 2, there are three possible exogenous wages: w_{2L} with probability P_L, w_{2M} with probability P_M, and w_{2H} with probability P_H. That is, uncertainty about earnings exists in sector 2. According to compensating differential

[9]With no variation in labor supply in this model, the results are consistent with the results in the core model, which has a continuous distribution of realized earnings and a continuous tax function.

theory, a higher variance of wage needs to be compensated by a higher mean wage. Therefore, I assume $w_{2H} > w_{2M} > w_{2L} = w_1$.

Workers are heterogeneous in risk aversion. Those who are more risk averse will choose sector 1, and those who are less risk averse will choose sector 2. Once workers sort into sectors based on their coefficient of relative risk aversion, they will decide how many hours to work based on the income taxation structure. I assume workers cannot move between sectors when income taxation becomes more progressive or less progressive. Following Eichenbaum et al. (1988) and French (2005), the utility function is given by

$$U(c, l) = \frac{(c^{\lambda} l^{1-\lambda})^{1-\theta}}{1-\theta} \tag{14}$$

where c is consumption, l is leisure, and θ is the coefficient of relative risk aversion.

4.2 Leisure Choice

4.2.1 Progressive-Tax Case

In the progressive tax regime, there is a piecewise linear tax code with three rates: t_L, t_M, and t_H. Workers in sector 1 will be taxed at t_L. Workers in sector 2 will be taxed at t_L, t_M, or t_H based on their realized earnings.

The consumption and the total tax paid by worker i in sector 1 is

$$c_{i1} = (1 - t_L) w_1 (H - l_1) + B_p, \tag{15}$$

where H is the time endowment and B_p is the lump-sum rebate from the tax revenue[10].

[10] In this model, I rebate the tax revenue to every worker in an equal amount. Therefore, on average, no income effect imposed by the progressive tax will exist on the labor supply decision. That is, the substitution effect shows the decrease in labor supply if the income tax is more progressive.

The optimal level of leisure that worker i will choose is given by the first order condition: $\frac{\partial u}{\partial l_1} = 0$. After some simple algebra, the amount of leisure that worker i will choose and the corresponding utility level are

$$
\begin{aligned}
l_1(t_L) &= (1-\lambda)\Big[H + \frac{B_p}{(1-t_L)w_1}\Big], \\
U_{i1,P} &= \frac{\Big\{\Big[(1-t_L)w_1\lambda H + \lambda B_p\Big]^{\lambda}\Big[(1-\lambda)(H + \frac{B_p}{(1-t_L)w_1})\Big]^{1-\lambda}\Big\}^{1-\theta_i}}{1-\theta_i}.
\end{aligned} \tag{16}
$$

For worker i in sector 2, the consumption function is

$$
c_{i2} = \begin{cases} (1-t_L)w_{2L}(H-l_{2L}) - \gamma + B_p & \text{with } P_L \\ (1-t_L)Y_L + (1-t_M)[w_{2M}(H-l_{2M}) - Y_L] - \gamma + B_p & \text{with } P_M \\ (1-t_L)Y_L + (1-t_M)(Y_M - Y_L) + (1-t_H)[w_{2H}(H-l_{2H}) - Y_M] - \gamma + B_p & \text{with } P_H, \end{cases}
$$

where $Y_L = w_{2L}(H - l_{2L})$ is the cutoff in the first tax bracket and $Y_M = w_{2M}(H - l_{2M})$ is the cutoff in the second tax bracket.

Taking first-order conditions, the optimal level of leisure that worker i will choose and the corresponding indirect utility levels are

$$
\begin{aligned}
l_{2L}(t_L) &= \frac{1-\lambda}{(1-t_L)w_{2L}}\Big[(1-t_L)w_{2L}H - \gamma + B_p\Big], \\
l_{2M}(t_M) &= \frac{1-\lambda}{(1-t_M)w_{2M}}\Big[(1-t_M)w_{2M}H + (t_M - t_L)Y_L - \gamma + B_p\Big], \\
l_{2H}(t_H) &= \frac{1-\lambda}{(1-t_H)w_{2H}}\Big[(1-t_H)w_{2H}H + (t_M - t_L)Y_L + (t_H - t_M)Y_M - \gamma + B_p\Big];
\end{aligned} \tag{17}
$$

$$U_{i2,L} = \frac{\left\{\left[(1-t_L)w_{2L}\lambda H - \lambda\gamma + \lambda B_p\right]^{\lambda}(l_{2L}(t_L))^{1-\lambda}\right\}^{1-\theta_i}}{1-\theta_i},$$
$$U_{i2,M} = \frac{\left\{\left[(1-t_L)Y_L + (1-t_M)[w_{2M}(H-l_{2M}) - Y_L] - \gamma + B_p\right]^{\lambda}(l_{2M}(t_M))^{1-\lambda}\right\}^{1-\theta_i}}{1-\theta_i},$$
$$U_{i2,H} = \frac{\left\{\left[(1-t_L)Y_L + (1-t_M)(Y_M - Y_L) + (1-t_H)[w_{2H}(H-l_{2H}) - Y_M] - \gamma + B_p\right]^{\lambda}(l_{2H}(t_H))^{1-\lambda}\right\}^{1-\theta_i}}{1-\theta_i}. \tag{18}$$

Therefore, the expected utility under the progressive tax regime in sector 2 can be expressed as

$$E[U_{i2,P}] = U_{i2,L} * P_L + U_{i2,M} * P_M + U_{i2,H} * P_H.$$

4.2.2 Flat-Tax Case

In the flat tax regime, $t_L = t_M = t_H = \tau$. Consumption for worker i in sector 1 is then

$$c_{i1} = (1-\tau)w_1(H - l_1) + B_\tau, \tag{19}$$

where B_τ is the lump-sum rebate from the total tax revenue.

The optimal level of leisure that worker i will choose and the corresponding utility are

$$l_1(\tau) = (1-\lambda)\left[H + \frac{B_\tau}{(1-\tau)w_1}\right],$$
$$U_{i1,F} = \frac{\left\{\left[(1-\tau)w_1\lambda H + \lambda B_\tau\right]^{\lambda}\left[(1-\lambda)(H + \frac{B_\tau}{(1-\tau)w_1})\right]^{1-\lambda}\right\}^{1-\theta_i}}{1-\theta_i}. \tag{20}$$

For worker i in sector 2, the consumption function is

$$c_{i2} = \begin{cases} (1-\tau)w_{2L}(H-l_{2L}) - \gamma + B_\tau & \text{with } P_L \\ (1-\tau)w_{2M}(H-l_{2M}) - \gamma + B_\tau & \text{with } P_M \\ (1-\tau)w_{2H}(H-l_{2H}) - \gamma + B_\tau & \text{with } P_H. \end{cases}$$

Taking first-order conditions, the optimal level of leisure that worker i will choose and the corresponding utility levels are

$$\begin{aligned} l_{2k}(\tau) &= (1-\lambda)\Big[H - \frac{\gamma}{(1-\tau)w_{2k}} + \frac{B_\tau}{(1-\tau)w_{2k}}\Big], \\ U_{i2,k}(\tau) &= \frac{\Big\{\Big[(1-\tau)w_{2k}\lambda H - \lambda\gamma + \lambda B_\tau\Big]^{\lambda}\Big[(1-\lambda)(H - \frac{\gamma}{(1-\tau)w_{2k}} + \frac{B_\tau}{(1-\tau)w_{2k}})\Big]^{1-\lambda}\Big\}^{1-\theta_i}}{1-\theta_i}, \end{aligned} \tag{21}$$

where $k \in \{L, M, H\}$. Thus, the expected utility can be expressed as

$$E[U_{i2,F}] = U_{i2,L} * P_L + U_{i2,M} * P_M + U_{i2,H} * P_H. \tag{22}$$

4.3 Defining the Tax Rebates

Under the flat tax regime, the total tax revenue collected is

$$E[R_\tau] = n(1-h_m)\tau w_1(H - l_1(\tau)) + nh_m P_k \sum_{k=L}^{H} \tau w_{2k}(H - l_{2k}(\tau)), \tag{23}$$

where $k \in \{L, M, H\}$.

Under the progressive tax regime, the total tax revenue collected is

$$
\begin{aligned}
E[R_p] =& n(1-h_m)t_L w_1(H - l_1(t_L)) \\
&+ nh_m\Big\{t_L w_{2L}(H - l_{2L}(t_L))P_L + \big[t_L Y_L + t_M[w_{2M}(H - l_{2M}(t_M)) - Y_L]\big]P_M \\
&+ \big[t_L Y_L + t_M(Y_M - Y_L) + t_H[w_{2H}(H - l_{2H}(t_H)) - Y_M]\big]P_H\Big\}.
\end{aligned}
\tag{24}
$$

where h_m is the proportion of workers in sector 2. This value will be calibrated from the current tax system. The expected tax revenue should be equal under the progressive tax regime and flat tax regime (i.e., $E[R_\tau] = E[R_p]$). The tax rebates are defined as $B_\tau = \frac{E[R_\tau]}{n}$ for the flat tax regime, and $B_p = \frac{E[R_p]}{n}$ for the progressive tax regime. Under the contant tax revenue condition, $B_\tau = B_p$. All the tax revenue is rebated to the workers and every worker gets the same amount of lump-sum rebate.

The above total amount of tax revenue is a function of optimal level of leisure. The optimal level of leisure is a function of the tax rebate. I use iteration to solve B_τ and B_p.

4.4 Welfare Analysis

The measure of welfare is determined by using the same approach as under the core model. The idea is to compensate with a certain amount of consumption to make worker i indifferent to a progressive tax regime or a flat tax regime. For workers in sector 1, the amount of consumption Δc_i needs to meet

$$
U_{i1,P}(c_{i1}, l_{1,P}) = U_{i1,F}(c_{i1} + \Delta c_i, l_{1,F}). \tag{25}
$$

For workers in sector 2, the amount of consumption Δc_i needs to meet

$$
E[U_{i2,P}(c_{i2}, l_{2,P})] = E[U_{i2,F}(c_{i2} + \Delta c_i, l_{2,F})]. \tag{26}
$$

In the aggregate level, there are no capital owners, and the total welfare measure is determined by aggregating the compensating consumptions for all workers and checking how it changes when the tax system is more progressive:

$$\Delta c = \sum_i \Delta c_i. \tag{27}$$

4.5 Calibration

I focus on working-age individuals, aged 20-60 in the year 2000, whose population weight is nonzero. Each observation in the sample is weighted by its PSID supplied sample weight. For the value of the time endowment, $H = 16(hours/day) * 5(days/week) * 52(weeks) = 4,160$. I drop individuals whose annual hours of work are above the time endowment. According to the CNEF-PSID codebook, "If the individual had positive wages in the previous year and worked at least 52 hours, then the individual was employed. Otherwise, the individual was not employed." Thus, I drop individuals whose annual hours of work are below 52 hours.

An individual's hourly wage is equal to the individual's labor earnings divided by work hours of the individual. I further drop observations of those whose hourly wage is below the federal minimum wage rate of $5.15/hour. The remaining size of the sample is 3,236 individuals.

In the sample, there are two groups: workers who had obtained at most a high school diploma in the year 2000, and workers who had attended at least some college in the year 2000. Wage is exogenous in my model. The pre-tax mean hourly wage in sector 1 is equal to the pre-tax mean hourly wage for all workers who had obtained at most a high school diploma in the year 2000: $16.66 (i.e., $w_1 = 16.66$). I assume that $w_1 = w_{2L} = 16.66$. The pre-tax mean hourly wage in sector 2 is equal to the pre-tax mean wage for all workers who had attended at least some college in the year 2000: $27.08 (i.e., $w_{2M} = 27.08$). Because the wage follows a lognormal distribution, thus $ln(w_{2M}) - ln(w_{2L}) = ln(w_{2H}) - ln(w_{2M})$. I can then calculate $w_{2H} = 43.99$. Given the values of w_{2L} and w_{2M}, I observe these two points at the 37.8 percentile and 67.3 percentile of the wage distribution. Hence, I set $P_L = 0.378$, $P_H = 0.327$, and $P_M = 1 - P_L - P_H = 0.295$.

I use the mean of annual hours of work divided by the time endowment to approximate the value of λ in my sample. I calculate a value of 0.488, which is between 1/3 in Kydland and Precott (1982) and 0.5 in French (2005).

For the values of t_H, I choose the statutory marginal tax rates 31%. The value of t_M is 23%, which is the average federal income tax rate in the year 2000. I assume $\tau = t_M = 23\%$. Based on the constant tax revenue condition under the progressive tax regime and flat tax regime, I can further impute t_L, which is around 20%.

I draw a three-parameter lognormal distribution (shape=0.3, scale=0.3, threshold=2.0), which approximates the targeted risk aversion distribution in the sample.

The allocation of workers in sector 2 (h_m) is the proportion of workers who had attended at least some college in the year 2000, which is 0.533. Since I know the distribution of θ and also the cumulative distribution function evaluated at θ_m, $h(\theta_m) = 0.533$, I can find the marginal worker's θ_m who is indifferent to choose sector 1 or sector 2 in the year 2000, $U_{m1} = E[U_{m2}]$. Using this equation, I can calibrate the only remaining unknown parameter,γ, the cost of accumulating higher human capital, which is \$10,739. The unit of γ is the annual earnings. Table 2 summarizes the calibration values for each parameter.

4.6 Results

I start at the point where marginal tax rates, $t_L = t_M = t_H = 23.0\%$. Then I make the tax system more progressive: increasing t_H in a increment of 1.5%. To keep the tax revenue constant, I search t_L to match each t_H. The relationship between t_L and t_H is shown in Figure 10. When $t_H > 41\%$, the labor supply distortion is large. The tax revenue collected from the high income bracket decreases in a large amount. Thus, the marginal tax rate t_L

eventually increases to keep the tax revenue constant, although it is still smaller than τ to keep the insurance effect working. Figure 11 presents how the more progressive tax system distorts the labor supply decision: leisure $l_{2H}(t_H)$ increases when t_H increases.

By using equations (25) and (26), individual i's welfare difference between the progressive tax and flat tax regimes can be captured by Δc_i. By using equation (27), the aggregate welfare difference between the progressive tax and flat tax regimes can be captured by Δc.

Since I assume workers are heterogeneous in risk aversion, even though they pay the same tax rate in sector 2, the insurance effect will be different for workers with different degree of risk aversion. Figures 12(a) and 12(b) display the hump shapes between Δc and t_H for the marginal worker (i.e., $\theta_{marg} = 3.38$) and the least risk averse worker (i.e., $\theta_{min} = 2.48$). Since the worker with $\theta_{marg} = 3.38$ is more risk averse than the worker with $\theta_{min} = 2.48$, his insurance effect will be larger than that for the least risk averse worker. Therefore, consumption needs to compensate more for him in the flat tax regime in order to match the expected utility level in the progressive tax regime. That is, Δc_{marg} is larger than Δc_{min} before the inflection point. In addition, as Figures 12(a) and 12(b) show, the efficient degree of progressivity for $\theta_{marg} = 3.38$ is $t_H = 0.335$ and $t_L = 0.218$; the efficient degree of progressivity for $\theta_{min} = 2.48$ is $t_H = 0.290$ and $t_L = 0.222$. That is, the efficient degree of progressivity is more progressive for the marginal worker because of larger insurance effect. When the tax system is very progressive, the distortion of labor supply and the reduction of expected net-of-tax returns to human-capital acquisition dominate the insurance effect; thus the marginal worker and least risk-averse worker are both worse off in the progressive tax regime, compared to flat tax regime.

Figure 12(c) shows the hump shape between Δc_{max} and t_H for the most risk-averse worker (i.e., $\theta_{max} = 5.83$). He is always better off in the progressive tax regime, compared to flat tax regime (i.e., $\Delta c_{max} > 0$), because the most risk-averse worker pays t_L, which is always smaller than τ. When keeping tax revenue constant, t_L first decreases than increases; thus his welfare gain first increases than decreases.

Figure 13 shows a hump shape between aggregate welfare change Δc and t_H and the efficient degree of progressivity is at $t_L = 21.7\%$ and $t_H = 36.5\%$, given $t_M = 23.0\%$. When the tax is more progressive compared to the flat tax, the insurance effect initially dominates the labor supply distortion and the reduction of expected net-of-tax returns to human-capital acquisition. After the tax becomes more progressive than the efficient point, the labor supply distortion and the reduction of expected net-of-tax returns to human-capital acquisition dominate the insurance effect.

5 Conclusion

The welfare analysis in this paper is based on efficiency instead of redistribution. Three effects are considered in unison: the insurance effect, the reduction of expected net-of-tax returns to human-capital acquisition, and the labor supply distortion. I use each worker's willingness to pay for the insurance being provided as a welfare measure on the individual level, which depends on each worker's degree of risk aversion. Based on the Kaldor-Hicks efficiency criterion, I further aggregate each individual's willingness to pay to find the efficient degree of progressivity. Therefore, the willingness to pay approach provides a fruitful lens for examining the efficient degree of progressivity for income taxation.

To characterize the efficient degree of progressivity, I construct models to examine how progressivity for income taxation influences workers with heterogeneous risk aversion to sort into two sectors with different levels of lifetime earnings uncertainty. In the core model, as in Thaler and Rosen (1976), there is a "hedonic wage locus", which indicates how the market allows individuals to substitute the mean level of the wage for its variability across sectors. However, in my general equilibrium framework, the variability-return locus varies with respect to changes in the tax rate. Furthermore, the shape of the indifference curve between the mean and variability of consumption is different for workers with different levels of risk aversion. In the extension of the core model, I further account for the labor supply response with respect to the degree of progressivity for income taxation. After calibrating the model, I find the efficient degree of progressivity is similar to that of the U.S. tax code.

The sorting mechanism in my paper is based on heterogeneous risk aversion. In future work, I will add an additional dimension of sorting: heterogeneous ability. The two dimensions will jointly determine how workers sort into two sectors: a sector with high uncertainty of lifetime earnings and a sector with low uncertainty of lifetime earnings. Moreover, I will further research how the joint dimensions of heterogeneous ability and heterogeneous risk aversion determine the efficient degree of progressivity.

Appendices

A Imputation of Risk Aversion

In the theoretical model, one of the key variables is the individual specific coefficient of risk aversion θ. Kimball, Sahm, and Shapiro (2008) have developed direct survey measures of risk tolerance based on hypothetical choices and appropriate econometric techniques for dealing with the inevitable measurement error in questionnaires. Kimball, Sahm, and Shapiro (2009) also present the risk tolerance imputations for the survey responses in the PSID. I follow their imputation methods based on the gambling questions in the 1996 PSID. The questions are as follows:

Suppose you had an occupation that guaranteed you income for life equal to your current total income. Also suppose that occupation was your/your family's only source of income. Then you are given the opportunity to take a new, and equally good, occupation with a 50-50 chance that it will double your income and spending power. But there is a 50-50 chance that it will cut your income and spending power by a third. Would you take the new occupation?

Individuals who answered that they would take this risky occupation were then asked about a riskier occupation:

Now, suppose the chances were 50-50 that the new occupation would double your/your family's income and 50-50 that it would cut it in half. Would you still take the new occupation?

In contrast, individuals who would not take the initial risky occupation were asked about a less risky occupation:

Now, suppose the chances were 50-50 that the new occupation would double your/your family's income and 50-50 that it would cut it by 20%. Then, would you take the new occupation?

Conditional on their first two responses, individuals were asked to consider a risky occupation with either a 75% downside risk or a 10% downside risk. These responses allow us to order individuals into six categories. Kimball, Sahm, and Shapiro (KSS) (2009) used maximum likelihood estimation and then impute the conditional expectation of risk aversion for each category c: θ_c. Thus, only one risk aversion in each category is available in their paper, which is shown in Table 3. However, I assume there is a continuous distribution of risk aversion in the whole population in the theoretical model, and I will make a further imputation based on their results. Guiso and Paiella (2005) show that aversion to losses is less pronounced among people with higher levels of education. Thomas, Armin, Huffman, and Uwe (2010) propose that exogenous personal characteristics, like age and gender, determine an individual's risk aversion. Thus, I use an individual's number of years of education (e), age (a), $age^2/100$ ($a^2/100$), male (m), children (c), and race (r) to predict their risk aversion based on the available six levels of risk aversion. The whole sample I use for my prediction is composed of employed people between 20 and 60 years old, whose population weights are nonzero. The imputation model is as follows:

$$\theta_c = \beta_0 + \beta_1 e + \beta_2 a + \beta_3 a^2/100 + \beta_4 m + \beta_5 c + \beta_6 m * c + \vec{\beta_7} r + \epsilon,$$

where $e \in \{1, 2, ..., 17\}, a \in \{20, 21, ..., 69\}, m \in \{1 = Male, 0 = Female\}, c \in \{1 =$

$HaveChildren, 0 = NoChildren\}$, r is a vector of dummies for race and $\vec{\beta_7}$ is a vector of coefficients.

Based on the above model, I run regressions and get the predicted value of θ_c on each category (e, a, m, c, $m * c$, r). Therefore, there will be a distribution of θ_c among the population. Table 4 reports the regression results for the whole sample and for subsamples of single and married individuals. Figure 14 shows that the shifted lognormal distribution of risk aversion among populations fits the data pretty well.

B Individual Consumption

The pre-tax wage for one unit of effective labor in sector 1 or sector 2 is

$$
\begin{aligned}
w_1 = \alpha_1\beta_1 L_1^{\beta_1-1} = \alpha_1\beta_1\Big[n(1-h(\theta_m))\int_0^{+\infty}\eta_{i1}dF(\eta_{i1})\Big]^{\beta_1-1},\\
w_2 = \alpha_2\beta_2 L_2^{\beta_2-1} = \alpha_2\beta_2\Big[nh(\theta_m)\int_0^{+\infty}\eta_{i2}dF(\eta_{i2})\Big]^{\beta_2-1}.
\end{aligned}
$$

The above equations are identical to

$$
\begin{aligned}
w_1 = \alpha_1\beta_1 L_1^{\beta_1-1} = \alpha_1\beta_1\Big[n(1-h(\theta_m))\Big]^{\beta_1-1},\\
w_2 = \alpha_2\beta_2 L_2^{\beta_2-1} = \alpha_2\beta_2\Big[nh(\theta_m)\Big]^{\beta_2-1}.
\end{aligned}
$$

If each worker is paid a wage equal to her productivity, then the wage for individual i in sector 1 or sector 2 is

$$
\begin{aligned}
w_{i1} = \eta_{i1}\alpha_1\beta_1\Big[n(1-h(\theta_m))\Big]^{\beta_1-1},\\
w_{i2} = \eta_{i2}\alpha_2\beta_2\Big[nh(\theta_m)\Big]^{\beta_2-1}.
\end{aligned}
$$

Therefore, the consumption for an individual who chooses to work in sector 1 or sector 2 is

$$
\begin{aligned}
c_{i1} = \eta_{i1}\alpha_1\beta_1(n(1-h(\theta_m)))^{\beta_1-1},\\
c_{i2} = \eta_{i2}\alpha_2\beta_2(nh(\theta_m))^{\beta_2-1} - \gamma,
\end{aligned}
$$

where γ is the cost of accumulating human capital (i.e., the cost of higher productivity α_2).

C Constant Expected Tax Revenue Condition

C.1 Expected Tax Revenue in the Flat Tax Regime

Under a flat tax regime, the expected tax revenue collected from workers is

$$E_w[R_\tau] = \underbrace{\tau(n(1-h(\theta_{m,F}))) \int_0^{+\infty} \left[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,F})))^{\beta_1-1}\right] \frac{1}{\eta_{i1}\sigma_1\sqrt{2\pi}} e^{\frac{-((log(\eta_{i1})-\mu_1)^2)}{2\sigma_1^2}} d\eta_{i1}}_{\text{total tax revenue collected in sector 1}}$$

$$+ \underbrace{\tau(nh(\theta_{m,F})) \int_0^{+\infty} \left[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,F}))^{\beta_2-1}\right] \frac{1}{\eta_{i2}\sigma_2\sqrt{2\pi}} e^{\frac{-((log(\eta_{i2})-\mu_2)^2)}{2\sigma_2^2}} d\eta_{i2}}_{\text{total tax revenue collected in sector 2}}$$

where $\mu_1 = \frac{-\sigma_1^2}{2}$ and $\mu_2 = \frac{-\sigma_2^2}{2}$.

C.2 Expected Tax Revenue in the Progressive Tax Regime

Under a progressive tax regime, the total expected tax revenue collected from workers is

$$E_w[R_\rho] = \underbrace{n(1-h(\theta_{m,P})) \int_0^{+\infty} a\left[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,P})))^{\beta_1-1}\right] + \rho\left[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,P})))^{\beta_1-1}\right] log\left[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,P})))^{\beta_1-1}\right] f(\eta_{i1}) d\eta_{i1}}_{\text{total tax revenue collected in sector 1}}$$

$$+ \underbrace{nh(\theta_{m,P}) \int_0^{+\infty} a\left[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,P}))^{\beta_2-1}\right] + \rho\left[\eta_{i2}\alpha_2\beta_2(n(1-h(\theta_{m,P})))^{\beta_2-1}\right] log\left[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,P}))^{\beta_2-1}\right] f(\eta_{i2}) d\eta_{i2}}_{\text{total tax revenue collected in sector 2}}.$$

C.3 Relationship

The tax revenue neutral condition implies $E_w[R_\tau] = E_w[R_\rho]$. Therefore, the relationship between τ and $(\rho, a)$is

$$\tau^* = \frac{E_w[R_\rho]}{n(1-h(\theta_{m,F}))\int_0^{+\infty}\left[\eta_{i1}\alpha_1\beta_1(n(1-h(\theta_{m,F})))^{\beta_1-1}\right]f(\eta_{i1})d\eta_{i1} + nh(\theta_{m,F})\int_0^{+\infty}\left[\eta_{i2}\alpha_2\beta_2(nh(\theta_{m,F}))^{\beta_2-1}\right]f(\eta_{i2})d\eta_{i2}}.$$

where $f(\eta_{i1}) = \frac{1}{\eta_{i1}\sigma_1\sqrt{2\pi}}e^{\frac{-((log(\eta_{i1})-\mu_1)^2)}{2\sigma_1^2}}$, $f(\eta_{i2}) = \frac{1}{\eta_{i2}\sigma_2\sqrt{2\pi}}e^{\frac{-((log(\eta_{i2})-\mu_2)^2)}{2\sigma_2^2}}$, $\mu_1 = \frac{-\sigma_1^2}{2}$, and $\mu_2 = \frac{-\sigma_2^2}{2}$.

In addition, when the value of ρ is changed, the value of a needs to be changed to match $E_w[R_\rho]$ given the initial value of ρ and a.

D Solution Algorithm

The computational procedure used to solve the equilibrium allocations and further welfare calculations in different progressive tax regimes can be represented by the following algorithm:

1) Given all α_1, α_2, β_1, β_2, σ_1^2, σ_2^2, a_1 and ρ_1 (i.e., $a_1 = 0.23$ and $\rho_1 = 0$), the equilibrium allocation of workers $h(\theta_{m,P1})$ in the progressive tax regime can be solved through equation (6).

2) Once $h(\theta_{m,P1})$ is solved, the related tax revenue, $E_w[R_{\rho 1}]$, can be calculated.

3) Then I increase the value from ρ_1 to ρ_2 to make the tax system more progressive. I define an interval of $a_2 \in [\underline{a_2}, \overline{a_2}]$. For every possible value of a_2, I find the new equilibrium, $h(\theta_{m,P2})$, and then $E_w[R_{\rho 2}]$. I search the value of a_2 in the interval to minimize the distance between $E_w[R_{\rho 2}]$ and $E_w[R_{\rho 1}]$. Therefore, I could find the matched a_2 and unchanged ρ_2.

4) The compensating consumption for individuals can be calculated based on the above values, and total welfare change can be further analyzed.

5) I further increase ρ_2 to ρ_3 and do the procedures 3) and 4) again.

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Table 1: Calibrated Parameters

Parameters	Value
h^*	0.521
β_1	0.67
β_2	0.67
α_1	85.4
α_2	100.0
γ	4.97

Table 2: Calibrated Parameters

Parameters	Value
h_m	0.533
w_1	\$16.66
w_{2L}	\$16.66
w_{2M}	\$27.08
w_{2H}	\$43.99
P_L	0.378
P_M	0.295
P_H	0.327
λ	0.488
γ	\$10,739

Table 3: Imputation of Risk Preference in KSS (2009)

Response Category	Risk Tolerance	Risk Aversion
1	0.27	6.7
2	0.40	4.2
3	0.49	3.5
4	0.60	2.8
5	0.79	2.2
6	1.22	1.4

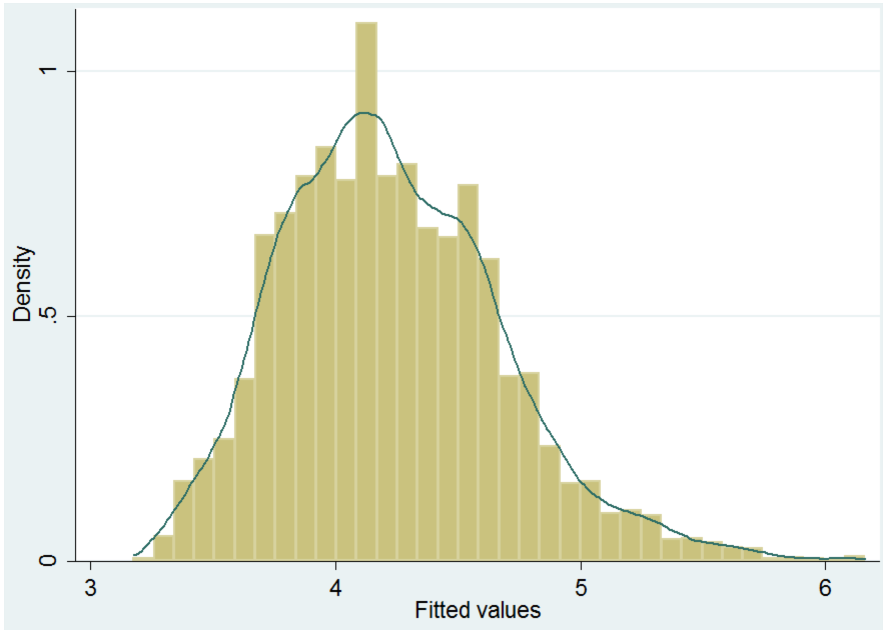
NOTE: This table comes from part of Table 1-Risk Tolerance in the PSID in Kimball, Sahm, and Shapiro (2009).

Table 4: Regression of θ_c on Personal Characteristics

	All	Single	Married
Years of education	-0.092***	-0.060**	-0.093***
	(0.012)	(0.029)	(0.015)
Age	-0.059***	-0.039	-0.034
	(0.022)	(0.053)	(0.031)
$Age^2/100$	0.112***	0.087	0.082**
	(0.027)	(0.072)	(0.038)
Male	-0.318***	-0.413***	-0.172*
	(0.072)	(0.134)	(0.103)
Children	0.040	0.015	0.039
	(0.077)	(0.206)	(0.105)
Male*children	0.025	0.638**	-0.110
	(0.103)	(0.319)	(0.134)
Black	0.414***	0.280*	0.325**
	(0.083)	(0.157)	(0.134)
American Indian, Aleut, Eskimo	0.317*	0.494	0.441*
	(0.186)	(0.466)	(0.249)
Asian, Pacific Islander	-0.137	-1.003	-0.083
	(0.476)	(0.876)	(0.575)
Hispanic	-0.185	0.010	0.126
	(0.414)	(0.898)	(0.534)
Other race	0.291	0.200	0.424
	(0.210)	(0.468)	(0.275)
Constant	5.932***	5.004***	5.394***
	(0.422)	(0.965)	(0.611)
Observations	4747	949	2793

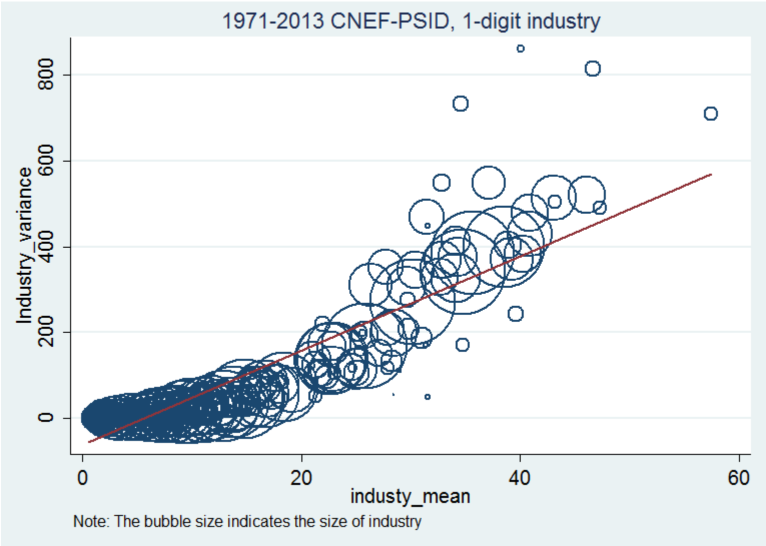
NOTE: A larger value of θ implies more risk aversion. Significant codes: 0.01 '***' 0.05 '**' 0.1 '*'.

Figure 1: The Distribution of Risk Aversion



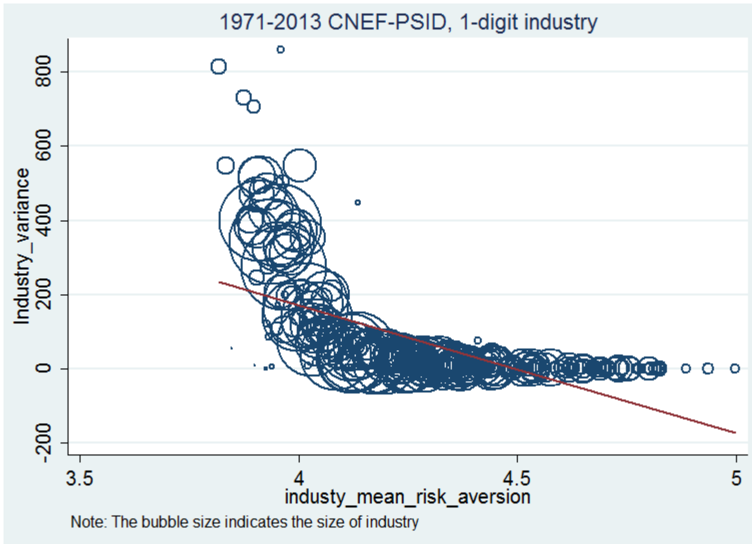
Note: The x-axis indicates the coefficient of relative risk aversion. A higher number means a worker is more risk averse.

Figure 2: The Relationship between Average Wage and Unobserved Variance of Wages Across Industries



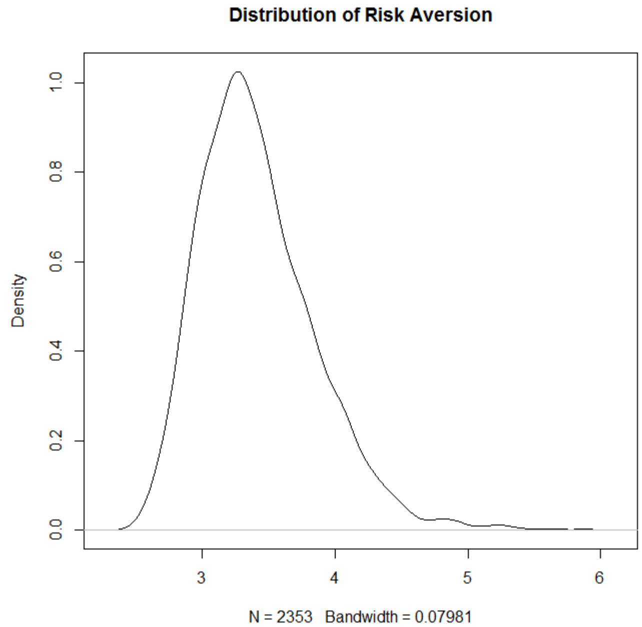
Note: The x-axis measures the mean real wage of workers in each industry each year. The y-axis measures the unobserved variance of wages in each industry each year by using residuals from the Mincer equations. The bubble indicates the size of the industry. The figure shows that a higher average wage compensates for a higher unobserved variance of wages in each industry each year.

Figure 3: The Relationship between Average Risk Aversion of Workers and Unobserved Variance of Wages Across Industries



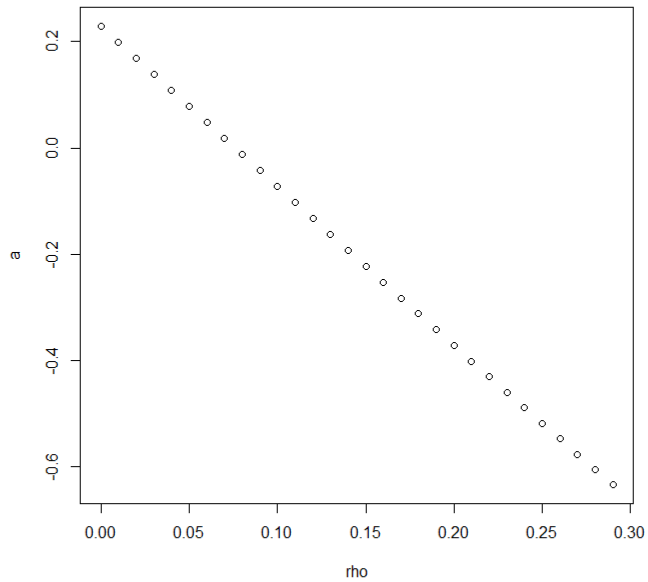
Note: The x-axis measures the average coefficient of relative risk aversion of full-time workers in each industry each year. A higher number means workers, on average, are more risk averse. The y-axis measures the unobserved variance of wages in each industry each year by using residuals from the Mincer equations.The bubble indicates the size of the industry. The figure shows that, on average, workers who are more risk averse will sort into industries that have a smaller unobserved variance of wages each year.

Figure 4: The Distribution of Risk Aversion in the Sample



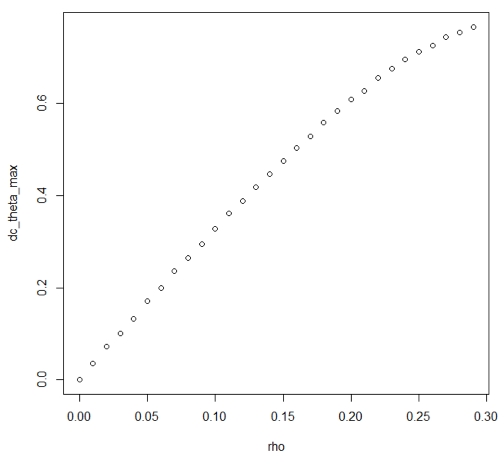
Note: Figure 4 shows the distribution of the coefficient of relative risk aversion for full-time workers in the sample in the year of 2000.

Figure 5: The Relationship between ρ and a for Constant Tax Revenue

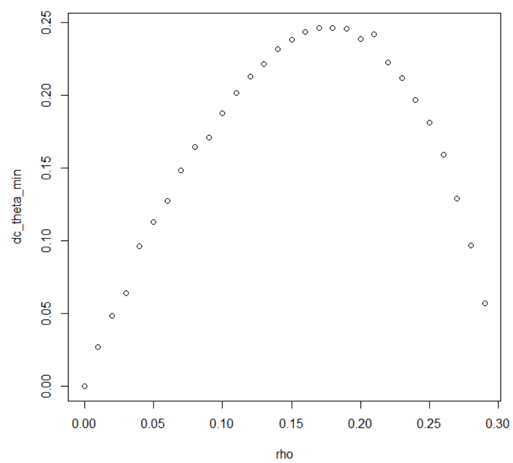


Note: There are two parameters, ρ and a, in the tax functional form I assume. In order to keep the tax revenue constant, when ρ is increased to make the tax system more progressive, a must be adjusted correspondingly. The tax revenue neutral relationship between ρ and a is shown here.

Figure 6: Individual Welfare Change when Tax is More Progressive

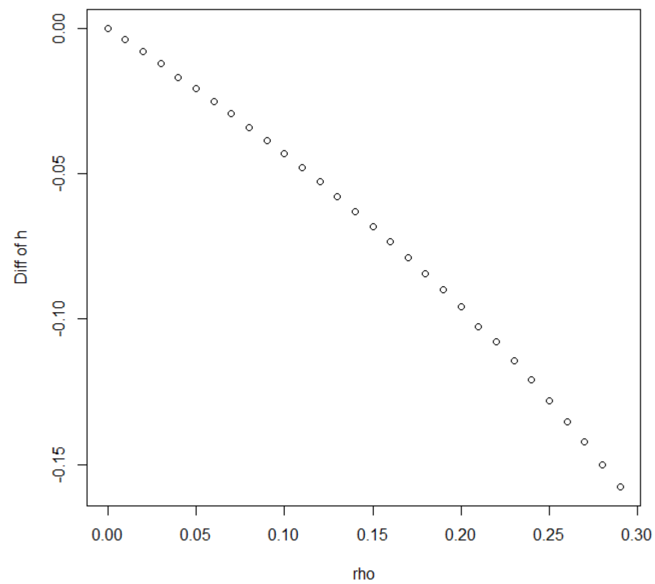


(a) Welfare Change for the Most Risk Averse Worker



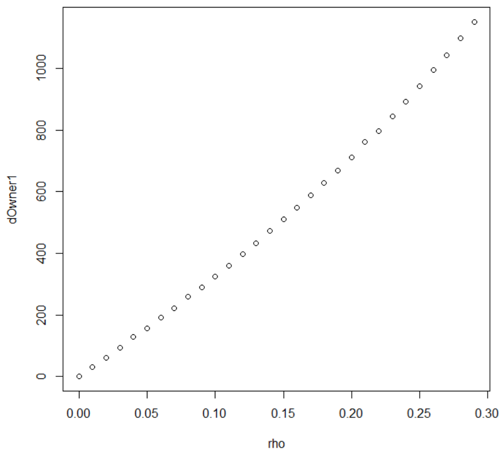
(b) Welfare Change for the Least Risk Averse Worker

Figure 7: Difference in Proportion of Workers in Sector 2: Progressive vs. Flat Tax Rate

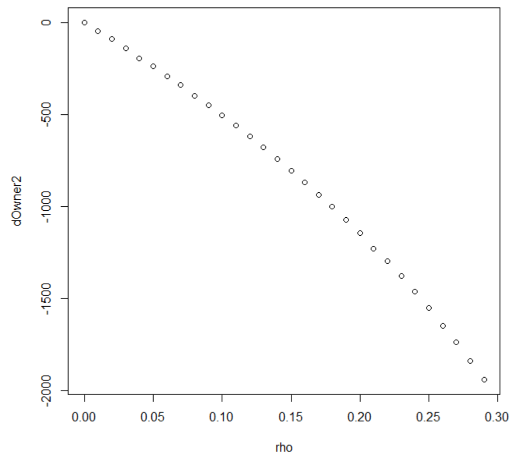


Note: When the tax rate is more progressive (i.e., the parameter in the tax function, ρ, is larger), workers are moving from sector 2 to sector 1.

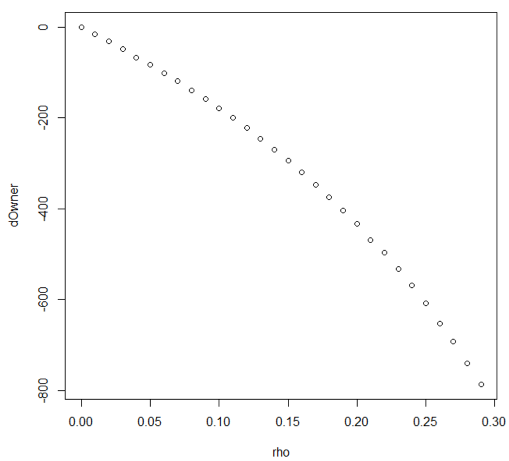
Figure 8: Welfare Change for the Capital Owners When the Tax is More Progressive



(a) Welfare Change for Owner in Sector 1



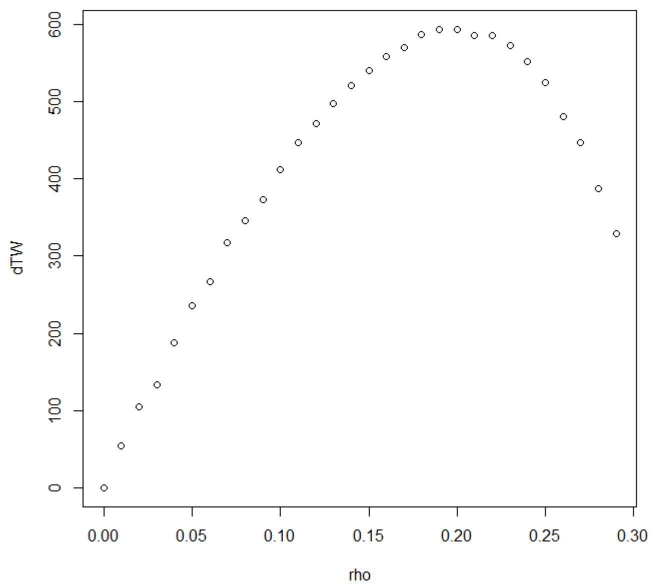
(b) Welfare Change for Owner in Sector 2



(c) Welfare Change for Both Owners

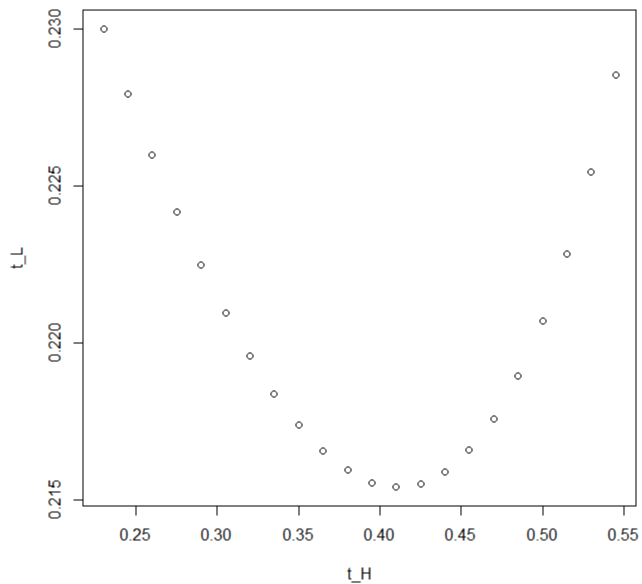
Note: Figure 8 shows the welfare change for capital owners when the tax system is more progressive, that is, the parameter in the tax function, ρ, is larger.

Figure 9: Welfare Change for Workers and Owners in the Aggregate Level



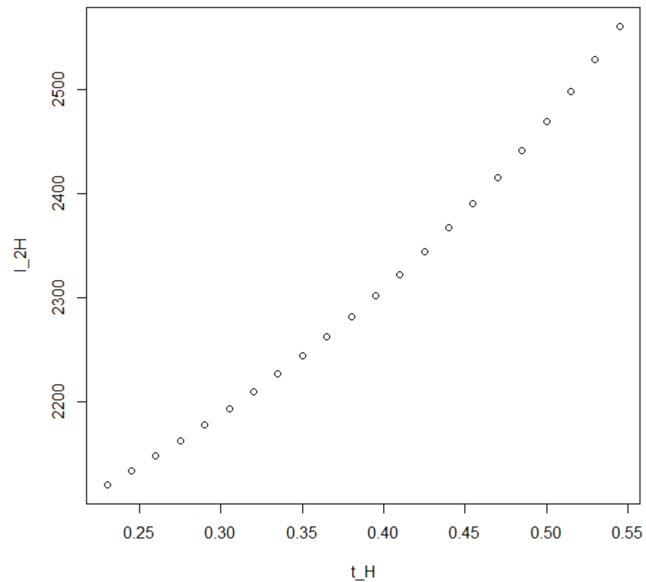
Note: Figure 9 shows shows the hump relationship between the change in welfare for the whole economy and the degree of progressiveness. When the parameter in the tax function, ρ, is larger, the tax system is more progressive. Thus, there is an efficient degree of progressivity.

Figure 10: The Relationship between Marginal Tax Rate t_L and Marginal Tax Rate t_H



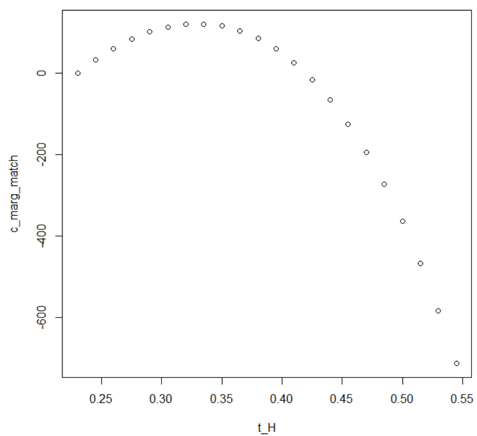
Note: Under the more progressive tax system, when t_H increases, t_L needs to be adjusted in order to keep the tax revenue constant. In addition, the flat tax regime indicates $t_H = t_L$, which is shown at the initial point in Figure 10.

Figure 11: The Relationship between Leisure l_{2H} and Marginal Tax Rate t_H

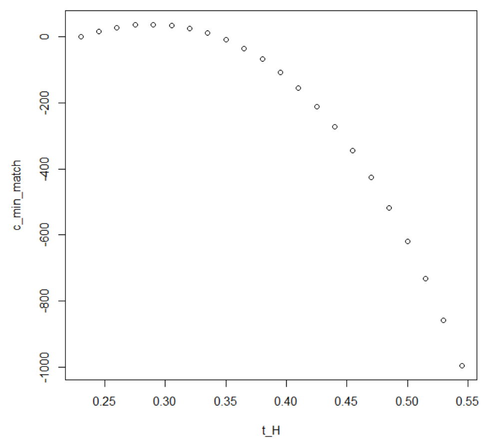


Note: When the tax system is more progressive (i.e., the marginal tax rate t_H increases), the labor supply distortion is larger (i.e., yearly hours of leisure, $l_{2H}(t_H)$, increases).

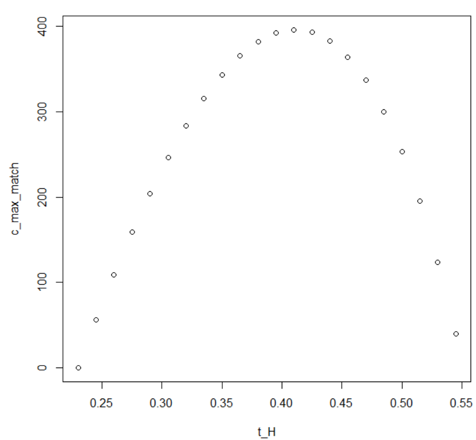
Figure 12: Individual Welfare Change with Respect to the Individual's Degree of Risk Aversion



(a) Welfare Change for θ_{marg} in Sector 2

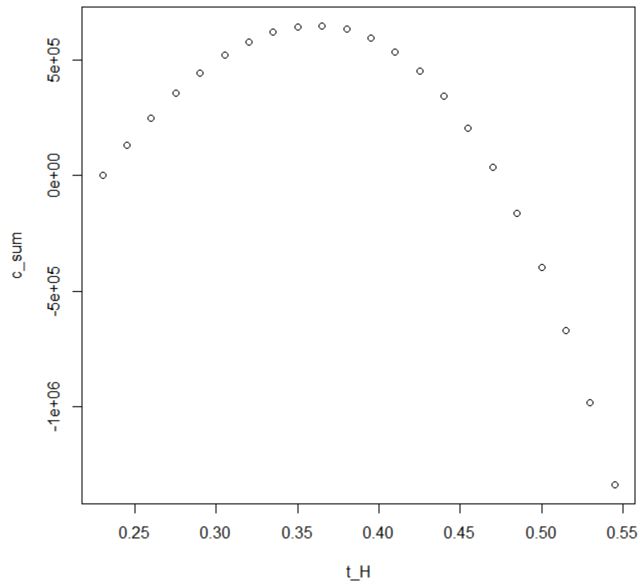


(b) Welfare Change for θ_{min} in Sector 2



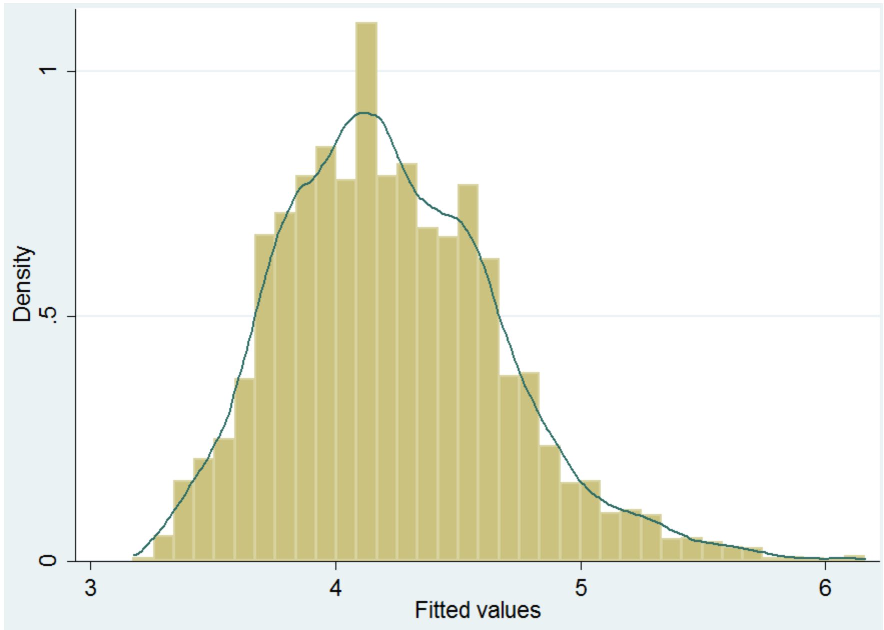
(c) Welfare Change for θ_{max} in Sector 1

Figure 13: The Relationship between Aggregate Welfare Change and Marginal Tax Rate t_H



Note: As the tax system becomes more progressive, the aggregate welfare first increases and then decreases after an inflection point. The efficient degree of progressivity is at the marginal tax rates $t_L = 21.7\%$ and $t_H = 36.5\%$, given $t_M = 23.0\%$.

Figure 14: The Distribution of Risk Aversion



Note: The x-axis indicates the coefficient of relative risk aversion. A higher number means a worker is more risk averse.