Preference Difference, Generalized Social Marginal Welfare Weights, and the Two Approaches for Optimal Tax Theory*

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Abstract

Many reasons can result in that the government's and individuals' preferences differ. We make two key contributions in this paper. First, we parameterize the relationship between the preference difference and the generalized social marginal welfare weights (Saez and Stantcheva, 2016) by following the definition of the latter. Second, we use an optimal tax example with a simple preference difference to demonstrate that one can take the traditional approach for optimal tax theory (Mirrlees, 1971) to derive the optimal marginal income tax as a function of the preference difference. Our work suggests that the traditional and the new approaches for optimal tax theory, pioneered by Mirrlees (1971) and Saez and Stantcheva (2016), respectively, are essentially dual.

Key words: Generalized Social Marginal Welfare Weights; Optimal Taxation; Preference Difference
JEL classification: H2

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I. Introduction

On one hand, many reasons can result in that the government's and individuals' preferences differ. A selective sample includes, for example, merit goods (Musgrave, 1959; Sandmo, 1983; Besley, 1988), specific egalitarianism (Tobin, 1970), paternalism (Thaler and Sunstein, 2003; O'Donoghue and Rabin, 2006), excluding warm glow from social welfare (Diamond, 2006; Andreoni, 2006; An, 2015)[1], tax sheltering (Chetty, 2009), equal sacrifice (Weinzierl, 2014), rent-seeking (Piketty et al., 2014; Rothschild and Scheuer, 2016; Lockwood et al., 2017), and political economy reasons (Bierbrauer et al., 2017).

On the other hand, Saez and Stantcheva (2016) first propose a new measure called the generalized social marginal welfare weights (GSMWW), and then pioneer a new approach for optimal tax theory to derive the optimal marginal income tax as a function of GSMWW, but two issues might need more elaboration and clarification. First, although they note that GSMWW can be derived from the most prominent alternatives to welfarism, it is defined up to a constant. Second, in spite of the fact that Weinzierl (2014) remarks that the traditional and the new approaches for optimal tax theory, pioneered by Mirrlees (1971) and Saez and Stantcheva (2016), respectively, "can be seen as two sides of the same coin," he stops short of elaborating it.

This paper makes two key contributions to address the aforementioned two issues. First, we follow the definition of GSMWW to derive the functional relationship between it and the preference difference.

Second, we use an optimal tax example with a simple preference difference to demonstrate that one can take the traditional approach to derive the optimal marginal income tax

[1] The warm glow model of charitable giving is developed by Andreoni (1989; 1990).

as a function of the preference difference.[2] Our example generalizes Diamond (1998)'s optimal income taxation model to allowing for the possibility of a simple preference difference.[3] We employ his model as our benchmark for two thoughtful reasons. The first reason is that, relative to Mirrlees (1971)'s pioneering optimal income taxation model, a key assumption made by him is that individuals' utility function is quasi-linear in consumption so that there are no income effects on labor supply, which greatly simplifies the theoretical analysis. The second, and more important, reason is that we want to be consistent with Saez and Stantcheva (2016) who make the same assumption of a quasi-linear utility function when they take the new approach to derive the optimal marginal income tax as a function of GSMWW.

Our two contributions combined suggest that the traditional and the new approaches for optimal tax theory are essentially dual. Hence, depending on particular contexts, one approach might be better or worse than the other, but no universal conclusion can be drawn.

Our work is related, and consistent, with another line of research on the sufficiency of using the elasticity of taxable income (ETI), namely, the effect of taxation on the reported taxable income, to calculate the deadweight loss (DWL). In a pioneering and influential study, Feldstein (1999) argues that ETI is a sufficient statistic to calculate DWL, and the etiology of the elasticity — whether it is, for example, labor supply, work effort, or evasion — is irrelevant. Chetty (2009) shows that tax sheltering would invalidate the sufficiency of ETI because it would generate fiscal externalities. An (2015) shows that excluding warm glow from social welfare (Diamond, 2006; Andreoni, 2006) also renders ETI insufficient to calculate DWL. The results of

[2]To avoid that our simple preference difference might mislead the reader, we clarify and emphasize at the very beginning that both GSMWW and preference difference are general concepts. The determinants of GSMWW can be quite general and complex. As discussed in the first paragraph of this paper, the reasons for preference difference can also be very broad and complicated.

[3]Kanbur et al. (2006) and Blomquist and Micheletto (2006) did similar extensions of Mirrlees (1971)'s pioneering optimal income taxation model and Stiglitz (1982)'s two-type optimal income taxation model, respectively, but clearly, both have different focuses from ours.

his numerical simulations suggest that using ETI to calculate DWL would result in underestimating the DWL by at least 10 percent.[4] An (2017) generalizes both Chetty (2009) and An (2015) to show that ETI is not sufficient to calculate DWL in the presence of preference difference, which is meaningful because, as discussed at the beginning of this paper, lots of other reasons can cause the government's and individuals' preferences to diverge.

Farhi and Gabaix (2015) explore an optimal income taxation model in the context of decision vs. experienced utility.[5] In terms of mathematical form, our optimal tax example with a simple preference difference is a variant of their model. This is not surprising since decision vs. experienced utility is clearly a specific case of preference difference. Nevertheless, there are two key differences. First, we interpret our example in the more general context of preference difference, rather than in the specific context of decision vs. experienced utility. Second, and more importantly, we combine it with our first contribution to illustrate the dual relationship of the two approaches for optimal tax theory. To some extent, we are "putting new wine into an old bottle."

The remainder of this paper is organized as follows. Sections II and III present our two key contributions, respectively. Section IV combines the two contributions to discuss the relationship of the two approaches for optimal tax theory. Finally, Section V briefly concludes.

II. Preference Difference versus GSMWW

Individuals are heterogeneous only in one dimension, namely, skill. An individual indexed by skill n has a marginal product equal to n. The distribution of skill is written as

[4]This suggests that preference difference is important not only conceptually, but also quantitatively, which hence strengthens the motivations for this research.

[5]See also Gerritsen (2016) and Lockwood (2017).

$F(n)$, with density $f(n)$. It is assumed that the distribution of skill is single-peaked. The density is assumed to be positive and continuous in the range of $[n_{\min}, n_{\max}]$.

Individual n has a utility function $u^n\left(c(n), l(n)\right)$, where $c(n)$ denotes his consumption, $l(n)$ denotes his labor supply, $u_c^n \geq 0$ and $u_l^n \leq 0$. His pre-tax income is equal to his labor income $nl(n)$. The total income tax for income $nl(n)$ is $T\left(nl(n)\right)$.[6] His consumption is equal to the after-tax income, namely, $c(n) = nl(n) - T\left(nl(n)\right)$.

The government's and individuals' preferences could differ for various reasons. We use $\overline{u^n}\left(c(n), l(n)\right)$ to denote the government's preference for individual n. In general, $\overline{u^n}\left(c(n), l(n)\right) \neq u^n\left(c(n), l(n)\right)$ with preference difference.

In the presence of preference difference, the government's preference for individual n is employed to evaluate the social welfare. The social welfare function can thus be stated as

$$\begin{aligned} W &= \int_{n_{\min}}^{n_{\max}} G\left(\overline{U^n}\left(c(n), l(n)\right)\right) f(n)\,dn \\ &= \int_{n_{\min}}^{n_{\max}} G\left(U^n\left(c(n), l(n)\right) + \left(\overline{U^n}\left(c(n), l(n)\right) - U^n\left(c(n), l(n)\right)\right)\right) f(n)\,dn \end{aligned}, \qquad (1)$$

where $U^n\left(c(n), l(n)\right)$ is the utility attained by individual n with his own preference, $\overline{U^n}\left(c(n), l(n)\right)$ is the utility attained by individual n with the government's preference for him, and $G(\bullet)$ is an increasing and strictly concave function of utility, with G independent of n.

By Saez and Stantcheva (2016), GSMWW represents "the value that society puts on providing an additional dollar of consumption to any given individual." That is, it measures how

[6] As the government can only observe earnings, not labor supply or skill, it is restricted to setting taxes as a function only of earnings.

much society values the marginal consumption of any given individual. Mathematically, we have:

$$g^n \equiv \frac{dW}{dc(n)}, \qquad (2)$$

where g^n denotes the GSMWW for individual n.

From Equations (1) and (2), and by simple algebra and decomposition, we have:

$$g^n = G'\left(U^n + \left(\overline{U^n} - U^n\right)\right) * \left(U^n_{c(n)} + \left(\overline{U^n_{c(n)}} - U^n_{c(n)}\right)\right), \qquad (3)$$

where $U^n_{c(n)}$ denotes the partial derivative of U^n with respect to $c(n)$, and $\overline{U^n_{c(n)}}$ denotes the partial derivative of $\overline{U^n}$ with respect to $c(n)$, respectively.

In Equation (3), the preference difference is captured by both $\left(\overline{U^n} - U^n\right) \neq 0$ and $\left(\overline{U^n_{c(n)}} - U^n_{c(n)}\right) \neq 0$. Thus, we have parameterized the relationship between the preference difference and GSMWW by following the definition of the latter, which is the first key contribution of this paper.

III. An Optimal Tax Example with a Simple Preference Difference

The optimal income taxation problem pioneered by Mirrlees (1971) is that the government chooses an income tax schedule to maximize the social welfare, subject to two constraints. The first one is the government budget constraint. The second one is that individuals optimize in their choice of labor supply, taking as given the income tax schedule chosen by the government. The government is assumed to have only one policy instrument, namely, income taxation. Individuals are assumed to be heterogeneous only in one dimension, namely, skill, with an individual indexed by skill n having a marginal product equal to n. The government is restricted to setting taxes as a function only of earnings because it can only observe earnings, not labor supply or skill.

Relative to Mirrlees (1971)'s groundbreaking optimal income taxation model, a key assumption made by Diamond (1998) is that individuals' utility function is quasi-linear in consumption, namely, $u\left(c(n), l(n)\right) = c(n) + v\left(1 - l(n)\right) = nl(n) - T\left(nl(n)\right) + v\left(1 - l(n)\right)$. This assumption suggests that there are no income effects on labor supply, which greatly simplifies the theoretical analysis.

Denoting the government expenditures as E, the government budget constraint can be stated as:

$$\int_{n_{\min}}^{n_{\max}} \left(u\left(c(n), l(n)\right) - v\left(1 - l(n)\right)\right) f(n)\, dn \le \int_{n_{\min}}^{n_{\max}} nl(n) f(n)\, dn - E, \qquad (4)$$

where the left-hand side is aggregate consumption, and the right-hand side is aggregate production minus government expenditures. Thus, Equation (4) says that aggregate consumption be less than aggregate production minus government expenditures.

Each individual is assumed to choose his own labor supply to maximize his own utility, taking $T\left(nl(n)\right)$ as given. The first-order condition for individual choice can be written as:

$$v'\left(1 - l(n)\right) = n\left(1 - T'\left(nl(n)\right)\right), \qquad (5)$$

where T' is the marginal income tax rate.

Using Equation (5), the change in consumption with skill satisfies:

$$c'(n) = \left(l(n) + nl'(n)\right)\left(1 - T'\right) = \frac{\left(l(n) + nl'(n)\right) v'\left(1 - l(n)\right)}{n}. \qquad (6)$$

With the quasi-linear utility function, one can calculate the derivative of u with respect to n:

$$u'\left(c\left(n\right),l\left(n\right)\right)=c'\left(n\right)-v'\left(1-l\left(n\right)\right)l'\left(n\right)=\frac{l\left(n\right)v'\left(1-l\left(n\right)\right)}{n}. \qquad (7)$$

For later use when deriving the optimal income tax schedule, it is convenient to note that for the quasi-linear utility function, the elasticity of labor supply evaluated at the chosen labor supply of an individual of skill n, $e\left(n\right)$, satisfies:

$$e\left(n\right)=\frac{-v'\left(1-l\left(n\right)\right)}{l\left(n\right)v''\left(1-l\left(n\right)\right)}. \qquad (8)$$

Since the wage equals the skill level, this is the elasticity with respect to the wage, evaluated at the labor supply level that is chosen by someone with skill n.

Now, suppose that the government's and individuals' preferences could differ. For analytical convenience, our optimal tax example assumes a simple preference difference:

$$\bar{u}\left(c\left(n\right),l\left(n\right)\right)=c\left(n\right)+\left(1+\delta\right)v\left(1-l\left(n\right)\right)=u\left(c\left(n\right),l\left(n\right)\right)+\delta v\left(1-l\left(n\right)\right), \qquad (9)$$

where δ measures the preference difference between the government and individuals. In general, $\delta \neq 0$, and thus $\delta v\left(\bullet\right)\neq 0$ in the presence of preference difference. Diamond (1998) assumes that $\delta = 0$, which means that the government's and individuals' preferences are identical, that is, the government respects individuals' preferences. Hence, his model is a special case of our example with preference difference.

In the presence of preference difference, the government's preferences for individuals are employed to evaluate the social welfare. Hence, the optimal tax problem of the government can be written as:

$$\begin{aligned}
&\max \int_{n_{\min}}^{n_{\max}} G\left(\bar{u}\left(c(n), l(n)\right)\right) f(n)\, dn = \int_{n_{\min}}^{n_{\max}} G\left(u\left(c(n), l(n)\right) + \delta v\left(1-l(n)\right)\right) f(n)\, dn,\\
&\textit{subject to}:\\
&\int_{n_{\min}}^{n_{\max}} \left(u\left(c(n), l(n)\right) - v\left(1-l(n)\right)\right) f(n)\, dn \le \int_{n_{\min}}^{n_{\max}} nl(n) f(n)\, dn - E;\\
&u'\left(c(n), l(n)\right) = \frac{l(n)v'\left(1-l(n)\right)}{n}.
\end{aligned} \qquad (10)$$

When $\delta = 0$, Equation (10) is exactly reduced to the optimal tax problem solved by Diamond (1998) for the special case of no preference difference:

$$\begin{aligned}
&\max \int_{n_{\min}}^{n_{\max}} G\left(u\left(c(n), l(n)\right)\right) f(n)\, dn,\\
&\textit{subject to}:\\
&\int_{n_{\min}}^{n_{\max}} \left(u\left(c(n), l(n)\right) - v\left(1-l(n)\right)\right) f(n)\, dn \le \int_{n_{\min}}^{n_{\max}} nl(n) f(n)\, dn - E;\\
&u'\left(c(n), l(n)\right) = \frac{l(n)v'\left(1-l(n)\right)}{n}.
\end{aligned} \qquad (11)$$

Treating $u\left(c(n), l(n)\right)$ as a state variable and $l(n)$ as a control variable, Equation (10) is a standard optimal control problem. One can basically follow the routine procedures to solve this optimal control problem, and derive the first-order condition for the optimal tax:

$$\frac{T'}{1-T'} = \frac{\delta G'(n)}{\lambda} + \frac{\left(1+\frac{1}{e(n)}\right)\int_{n}^{n_{\max}} \left(\lambda - G'(n)\right) dF(n)}{\lambda n f(n)}, \qquad (12)$$

where λ is the multiplier with respect to the government budget constraint.

Two points can be made from Equation (12). First, when $\delta = 0$, Equation (12) is exactly reduced to the first-order condition for the optimal tax derived by Diamond (1998) for the special case of no preference difference:

$$\frac{T'}{1-T'} = \frac{\left(1+\frac{1}{e(n)}\right)\int_{n}^{n_{\max}} \left(\lambda - G'(n)\right) dF(n)}{\lambda n f(n)}. \qquad (13)$$

Second, from Equation (12), it is straightforward to prove that when δ increases, the optimal marginal income tax rate T' increases. This result intuitively makes sense. When δ increases, it means that the government places a larger weight on individuals' utility derived from leisure, and hence it should set a higher marginal income tax rate so as to discourage labor supply.

Thus, we have used an optimal tax example with a simple preference difference to demonstrate that one can take the traditional approach, pioneered by Mirrlees (1971), to derive the optimal marginal income tax as a function of the preference difference, which is the second key contribution of this paper.

IV. The Traditional versus the New Approach for Optimal Tax Theory

Figure 1 depicts the general picture of our idea. It consists of three key points. First, it visualizes the functional relationship between the preference difference and GSMWW, as shown by our first key contribution. Second, it indicates that one can take the traditional approach to derive the optimal marginal income tax as a function of the preference difference, as demonstrated by our second contribution. Finally, it points out that one can employ the new approach to derive the optimal marginal income tax as a function of GSMWW, as illustrated by Saez and Stantcheva (2016).

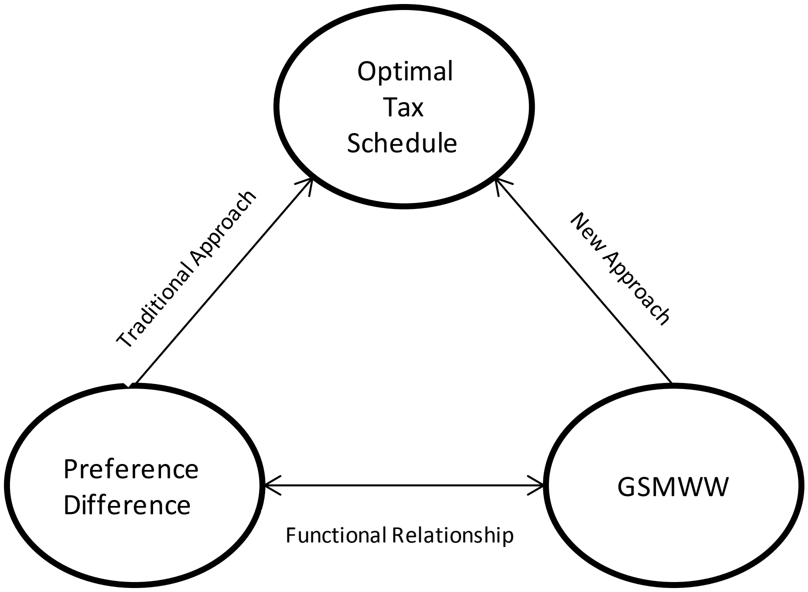
[Figure 1 here]

From Figure 1, it is clear to see that the two approaches for optimal tax theory are essentially dual. Hence, no universal conclusion can be drawn regarding which approach is better or worse than the other; the results of comparison likely depend on specific contexts.

V. Conclusion

Numerous reasons can cause the government's and individuals' preferences to differ. We make two key contributions in this paper. First, we have followed the definition of GSMWW to derive the functional relationship between it and the preference difference. Second, we have used an optimal tax example with a simple preference difference to demonstrate that can one take the traditional approach for optimal tax theory to derive the optimal marginal income tax as a function of the preference difference. Our work suggests a dual relationship of the two approaches for optimal tax theory.

Figure 1. Preference Difference vs. GSMWW and Traditional vs. New Approach



Note: This figure depicts the general picture of our idea. It consists of three key points. First, it visualizes the functional relationship between the preference difference and GSMWW, as shown by our first contribution. Second, it indicates that one can take the traditional approach to derive the optimal marginal income tax as a function of the preference difference, as demonstrated by our second contribution. Finally, it points out that one can employ the new approach to derive the optimal marginal income tax as a function of GSMWW, as illustrated by Saez and Stantcheva (2016). This figure suggests that the two approaches for optimal tax theory are essentially dual.

REFERENCES

An, Z., "On the Sufficiency of Using the Elasticity of Taxable Income to Calculate Deadweight Loss: The Implications of Charitable Giving and Warm Glow," *International Tax and Public Finance*, 2015, 22, 1040-1047.

An, Z., "When Preferences Differ: ETI Insufficient to Calculate DWL," *Tax Notes*, 2017, 155(6), 825-827.

Andreoni, J., "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence," *Journal of Political Economy*, 1989, 97(6), 1447-1458.

Andreoni, J., "Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving," *Economic Journal*, 1990, 100(401), 464-477.

Andreoni, J. "Philanthropy." In *Handbook of the Economics of Giving, Altruism and Reciprocity*. Edited by Serge-Christophe Kolm and Jean Mercier Ythier, 1201-1269. 2006. Amsterdam: Elsevier/North-Holland.

Besley, T., "A Simple Model for Merit Good Arguments," *Journal of Public Economics*, 1988, 35, 371-383.

Bierbrauer, F., A. Tsyvinski, and N. D. Werquin, "Taxes and Turnout," *NBER Working Paper No. 24123*, 2017.

Blomquist, S., and L. Micheletto, "Optimal Redistributive Taxation when Government's and Agents' Preferences Differ," *Journal of Public Economics*, 2006, 90(6), 1215-1233.

Chetty, R., "Is the Taxable Income Elasticity Sufficient to Calculate Deadweight Loss? The Implications of Evasion and Avoidance," *American Economic Journal: Economic Policy*, 2009, 1(2), 31-52.

Diamond, P., "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 1998, 88(1), 83-95.

Diamond, P., "Optimal Tax Treatment of Private Contributions for Public Goods with and without Warm Glow Preferences," *Journal of Public Economics*, 2006, 90, 897-919.

Farhi, E., and X. Gabaix, "Optimal Taxation with Behavioral Agents," *NBER Working Paper No. 21524*, 2015.

Feldstein, M., "Tax Avoidance and the Deadweight Loss of the Income Tax," *Review of Economics and Statistics*, 1999, 81(4), 674-680.

Gerritsen, A., "Optimal Taxation when People Do Not Maximize Well-Being," *Journal of Public Economics*, 2016, 144, 122-139.

Kanbur, R., J. Pirttila, and M. Tuomala, "Non-welfarist Optimal Taxation and Behavioral Public Economics," *Journal of Economic Surveys*, 2006, 20(5), 849-868.

Lockwood, B. B., "Optimal Income Taxation with Present Bias," *Working Paper*, 2017.

Lockwood, B. B., C. G. Nathanson, and E. G. Weyl, "Taxation and the Allocation of Talent," *Journal of Political Economy*, 2017, 125(5), 1635-1682.

Mirrlees, J. A., "An Exploration in the Optimal Theory of Income Taxation," *Review of Economic Studies*, 1971, 38(2), 175-208.

Musgrave, R. A. *The Theory of Public Finance*. 1959. New York: McGraw-Hill.

O'Donoghue, T., and M. Rabin, "Optimal Sin Taxes," *Journal of Public Economics*, 2006, 90, 1825-1849.

Piketty, T., E. Saez, and S. Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities," *American Economic Journal: Economic Policy*, 2014, 6(1), 230-271.

Rothschild, C., and F. Scheuer, "Optimal Taxation with Rent-Seeking," *Review of Economic Studies*, 2016, 83(3), 1225-1262.

Saez, E., and S. Stantcheva, "Generalized Social Marginal Welfare Weights for Optimal Tax Theory," *American Economic Review*, 2016, 106(1), 24-45.

Sandmo, A., "Ex Post Welfare Economics and the Theory of Merit Goods," *Economica*, 1983, 50, 19-33.

Stiglitz, J. E., "Self-selection and Pareto Efficient Taxation," *Journal of Public Economics*, 1982, 17(2), 213-240.

Thaler, R. H., and C. R. Sunstein, "Libertarian Paternalism," *American Economic Review*, 2003, 93, 175-179.

Tobin, J., "On Limiting the Domain of Inequality," *Journal of Law and Economics*, 1970, 13, 263-277.

Weinzierl, M., "The Promise of Positive Optimal Taxation: Normative Diversity and a Role for Equal Sacrifice," *Journal of Public Economics*, 2014, 118, 128-142.