Interest deductibility and capital investment

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Overview

In partial equilibrium, value-maximizing firm
- investment, dividends, tax liability
- responses to tax rate, inflation rate changes

Findings:

1% increase in corporate tax rate decreases capital stock by 0.098%.
- Depreciation and interest deductions largely mitigate the negative effect of higher tax rate on capital.

1% increase in inflation rate increases capital stock by 1.057%.
- Higher inflation encourages more capital investment when partially financed by debt.
This work

- Detailed modeling of corporate taxation that reflects the current U.S. tax code.
- Tax shield (interest deductibility) determines the firm’s optimal debt policy.
- Calibrate the model to match observed corporate credit spread, leverage ratio.
Firm’s problem

A representative firm owns capital, takes as given wage, $w_t$, the discount factor, $\frac{1}{1+R_t}$. It chooses labor, $l_t$, next period’s capital, $k_{t+1}$, debt, $b_{t+1}$.

$$V_t(k_t, b_t) = \max\{l_t, k_{t+1}, b_{t+1}\} d_t(l_t, k_t, k_{t+1}, b_t, b_{t+1}) + \frac{1}{1+R_t} V_{t+1}(k_{t+1}, b_{t+1})$$

subject to

$$0 < l_t \leq L_t(w_t)$$

$$b_{t+1} \geq 0, k_{t+1} > 0$$
Dividends

\[ d_t = y_t \]

\[ - w_t l_t \]

\[ - i_t \]

\[ - C_k(k_t, k_{t+1}) \]

\[ - \zeta_t \text{otherExp}_t y_t \]

\[ + b_{t+1} \]

\[ - (1 + \rho_t) b_t \]

\[ - T_t \]

(Output)  (Labor cost)  (Investment)  (Capital adjustment cost)  (Other expenses)  (Business debt proceeds)  (Business debt repayment)  (Corporate taxation)
Corporate income tax liability

The total amount of corporate income tax liability is:

\[ T_t = \tau_t \Pi_t^{\text{taxable}} - \zeta_t^{\text{invCred}} i_t - \zeta_t^{\text{otherCred}} y_t \]

where taxable income is:

\[ \Pi_t^{\text{taxable}} = y_t \]

\( y_t \) (Sales)

\( - w_t l_t \) (Labor deduction)

\( - \phi_t^{\text{int}} \rho_t b_t \) (Interest deduction)

\( - x_t \) (Depreciation deduction)

\( - C_k(k_t, k_{t+1}) \) (Capital installation deduction)
Tax shield

Tax shield is deductible amount $\phi^{\text{int}}_t$ of nominal interest payment:

$$\phi^{\text{int}}_t \rho_t b_t = \phi^{\text{int}}_t (R_{t-1} b_t + \nu(b_t, k_t))$$

The convex leverage cost function $\nu(\cdot)$ is defined by:

$$\nu(b_t, k_t) = \frac{1}{\nu_1} \left( \frac{b_t}{\nu_2 k_t} \right)^{\nu_1} \nu_2 k_t$$
Firm chooses debt to maximize tax benefit net of leverage cost:

\[ b_{t+1}^* = \left( \frac{\tau_{t+1} \phi_{t+1}^{int} R_t}{1 - \tau_{t+1} \phi_{t+1}^{int}} \right)^{\frac{1}{\nu_1-1}} \nu_2 k_{t+1} \]

- \( \nu_1 = 2.92 \): elasticity of corporate credit spread w.r.t. debt to fixed assets using quarterly data between 1953Q2 and 2020Q3.
- \( \nu_2 = 6.90 \): given \( R = 2.34\% \), debt to fixed assets ratio averaged between 2018Q1 and 2020Q3 (46.3%).

\[ \text{Difference between Moody’s seasoned AAA corporate bond yield and 20-year Treasury constant maturity rate.} \]
A marginal increase in corporate tax rate

- $\tau = 21\%, \ R = 2.34\%$

<table>
<thead>
<tr>
<th>$\phi^{\text{exp}} = 0, \phi^{\text{int}} = 0$</th>
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<tbody>
<tr>
<td>$\frac{dk}{d\tau}$</td>
<td>-2.129</td>
<td>-1.922</td>
<td>-0.393</td>
</tr>
<tr>
<td>$\frac{k}{dk}$</td>
<td>-0.141</td>
<td>-0.129</td>
<td>-0.038</td>
</tr>
<tr>
<td>$\frac{k}{dCTR}$</td>
<td>0.097</td>
<td>0.098</td>
<td>0.038</td>
</tr>
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Higher tax rate increases cost of capital, $\frac{r}{1-\tau}$, lowers capital stock.
A marginal increase in corporate tax rate

- $\tau = 21\%, \ R = 2.34\%$

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<td>0.034</td>
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- Cost of capital increases by less, $\frac{r}{1-\tau} - \frac{\tau \phi^{exp} r}{1-\tau}$, since the firm is able to deduct investment over time and $\phi^{exp}$ is high.
A marginal increase in corporate tax rate

\[ \tau = 21\%, \quad R = 2.34\% \]

\[
\begin{array}{cccccc}
\phi^\text{exp} = 0, \phi^\text{int} = 0 & \phi^\text{exp} = 0, \phi^\text{int} \neq 0 & \phi^\text{exp} \neq 0, \phi^\text{int} = 0 & \phi^\text{exp} \neq 0, \phi^\text{int} \neq 0 \\
\frac{dk}{d\tau} & -2.129 & -1.922 & -0.393 & -0.098 \\
\frac{dk}{dd} & -0.141 & -0.129 & -0.038 & -0.029 \\
\frac{d\tau}{dC_{TR}} & 0.097 & 0.098 & 0.038 & 0.034
\end{array}
\]

Cost of capital increases by less, \(\frac{r}{1-\tau} - \frac{\tau \phi^\text{int} r}{1-\tau} - \frac{\tau \phi^\text{int} \pi}{1-\tau}\), since the firm is able to deduct nominal interest expense.
A marginal increase in corporate tax rate

- $\tau = 21\%, \ R = 2.34\%$

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</tr>
<tr>
<td>$\frac{dCTR}{d\tau}$</td>
<td>0.097</td>
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- Deduction of inflation component of nominal interest rate and deduction of investment largely offset the increase in cost of capital,

$$\frac{r}{1-\tau} - \frac{\tau \phi^{\text{int}} r}{1-\tau} - \frac{\tau \phi^{\text{int}} \pi}{1-\tau} - \frac{\tau \phi^{\exp} r}{1-\tau}.$$
A marginal increase in corporate tax rate

- $\tau = 21\%$, $R = 2.34\%$

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Smaller decline in capital stock and lower tax liability restore dividends.
A marginal increase in corporate tax rate

- \( \tau = 21\% \), \( R = 2.34\% \)

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<td>0.098</td>
<td><strong>0.038</strong></td>
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- Higher tax rate raises corporate income taxes for each dollar of profit.
- Depreciation and interest deductions increase, tax revenue decreases.
- Firm changes its investment and debt policies, which affects tax revenue.
A marginal increase in inflation rate

\( \tau = 21\%, \ R = 2.34\% \)

\[
\begin{array}{c|cccc}
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\hline
\frac{dk}{d\pi} & 0 & 1.690 & -1.026 & 1.057 \\
\frac{dk}{dd} & 0 & 0.056 & -0.046 & 0.011 \\
\frac{dk}{dCTR} & 0 & 0.029 & 0.045 & 0.041 \\
\frac{dk}{d\pi} & 0 & 0 & 0 & 0 \\
\end{array}
\]

- Inflation has no effect on cost of capital, \( \frac{r}{1-\tau} \). Capital stock, dividends and corporate tax revenue remain the same.
A marginal increase in inflation rate

- \( \tau = 21\% \), \( R = 2.34\% \)

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<tr>
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<td>0</td>
<td>0.029</td>
<td>0.045</td>
</tr>
<tr>
<td>( \frac{d\pi}{d\pi} )</td>
<td>0</td>
<td>0</td>
<td>0.041</td>
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Inflation decreases the present value of tax deduction of investment, increases cost of capital, \( \frac{r}{1-\tau} - \frac{\tau \phi^{\text{exp}} r}{1-\tau} \), and lowers capital stock.

- Dividends drop: lower production and lower depreciation deduction.
- Tax revenue increases: lower depreciation deduction offsets lower corporate profits.
A marginal increase in inflation rate

- $\tau = 21\%, \ R = 2.34\%$

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Higher inflation lowers cost of capital, $r \frac{r}{1-\tau} - \frac{\tau \phi^{int} r}{1-\tau} - \frac{\tau \phi^{int} \pi}{1-\tau}$, and leads to higher capital stock.

- Dividends increase: higher production, higher interest deduction offset higher interest on debt.

- Tax revenue increases: higher tax revenue from interest income, corporate profits outweigh higher interest deduction.
A marginal increase in inflation rate

\[ \tau = 21\%, \ R = 2.34\% \]

\[
\begin{align*}
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\frac{dk}{d\pi} & = 0 & 1.690 & -1.026 & 1.057 \\
\frac{dk}{dd} & = 0 & 0.056 & -0.046 & 0.011 \\
\frac{dk}{d\text{CTR}} & = 0 & 0.029 & 0.045 & 0.041 \\
\frac{dk}{d\pi} & = 0 & & & \\
\end{align*}
\]

Capital stock expands as higher deduction of inflation component of nominal interest dominates lower present value of depreciation deduction,

\[
\frac{r}{1-\tau} - \tau \phi^{\text{int}} \frac{r}{1-\tau} - \tau \phi^{\text{int}} \frac{\pi}{1-\tau} - \tau \phi^{\text{exp}} \frac{r}{1-\tau}.
\]

Dividends increase: higher production, higher interest deduction outweigh lower depreciation deduction, higher interest on debt.

Tax revenue increases: higher tax revenue from interest income, corporate profits, lower depreciation deduction outweigh higher interest deduction.
Conclusion

- Changing statutory corporate tax rate alone does not have a big impact → other tax provisions matter.
- **Higher tax rate**: small negative effect on capital stock.
- **Higher inflation**: capital stock expands when partially financed by debt.
Appendix: Dividends

\[ \begin{align*}
    y_t &= A_t k_t^{\alpha_k} l_t^{\alpha_l} \\
    i_t &= k_{t+1} - (1 - \delta)k_t \\
    C_k(k_t, k_{t+1}) &= \frac{\eta}{2} \left( \frac{k_{t+1} - (1 - \delta)k_t}{k_t} \right)^2 k_t
\end{align*} \]
Appendix: Depreciation deduction

In the present year $t$, the firm’s depreciation deduction is:

$$x_t = \sum_{s=0}^{40} \psi_s l_{t-s} (1 - \zeta_{t-s}^{invCred})$$

- $\psi_s$ is the allowed depreciation in schedule year $s$.

In steady state, present value of tax deduction of a dollar of current investment:

$$\phi^{exp} = \sum_{s=0}^{40} \frac{1}{(1 + R)^s} \psi_s (1 - \zeta_{invCred})$$
## Appendix: Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>adjustment cost</td>
<td>$\eta$</td>
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<tr>
<td>economic depreciation</td>
<td>$\delta$</td>
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<tr>
<td>capital share</td>
<td>$\alpha_k$</td>
<td>0.34</td>
</tr>
<tr>
<td>labor share</td>
<td>$\alpha_l$</td>
<td>0.66</td>
</tr>
<tr>
<td>TFP</td>
<td>$A$</td>
<td>1</td>
</tr>
<tr>
<td>leverage sensitivity</td>
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</tr>
<tr>
<td>leverage scaling</td>
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<td>6.90</td>
</tr>
<tr>
<td>corporate tax rate</td>
<td>$\tau$</td>
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</tr>
<tr>
<td>depreciation deduction</td>
<td>$\phi^{exp}$</td>
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</tr>
<tr>
<td>interest deduction</td>
<td>$\phi^{int}$</td>
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</tr>
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<td>other expenses</td>
<td>$\zeta_{otherExp}$</td>
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</tr>
<tr>
<td>investment credit</td>
<td>$\zeta^{invCred}$</td>
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<td>other tax credit</td>
<td>$\zeta_{otherCred}$</td>
<td>0.012</td>
</tr>
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<td>interest rate</td>
<td>$R$</td>
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<tr>
<td>inflation rate</td>
<td>$\pi$</td>
<td>0.02</td>
</tr>
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</table>

**Table:** Parameter description and value
Appendix: Optimal 1

To condense notation, define in the steady state:

$$\chi_{div} = 1 - \zeta^{otherExp} - \tau + \zeta^{otherCred}$$

$$\chi_{debt} = \left( \frac{\tau \phi^{int} R}{1 - \tau \phi^{int}} \right)^{\frac{1}{\nu_1 - 1}} \nu_2$$

The first-order condition with respect to the firm’s labor demand implies that in the steady state,

$$l = \left[ \frac{w(1 - \tau)}{\alpha_l A k^{\alpha_k} \chi_{div}} \right]^{\frac{1}{\alpha_l - 1}}$$
Appendix: Optimal $k$

The first-order condition with respect to $k_{t+1}$ implies:

$$
\frac{\partial d_t}{\partial k_{t+1}} + \frac{1}{1 + R_t} \frac{\partial d_{t+1}}{\partial k_{t+1}} + \ldots + \frac{1}{(1 + R_t)(1 + R_{t+40})} \frac{\partial d_{t+41}}{\partial k_{t+1}} = 0
$$

and it follows that in the steady state,

$$
- \eta(1 - \tau)\delta - 1 + \tau \phi^{\exp} + \zeta^{\text{invCred}} + \chi_{\text{debt}} + \frac{1}{1 + R} \left[ \alpha_k A k^{\alpha_k - 1} l^{\alpha_l} \chi_{\text{div}} (1 + \pi) \right]
$$

$$
- \frac{\eta}{2} (1 - \tau) \left( (1 - \delta)^2 - 1 \right) (1 + \pi) + (1 - \delta)(1 - \tau \phi^{\exp} - \zeta^{\text{invCred}})(1 + \pi)
$$

$$
- \chi_{\text{debt}} - \chi_{\text{debt}} (1 - \tau \phi^{\text{int}}) R - (1 - \tau \phi^{\text{int}}) \frac{\nu_2^{1 - \nu_1}}{\nu_1} \left( \frac{b}{k} \right)^{\nu_1}
$$

where $\pi$ is the inflation rate.
Appendix: Comparative statics

To a first-order approximation by using the implicit function theorem, the change in $k$ and $b$ caused by a small change in $\tau$ satisfies:\(^2\)

$$\begin{bmatrix}
\frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial b} \\
\frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial b}
\end{bmatrix}
\begin{bmatrix}
dk \\
\db
\end{bmatrix}
= \begin{bmatrix}
-\frac{\partial f_1}{\partial \tau} d\tau \\
-\frac{\partial f_2}{\partial \tau} d\tau
\end{bmatrix}$$

When $\phi^\text{int} = 0$, $b = 0$. The change in $k$ satisfies:

$$\frac{dk}{k} = - \left(\frac{\partial f_1}{\partial k} k\right)^{-1} \frac{\partial f_1}{\partial \tau}$$

- $f_1$: first-order condition with respect to $k_{t+1}$.
- $f_2$: optimal debt equation.

\(\frac{\partial f_1}{\partial k}\) is the partial derivative of $f_1$ with respect to $k$. 
Appendix: Comparative statics

Using Cramer’s rule,

\[ dk = \begin{vmatrix} -\frac{\partial f_1}{\partial \tau} d\tau & \frac{\partial f_1}{\partial b} \\ -\frac{\partial f_2}{\partial \tau} d\tau & \frac{\partial f_2}{\partial b} \end{vmatrix} \begin{vmatrix} \frac{\partial f_1}{\partial k} \\ \frac{\partial f_2}{\partial k} \end{vmatrix} \]

After some algebra, we have:

\[ \frac{dk}{k} d\tau = -\frac{\partial f_1}{\partial \tau} \frac{\partial f_2}{\partial b} + \frac{\partial f_1}{\partial b} k \frac{\partial f_2}{\partial \tau} \]

\[ \frac{dk}{d\tau} = \frac{\partial f_1}{\partial k} k \frac{\partial f_2}{\partial b} - \frac{\partial f_1}{\partial b} k \frac{\partial f_2}{\partial k} \]
Appendix: Comparative statics

Similarly,

\[
\frac{db}{k \, d\tau} = -\frac{\partial f_1}{\partial k} \frac{\partial f_2}{\partial b} - \frac{\partial f_1}{\partial b} \frac{\partial f_2}{\partial k} + \frac{\partial f_1}{\partial \tau} \frac{\partial f_2}{\partial k} \quad \frac{\partial f_1}{\partial k} \frac{\partial f_2}{\partial \tau} - \frac{\partial f_1}{\partial \tau} \frac{\partial f_2}{\partial k}
\]

which implies that,

\[
\frac{db}{k} = -\frac{\partial f_1}{\partial k} k \frac{\partial f_2}{\partial \tau}
\]
Appendix: Comparative statics

The comparative static derivative of $k$ (in percentage) and $b$ with respect to $\pi$ can be analogously calculated as:

\[
\frac{dk}{k} \frac{d\pi}{d\pi} = -\frac{\partial f_1}{\partial \pi} \frac{\partial f_2}{\partial b} + \frac{\partial f_1}{\partial b} k \frac{\partial f_2}{\partial k} + \frac{\partial f_1}{\partial k} k \frac{\partial f_2}{\partial b} - \frac{\partial f_1}{\partial b} k \frac{\partial f_2}{\partial k}
\]  

(1)

and

\[
\frac{db}{k} \frac{d\pi}{d\pi} = -\frac{\partial f_1}{\partial k} \frac{\partial f_2}{\partial b} + \frac{\partial f_1}{\partial b} \frac{\partial f_2}{\partial k} + \frac{\partial f_1}{\partial k} k \frac{\partial f_2}{\partial b} - \frac{\partial f_1}{\partial b} k \frac{\partial f_2}{\partial k}
\]

(2)