INDIVIDUAL SAVINGS ACCOUNTS PROPOSALS: BUDGETARY AND ECONOMIC EFFECTS*

Jane Gravelle, Congressional Research Service

BOTH THE ADMINISTRATION AND THE PRESIdent's Advisory Panel on Tax Reform (2005) have proposed changes in individual retirement accounts (IRAs), replacing current accounts with two accounts, for retirement and for other purposes. Plans would increase limits on annual contributions, lift income ceilings, and are Roth-style (back-loaded) IRAs. The Administration's non-retirement account is unrestricted, while the Panel's account would restrict qualified withdrawals to education, health, and new home purchase. Earlier Administration and the Panel proposals limit contributions to $10,000 for each plan; more recent Administration proposals have a $5,000 limit (now available, as a result of 2006 legislation). The Advisory Panel also proposed IRAs under their consumption tax, presumably because they kept an add-on tax on interest, dividends, and capital gains.

Evaluating these proposals involves not only evaluating IRAs, but also their form. The traditional (front-loaded, or deductible) IRA, the only form until 1997, is similar to a pension plan: contributions are deducted, earnings are not taxed, and withdrawals are taxed. The Roth-style, or back-loaded approach, simply exempts the earnings. Both result in an effective tax rate on earnings of zero with a fixed discount rate and a fixed tax rate. Although there are income limits for both types, higher income individuals can open IRAs that allow deferral, but not forgiveness, of tax by accumulating funds in a nondeductible IRA and paying tax on the payments reflecting earnings. The timing of the tax benefits differs between the two types: while both benefit the taxpayer (and cost the government) with the earnings exclusion, the front-loaded IRA has an up-front benefit and an eventual payment of taxes. This timing has implications for many aspects of IRAs: the budgetary cost, the savings behavior of the taxpayer, the magnitude of the benefit, the sharing of risk and return between the government and taxpayer, the possibilities for creating non-zero tax rates with traditional IRAs if tax rates change, and the effective penalty structure.

The first part of the paper discusses form, whether for an expansion or as a change in the existing IRAs. The second part discusses more general issues surrounding IRAs.

COMPARING THE FRONT-LOADED AND BACK-LOADED FORMS

The two forms are equivalent under certain assumptions, effectively eliminating the tax on earnings: with constant tax rates, no early withdrawal, and no effect of the ceiling. To illustrate, assume a 25 percent tax rate, a $1,000 contribution, a 10 percent interest rate, and a 20-year holding period. In a taxable account, taxes are paid each year for an after-tax return of 7.5 percent and an amount after 20 years of $4,247.85 ($1,000 times 1.075^{20}). In a back-loaded IRA, the taxpayer pays no tax and the account is $6,727.50 ($1,000 times 1.10^{20}). The front-loaded account can yield the same outcome as the back-loaded, but the amount in the account must be larger (divided by 1 minus the tax rate). Thus, the contribution is $1,000/(1 – .25) or $1,333.33, the taxpayer receives a saving of $333.33 (–0.25 times $1,333.33), with a net cost of $1,000. After 20 years, the account is $8,970 ($1,333.33 times 1.10^{20}). After a tax of 25 percent, the amount would be $6,727.50, as in the back-loaded account.

The taxpayer needs to set aside a larger amount for the front-loaded account. If the same ceilings are allowed, and a taxpayer is at the ceiling (as about two thirds are according to data in Bryant and Sailer, 2006), the taxpayer has a smaller benefit in the front-loaded account. He or she could invest the tax savings in a taxable account but the combination of those accounts would yield a smaller future benefit of $6,107.59 ($1,000 times 1.10^{20} times 0.75 plus $250 times 1.075^{20}).

The front-loaded account can differ from the back-loaded if tax rates vary over time. If the rate is higher at contribution than at withdrawal, the tax

*The views in this paper do not represent the views of the Congressional Research Service or the Library of Congress.

rate is negative. For example, a zero tax at withdrawal yields $8,970, an 11.59 percent return and a negative 16 percent tax rate (0.10 – 0.1159)/.10. A 35 percent rate at withdrawal yields $5,830, a 9.2 percent return and 8 percent tax rate. Burman, Gale, and Weiner (2001) found a –11 percent tax for contributions made in 1982 and withdrawn in 1995 holding the tax law constant. It is possible, however, for a front-loaded account to yield less than a taxable account, especially when taxable accounts are invested in tax-favored assets.

Finally, as discussed later, if amounts are withdrawn prematurely, and subject to penalties, the benefits (or penalties, with shorter holding periods) will differ. The timing of these tax benefits is also important to budget discipline, savings response, and tax complexity.

Budget Issues in the Form of IRAs

IRAs began in 1974 for those without pensions and their front-loaded tax benefits were similar to pensions. In 1981 they were made universally available, but in 1986 were restricted to lower and moderate income individuals, along with those in an employer plan. Higher income individuals could defer taxes in an IRA (called a nondeductible IRA), by putting nondeductible amounts into an IRA and paying taxes on benefits in excess of contributions when withdrawn (much like an annuity). Penalties applied for early withdrawal from IRAs, and mandatory distributions were required.

Discussion began quickly, however, about restoring IRAs, and allowing a "back-loaded" account where the revenue cost would not occur upfront. It was generally recognized that the interest in back-loaded accounts was motivated by budget issues. Legislation in 1997 increased income limits slightly for traditional IRAs (phased out for incomes between $50,000 and $60,000 for joint returns), but significantly for new "back-loaded" Roth IRAs (phased out between $150,000 and $160,000). Both were allowed a penalty-free early withdrawal for a home purchase; the Roth also allowed a withdrawal for disability. Individuals could roll over existing IRAs into Roths (paying the tax) if income was less than $100,000. These forms of IRAs have two effects on the budget. First, for any expansion of coverage, the cost for a Roth is very small in the short run relative to the long run. Second, substituting Roth-style IRAs for traditional existing or planned IRAs would gain revenue in the present and lose it in the future.

These patterns are shown in Figure 1 (showing the pattern for new IRA revenue costs) and Figure 2 (showing the gain from switching from a front-loaded to a back-loaded IRA) for an illustrative example that assumes that contributions grow in an account for 15 years and are paid out as an annuity

Figure 1

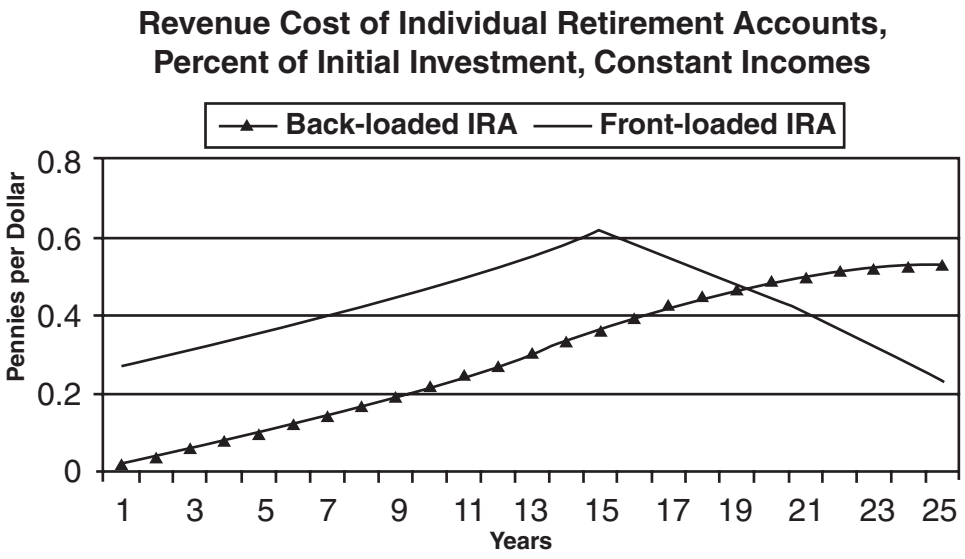
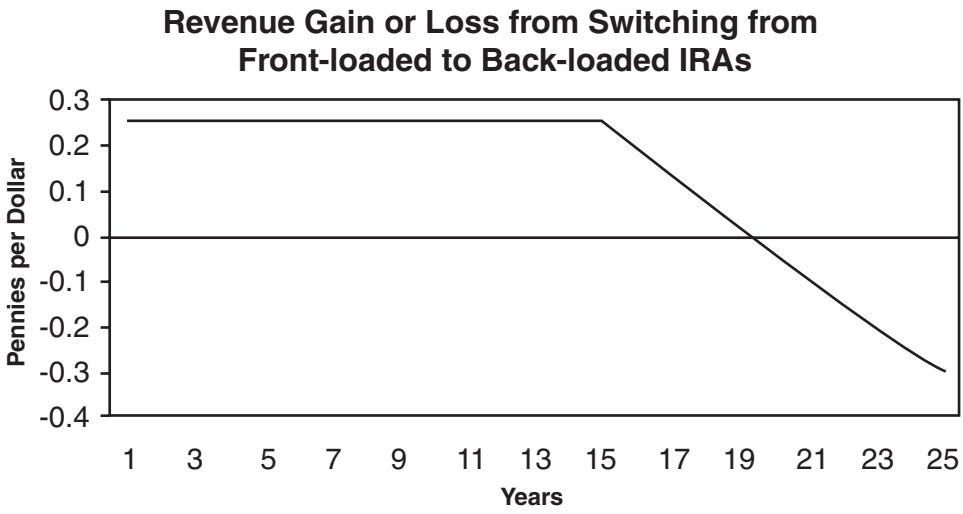


Figure 2



over 10 years. (See appendix for calculations.) In the first year, earnings from a Roth IRA per dollar of contributions is r, where r is the interest rate; in the second year, the value is $r(1 + r) + r$, or slightly more than double. Based on a 7 percent interest rate and a 3 percent growth rate (g), keeping all measures constant relative to GDP, in the first year revenue costs are 3 percent of their steady-state value (reached in 25 years). By the 5th year, they are 18 percent; by the 10th year, 41 percent. The first 5 years' cost is 11 percent of the steady state cost; over 10 years, one-fifth.

Front-loaded IRAs' initial cost is much closer to the steady state (higher or lower depending on the assumptions and slightly higher in this example). The cost rises until the withdrawals start and then declines. In the illustration, revenue costs in the first year are 20 percent larger than their steady-state value. By the 5th year, they are 56 percent larger and by the 10th year, they are twice as large. On average, over 4 years, the cost is 38 percent larger; over 10 years, 62 percent larger.

The second figure plots the value per dollar of contribution of switching from a front-loaded account to a back-loaded account.

To minimize the cost of an IRA expansion during the budget horizon, the back-loaded IRA is more attractive. And, if one simultaneously expands IRAs via a back-loaded account and also switches existing front-loaded IRAs to back-loaded ones, it is possible to raise revenue in the short run, as in the Administration's proposals. It was estimated to gain $1.5 billion over 10 years, even though Gravelle and Shvedov (2006) estimated the long run 10-year cost to be between $300 billion to $500 billion! This difference was more pronounced (although not separately reported) in the Panel's consumption tax, which shifted both IRAs and 401(k)s to back-loaded form. Revenue estimates for the administration proposal also assume some displacement of 401(k) contributions by the expanded IRAs. Also, there are significant rollovers from 401(k) plans, the largest source of IRA contributions, and if these rollovers continued, tax would be collected. In 2002, contributions to all IRAs was $42 billion, while rollovers, largely from employer plans, were $204 billion.

These proposals also allow a voluntary rollover of existing IRAs into back-loaded forms. In 1998 Roth assets were $57 billion (including both new contributions and rollovers), while there were $160 billion of rollovers from employee plans to traditional IRAs, $11.9 billion of new contributions, and a remaining stock of traditional IRAs of $1.9 trillion (Investment Company Institute, 2006).

The Effect of Form on Savings Behavior

The objective of the IRA is to induce additional private savings for retirement. The form of the tax benefits, in theory, should lead to different sav-

ings incentives, depending on how capable and informed taxpayers are. This discussion does not address whether private savings is increased or decreased; rather, it compares the expected difference in savings effects between the two forms.

Standard Optimizing Model

In the standard model, the front-loaded plan produces greater savings. Since contribution to a front-loaded IRA trades off the up-front deduction for a future tax, individuals should save more with a front-loaded IRA to pay that tax. Holding other taxes and spending constant, the additional short-run cost and consequent reduction in government saving of a front-loaded IRA compared to a back-loaded one should be exactly offset by the increase in private savings, leaving national savings unaffected by the form of the benefit. If shifting from a front-loaded to a back-loaded IRA is not accompanied by a reduction in the deficit, overall national savings should fall, and the proposals would decrease both private and national savings.

Rule of Thumb Models

This conclusion presumes individuals are rational, capable, informed, and forward looking. A case can be made that savings decisions are too complex to make an optimal savings choice (Thaler, 1994). What determines savings behavior? One possibility would be a rule of thumb, such as a fixed percentage of saving, or a fixed amount of assets upon retirement. With a reduction in tax, aggregate saving would not change (with the fixed saving rate) or would decline (as the tax benefit allows less saving to reach a fixed target). If individuals recognize that front-loaded IRA savings are not, effectively, as large as back-loaded IRA savings, because of the tax on withdrawal, front-loaded IRAs would lead to more private savings. Otherwise, savings would be similar.

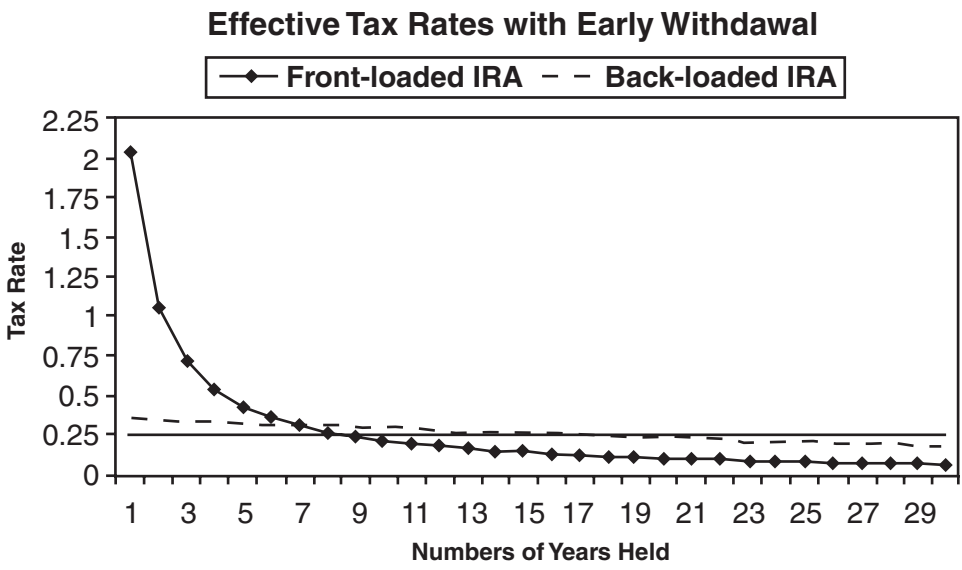
Mental Accounts, Up-Front Deduction and Penalty Structures

Another issue discussed by Thaler (1994) is the possibility that individuals cannot discipline themselves to save. The front-loaded IRA may have advantages, if the attraction of the up-front tax saving helps induce people to save. There is some evidence that investors prefer the front-loaded form. Hrung (2004) reports that prior to the 1997 changes, 70.2 percent of returns were eligible for a deductible IRA; afterwards, 85.7 percent were eligible, with the increase unlikely due to traditional IRAs, where the income limit was increased only slightly. In 1997, 3.3 percent of returns had a deductible IRA; in 1998, 3.1 percent had a deductible IRA and 2.8 percent had a Roth IRA.

Penalties on early withdrawal (currently 10 percent) may help individuals maintain discipline but may discourage individuals from committing funds to IRAs. The penalty structure differs dramatically (and perhaps unintentionally) between the current front-loaded and back-loaded forms. Figure 3 shows the effective tax rates on front-loaded and back-loaded IRAs as a function of the holding period, if the taxpayer is in the 25 percent tax bracket. If the tax rate is u, the penalty p, the rate of return before tax r, and the holding period t, the taxpayer gives up $(1 - u)$ in the front-loaded account and receives $(1 - u - p)$ times $(1 + r)^t$. Thus to solve for the after-tax rate of return, $(1 + R)^t = (1 - u - p)(1 + r)^t/(1 - u)$. The effective tax rate is $(R - r)/r$. For the front-loaded IRA the additional tax compared to a qualified distribution is the penalty on both principal and interest. For the back-loaded IRA, R solves the equation, $(1 + R)^t = (1 - u - p)(1 + r)^t + u + p$. In this back-loaded case, the additional tax compared to a qualified distribution is the penalty and regular tax on the accumulated earnings (since principal has never been deducted).

As shown in Figure 3, the penalties are very high for assets that need to be withdrawn quickly from a front-loaded IRA and they apply to both the principal and interest. For the back-loaded account the penalties are not nearly as great. However, the back-loaded account imposed a more severe burden for premature withdrawals for amounts that have been in the account for a long time when one is close to eligibility without a penalty. All non-qualified withdrawals from a front-loaded account reduce proceeds by 13 percent compared to a qualified withdrawal; however, for the back-loaded account proceeds are reduced by 31 percent for amounts held for 30 years, and by 24 percent for amounts held for 20 years. The growing penalty occurs because early withdrawal exposes the return to a tax that would not have been imposed at all, and the share of the account that comprises return grows over time.

Figure 3



The penalty structure has mixed effects: the front-loaded approach discourages IRA investments of funds that might be needed for other purposes for younger people, but for older individuals, the back-loaded penalty discourages withdrawals. Which pattern leads to the smaller savings effects is not clear, although it would seem likely that the penalty in the front-loaded approach would be a greater barrier, since many risks have resolved themselves for older individuals, those individuals are likely to have other assets, and there is an exception for disability for the back-loaded account. This more favorable penalty structure has not, however, been reflected in a preference for back-loaded IRAs, suggesting that it may not play a very important role in taxpayers' decisions.

There is no particular reason to have such dramatically differing penalty structures, as the penalty could be altered to have the same effect. No one seems to have paid much attention to the differing penalty structures in the front-loaded and back-loaded accounts. The administration, however, has also proposed a no-strings-attached account (and the Panel one with penalties but which can be withdrawn for other purposes). Given the contribution limits, for many taxpayers savings will be available in accounts with no penalty.

Complexity, Transparency, Tax Planning, and Transition

While the front-loaded account appears to be superior in imposing budgetary discipline and increasing private savings more (or not decreasing it as much), back-loaded accounts are simpler for the taxpayer to deal with. There is no concern about differing tax rates and timing of contributions and withdrawals. The possibility of large penalties if the money is needed in the near term is much less, and the danger of ultimately paying more tax because of the penalty for early withdrawal is greatly decreased.

If both a front-loaded IRA and back-loaded form are available, as they are to some taxpayers, the taxpayer now faces a tax planning problem, especially if income is variable. When tax rates are low, a back-loaded IRA is preferable. Similarly, the option to roll over a front-loaded IRA into a back-loaded one, which currently exists, complicates tax planning. In general, therefore, the complexity issue suggests only one type of IRA should be allowed and in general that form would be a back-loaded account. But it would probably be necessary, in practice, to maintain front-loaded accounts at least for purposes of rollovers from 401(k) plans, which is the major source of con-

tributions to IRAs, since many taxpayers may not want to pay the tax on such a significant lump sum of money. Allowing an option for rollovers into a front-loaded versus back-loaded would continue to complicate choice. In that case, an IRA form consistent with 401(k) and other employer plans might be simpler, suggesting a front-loaded, not a back-loaded, IRA.

Summing Up the Comparison

The front-loaded IRA is superior for fiscal restraint and saving. The back-loaded IRA simplifies taxpayer planning, but choice complicates it more, suggesting the front loaded approach given rollovers, not the back-loaded approach in the proposals.

THE GENERAL EFFECT OF IRAS AND PROPOSED CHANGES

As with many tax issues, the fundamental standards for evaluating IRAs are equity and efficiency.

Equity and Distributional Issues

IRAs originated in 1974 due to horizontal equity concerns: to permit a tax-favored retirement plan to taxpayers not covered by employer plans. In 1981, the IRA was made universally available with a focus on increasing private saving, but fully deductible IRAs were disallowed for higher income taxpayers covered by employer plans in 1986, reflecting the distributional and revenue neutrality objectives. Thus, equity and distributional concerns have clearly played an important role in the development of IRAs.

Despite the current income limits and contribution ceilings, IRAs favor higher income individuals. In the bottom half of the income distribution, as shown in Table 1, less than 10 percent of eligible taxpayers participate and their contributions are smaller. In addition, the main source of IRA contributions is rollovers from 401(k) and similar employer plans and these plans are much more likely to cover high-income individuals. Thus, while the benefit of IRAs is not as concentrated in the aggregate towards higher income individuals as much as some capital income tax relief provisions (such as lower tax rates on capital gains and dividends), it does favor high incomes.

Efficiency Gains

It is important to return to the question of what type of economic model we have in mind. In the traditional economic model, individuals are rational, utility maximizing individuals and taxes on capital income create distortions by taxing marginal investments. Alternatively, we have the "Rule of Thumb" model and the "Mental Accounts" model.

Traditional Economic Model

In this world, IRAs don't make much sense, as they have several features that undermine their effectiveness at reducing distortions from taxing capital income.

First, by tacking on the IRA treatment to an existing, somewhat flawed, income tax system,

Table 1
Who Contributes to IRAs?

Income Class (thousands)	*Share of Total Returns*	*Percentage Eligible*	*Percentage of Eligible Contributors*	*Average ($1)*
None	1.3	28.5	16.6	2,095
Under 10	14.6	78.2	2.4	1,360
10-20	15.5	77.5	4.3	1,500
20-30	13.1	76.6	7.5	1,671
30-40	10.5	79.3	9.1	1,746
40-50	8.7	81.2	11.2	1,864
50-75	16.0	82.9	12.3	2,143
75-100	9.3	85.1	15.7	2.492
100-200	8.6	84.1	21.5	3.092
Over 200	2.5	73.2	22.7	5,410
Total	100.0	79.4	10.3	2,412

Source: Calculated from data in Bryant and Sailer (2006).

they have the potential to create negative tax rates, which not only result in revenue loss but also, like positive tax rates, cause distortions. Because nominal interest is deducted, debt-financed assets are taxed at a negative rate for businesses, of approximately 32 percent (Gravelle 2006). And debt-financed investment in owner-occupied homes has a huge negative tax rate because no imputed income is taxed and property taxes are also deductible. Yet the greatest benefits currently for IRAs are for interest bearing assets, which remain subject to ordinary tax rates (unlike dividends and capital gains where ordinary tax rates are already lower).

Second, IRAs have income caps and dollar caps. If a taxpayer is contributing to IRAs at the limit, there is no tax benefit on the marginal investment even if that benefit were desirable. Based on data reported by Bryant and Sailer (2006), two-thirds of both traditional and Roth IRA contributions were made at one of the limits for 2002 ($3,000 or $3,500). In addition, the income limit requires a phase out, which introduces implicit marginal tax rates over the phase-out range.

Thirdly, premature withdrawals from IRAs are subject to a penalty which, if the investment is not held very long, especially in the case of front-loaded IRAs, actually raises taxes. Thus, to take advantage of the tax benefits of IRAs, the taxpayer loses the liquidity of having savings that can also be used for unanticipated emergencies, unless an extra tax is paid. By paying this prospective price for the tax benefit, the value of it has been reduced.

If efficiency were the only objective, then the revenue lost by IRAs would be better spent on reductions in capital income tax rates on the most heavily taxed investments and without introducing liquidity constraints.

In this model of the world, some of the proposed changes may be desirable, but others are not. Evaluating these changes is difficult given the basic flaws of IRAs noted above, since it is not clear that making more of IRAs marginal (which is likely if contribution limits are increased) or more valuable (if no-strings accounts are allowed) are really desirable if they are increasing distortions. Lifting the income limits would largely be a windfall to high-income people who save at the limit. The Administration's "no-strings-attached" plan might have merit. Increasing the dollar limits would be more focused on marginal investments.

The Panel's plan is inferior to the Administration's in two respects. For the income tax plan, dividend and most capital gains taxes are removed and thus IRAs will largely benefit debt financed investments, the most lightly taxed investment, and lead to negative tax rates. The Advisory Panel's general account comes with restrictions. The Advisory Panel's proposals are not as distorting in the case of their consumption tax because businesses are no longer allowed to deduct interest and all financial income is subject to the same 15 percent tax rate. But substantial benefits are allowed for owner-occupied housing since the proposal retains a mortgage credit.

Rule of Thumb

Since the margin does not matter in this world, there are no efficiency costs of capital income taxes and IRAs don't matter. If individuals did not recognize the tax liability attached to front-loaded IRAs and ultimately had smaller retirement plans, whether they are bad or good depends on whether people were saving too much or too little in the first place.

Mental Accounts

Many economists enthusiastic about IRAs referred to mental accounts as a justification for IRAs' effectiveness. In this model, savers need the discipline of setting aside retirement funds in segregated accounts that cannot be accessed without paying a penalty. Some also argued that the up-front deduction acted as advertising. Given this model, one might argue that if IRAs increased saving it would lead to more economic efficiency by giving people a way to discipline themselves.

Could evidence on IRAs and saving shed any light on the type of behavior? A body of literature has focused on whether IRAs tend to increase savings. Some of the earlier works are summarized in Gravelle (1994) and in three articles published in a symposium in 1996 (Hubbard and Skinner; Poterba, Venti and Wise; Engen, Gale, and Scholz). Subsequent to these reviews, Thomas and Towe (1996) and Attanasio and DeLeire (2002) found little evidence that IRAs increased savings.

In addition to this statistical evidence, the rather low rate of participation in IRAs suggests that they may not be an important force in inducing additional savings. Thus, in none of the models does the IRA appear to be appealing on economic efficiency grounds.

CONCLUSION

IRAs don't appear to be very effective at achieving efficiency gains, and despite limits and caps, they still favor higher income individuals who are likely to save. There are better potential uses of both existing and proposed revenue losses, particularly in the case of the Advisory Panel's income tax plan. But perhaps the most questionable issue is the shift to back-loaded accounts that masks the eventual budget costs of the expansion of these plans, especially since there appears to be a case for providing IRAs in the front-loaded form.

References

Attanasio, Orazio and Thomas De Leire. The Effect of Individual Retirement Accounts on Household Consumption and Savings. *Economic Journal* 112 (July 2002): 504-538.

Bryant, Victoria L. and Peter J. Sailer. Accumulation and Distribution of Individual Retirement Arrangements. *SOI Bulletin* (Spring 2006): 233-247.

Burman, Leonard E., William G. Gale, and David Weiner. The Taxation of Retirement Saving: Choosing Between Front-Loaded and Back-Loaded Options. *National Tax Journal* 54 (September 2001): 689-702.

Engen, Eric M., William G. Gale, and John Karl Scholz. The Illusory Effects of Saving Incentives on Saving. *Journal of Economic Perspectives* 10 (Fall 1996): 113-138.

Gravelle, Jane G.
The Economic Effects of Taxing Capital Income. Cambridge, MA: MIT Press, 1994.
The Advisory Panel's Tax Reform Proposals. Report RL33545. Congressional Research Service, Library of Congress, Washington, D.C., July 13, 2006.

Gravelle, Jane G., and Maxim Shvedov. Proposed Savings Accounts: Economic and Budgetary Effects. Report RL32228. Congressional Research Service, Library of Congress, Washington, D.C, June 30, 2006.

Investment Company Institute. The U.S. Retirement Market, 2005. Washington, D.C., July, 2006.

Hrung, Warren. Information: The Introduction of Roths, and IRA Participation. Washington, D.C.: U.S. Department of the Treasury, 2004. Office of Tax Analysis Working Paper 91.

Hubbard, R. Glenn, and Jonathan S. Skinner. Assessing the Effectiveness of Savings Incentives. *Journal of Economic Perspectives* 10 (Fall 1996): 73-90.

Poterba, James, Steven Venti, and David A. Wise. How Retirement Savings Programs Increase Saving. *Journal of Economic Perspectives* 10 (Fall 1996): 91-112.

President's Advisory Panel on Federal Tax Reform. Simple, Fair, and Pro-Growth: Proposals to Fix America's Tax System. Washington, D.C., 2005.

Thaler, Richard H. Psychology and Savings Policies. *American Economic Review* 84 (May 1994): 186-192

Thomas, Alun, and Christopher Towe. U.S. Private Saving and the Tax Treatment of IRA/401(k)s: A Re-examination Using Household Saving Data. Washington, D.C.: International Monetary Fund, 1996. IMF Working Paper 96/87.

APPENDIX

This appendix presents the method of estimating the growth pattern of a back-loaded tax-favored savings account for projecting revenue. Each year money is put into an account for T' years and is withdrawn as a level annuity for the next $(T - T')$ years. Contributions are a constant share of output.

To measure the revenue cost for the Roth IRA, we estimate the cumulated value of assets in the fund. For t years in the future, t less than T', for each value of a dollar invested currently, there are e^{-gt} dollars invested that have grown at rate r, so the value of that vintage of accounts is $e^{(r-g)t}$. Integrating the value of assets yields a value at time t of:

(1) Cumulative value at time $t = (e^{(r-g)t} - 1)/(r - g)$.

For t greater than T' but less than T, one must take account of the value of funds that have been partially withdrawn. An annuity for a dollar that grows at rate r for T' years and then is withdrawn over $(T - T')$ years is $e^{(rT')}\, r/(1 - e^{-r(T-T')})$. The remaining value in the asset account is:

$$\textbf{(2)} \quad V(t) = \frac{e^{(rT')} r/(1 - e^{-r(T-t)})}{1 - e^{-r(T-T')}}$$

when t reaches T, the numerator becomes zero.

To cumulate these amounts over time, recognize that the value in (2) is multiplied by e^{-gt}, and integrate from T' to t, to obtain the cumulative value of (2) at time t, $C(t)$:

$$\textbf{(3)} \quad C(t) = [e^{rT'}/(1 - e^{-r(T-T')})]\{(e^{-gT'} - e^{-gt})/g - e^{-rT}\,(e^{(r-g)t} - e^{(r-g)T'})/(r - g)\}$$

For period t, t greater than T' but less than T, the total asset value is (1) plus (3), with t set at T' in (1). For t greater than T, substitute T for t in (3) and obtain the steady state (relative to GDP) results by adding (1) and (3).

The revenue loss for the back-loaded IRA per dollar of contribution is therefore the tax rate times the interest rate times the cumulative asset value. The front-loaded account has an additional revenue loss per dollar of contribution equal to the tax rate. There is also an offsetting gain, eventually, as amounts are withdrawn. If we set the second term in the curly brackets (the one divided by r-g) to zero and multiply the result by the interest rate, the formula gives us the cumulative value of withdrawals at time t, which are relevant to traditional IRA revenue losses.