A NOTE ON TAX EXPENDITURES AND BUSINESS CYCLE FLUCTUATIONS

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INTRODUCTION

One frequently praised feature of income taxes and government spending is that they act as "automatic stabilizers," (Baumol and Blinder, 2000, p. 604) reducing the sensitivity of the economy to shocks.[1] When incomes fall, an income tax implies that disposable income falls by less in absolute terms then the fall in income, as tax revenues decline.[2] Government grants, such as unemployment insurance, welfare, and Medicaid also rise in response to falling incomes, providing a further boost to disposable income. As a result, total expenditures fall by less than they would without the existence of an income tax and government grant programs. Stabilization of expenditures reduces the sensitivity of an economy to inevitable shocks to income. This stabilization occurs automatically; government tax and spending policies do not need to change to make disposable income less variable than income.

While automatic stabilization through income taxes and government spending are frequently praised (e.g., Auerbach and Feenberg, 2000; Cohen and Follette, 2000), the stabilization properties of the third major element of government revenues and expenditures—tax expenditures—have been mostly ignored. This paper aims to fill this gap, examining the stabilization properties of tax expenditures within the context of a simple Keynesian macroeconomic model. The article finds that, relative to government expenditures with similar aims, tax expenditures for expenditures whose demand varies positively with income act as automatic destabilizers, amplifying rather than dulling the impact of an economic shock.

To illustrate, consider first a negative shock to investment and output that hits an economy with direct government spending on public goods. In this economy, government spending is likely to be unaffected by the decrease in output. As a result, the negative shock to investment and output induces no secondary effect on public goods provisions.

Contrast this outcome with a tax expenditure for charitable giving. Donations to charity are a positive function of output.[3] As a result, a negative shock to investment and total output leads to lower charitable donations. Because each dollar of charitable donations receives a subsidy from the government, lower donations mean that the government subsidy of public goods via the charitable deduction decreases. Thus, subsidies for public goods go down in response to a negative shock to investment—exacerbating the initial effect of a negative shock to the economy.

From a macroeconomic stabilization perspective, tax expenditures for goods whose consumption rises with income ("normal" goods) are destabilizing relative to acyclical government expenditures on the same goods. Moreover, the degree of destabilization depends on the sensitivity of consumption to income (the income elasticity of the good). The higher the income elasticity for a good, the greater the degree of destabilization associated with a tax expenditure for that good. Tax expenditures for inferior goods, for which consumption decreases as income increases, stabilize the economy; implicit government spending on the good rises as incomes go down.

The destabilizing features of tax expenditures for normal goods are exacerbated when the degree of subsidy depends on an individual's marginal tax rate and marginal tax rates are progressive. In this case, negative shocks to income reduce the average tax subsidy for a good. This increases the true price of the reducing consumption of the good, and further reduces total expenditure in recessions. Phase outs of tax expenditures, by contrast, have stabilizing effects on consumption that are essentially the inverse of the destabilizing features described here.

The paper proceeds as follows. The first section reviews the literature on tax expenditures. The second section develops a Keynesian model of the economy, and uses the model to examine the stabilization properties of an economy that relies on government spending versus one that relies on tax expenditures. The third section concludes.

THEORIES OF TAX EXPENDITURES

Governments can provide public goods or subsidize externalities through a variety of mechanisms.

These include direct government spending, tax expenditures, or regulation. Tax expenditures are statutorily defined as "revenue losses attributable to provisions of the Federal tax laws which allow a special exclusion, exemption, or deduction from gross income or which provide a special credit, a preferential rate of tax, or a deferral of tax liability."[4] Tax expenditures are viewed as substitutes for government expenditures. The Joint Committee on Taxation states that, "special income tax provisions are referred to as tax expenditures because they may be considered to be analogous to direct outlay programs, and the two can be considered as alternative means of accomplishing similar budget policy objectives." (Joint Committee on Taxation, 1999, pp. 1-2) Prominent examples of tax expenditures include the deductibility from income of home mortgage interest[5] and charitable contributions and the exclusion of employer provided health benefits from income. The costs of tax expenditures have generally been rising. Tax expenditures in 2008 were estimated to equal approximately 7 percent of GDP (Tax Policy Center, 2008a). After the tax reform of 1986, by contrast, tax expenditures totaled approximately 5 percent of GDP.

The costs and benefits of tax expenditures relative to government spending have been carefully analyzed (Saez, 2004). On the positive side, tax expenditures do not crowd out private spending on public goods or externalities, while government spending may produce such crowd out. Tax expenditures also enable the government to avoid wasted spending on public goods. Because tax expenditures amplify public spending, they require some private sector "buy in." As a result, the government's implicit subsidy on tax expenditures is less likely to be wasted on low value goods than direct government spending (see Gruber, 2005, pp. 503-506).

Tax expenditures suffer from notable costs, however. Most prominently, tax expenditures are regressive (Surrey, 1985). A dollar of tax deductible charity donated by a taxpayer at the 35 percent marginal rate benefits from a 35 percent subsidy, while a dollar donated to the same charity by a lower income taxpayer in the 10 percent bracket enjoys a much smaller subsidy. Tax expenditures also often fail tests of horizontal equity—they treat similar taxpayers differently (Elkins, 2006). For example, a self-employed businessperson enjoys no subsidy for health insurance, while a similarly capable individual working for someone else enjoys a large subsidy for health insurance provided by their employer. Finally, tax expenditures involve considerable inframarginal subsidies, raising costs without changing incentives (Saez, 2004).

No papers in the lengthy literature on tax expenditures, however, consider tax expenditures' macroeconomic impact. The remaining sections of the paper examine tax expenditures from a macroeconomic lens and find that most tax expenditures have undesirable macroeconomic effects.

THE KEYNESIAN MODEL OF ECONOMIC FLUCTUATIONS

Model Setup

This paper develops a simple Keynesian model of fluctuations,[6] a model which has assumed renewed salience in the economic crisis of 2007-2009. According to this model:

(1) $Y = C + I + G.$

Total Output (Y) equals total expenditures on consumption (C), investment (I) and government spending (G). Consider an economy with no tax expenditures but with income taxes. Assume that government spending is fixed $G = \bar{G}$, and investment is subject to random shocks ("animal spirits") but is otherwise insensitive to other factors, $I = \bar{I} + \varepsilon_I$. Consumption is determined by a standard consumption function, where consumption is a function of disposable income,

(2) $C = \alpha + \beta(Y - \tau Y),$

where the tax rate $0 \leq \tau \leq 1$. Each subunit of consumption also has an associated consumption function, $C_i = \alpha_i + \beta_i(Y - \tau Y)$, where β_i can be transformed into the income elasticity for good i by dividing by C_I. Combining equations 1 and 2 yields

$$Y = C + I + G = \alpha + \beta(Y - \tau Y) + \bar{I} + \varepsilon_I + \bar{G}$$

The sensitivity of this economy to a shock to investment is given by

(3) $$\frac{\partial Y}{\partial \varepsilon_I} = \frac{1}{\left(1 - \beta + \beta\tau\right)}.$$

As is well known (Baumol and Blinder, 2000, p. 604), the existence of income taxes lowers the

Keynesian multiplier of a shock $1/(1 - \beta + \beta\tau) < 1/(1 - \beta)$. Income tax receipts go up in response to a positive shock and down in response to a negative shock, diminishing the impact of the shock on disposable income and therefore on consumption.

Note, however, what occurs if the government implements negative income taxes, $\tau < 0$ (which can be made revenue neutral via the imposition of a lump sum tax). Negative income taxes make the economy more unstable $1/(1 - \beta + \beta\tau) > 1/(1 - \beta)$ when $\tau < 0$. With negative income taxes, the government amplifies shocks to incomes and thereby destabilizes the economy. While negative income taxes are unusual, the U.S. government has implemented an Earned Income Tax Credit that gives negative income taxes in some ranges. Such a program destabilizes the economy.

Tax Expenditures for Consumption

Most tax expenditures do not take the form of negative income taxes. Instead, they take the form of subsidies for favored elements of consumption, such as health care, housing, or charitable contributions. For simplicity, I assume that expenditures on such items are fully tax deductible. That is, disposable income is given by $Y - \tau Y + \tau C_D$, where C_D denotes consumption on deductible goods.

Writing down consumption functions for deductible and nondeductible goods:

$$\textbf{(4)} \quad C_D = \alpha_D + \beta_D\left(Y - \tau Y + \tau C_D\right),$$

$$\textbf{(5)} \quad C_{ND} = \alpha_{ND} + \beta_{ND}\left(Y - \tau Y + \tau C_D\right),$$

and

$$\textbf{(6)} \quad Y = C_D + C_{ND} + \bar{I} + \varepsilon_I + \bar{G}.$$

Using these three equations to solve for $\partial Y^D/\partial \varepsilon_I$, the sensitivity of an economy with deductions for consumption yields.

Theorem 1a: For any normal good i, $\beta_i > 0$ a tax deduction for that good destabilizes the economy relative to government spending on good I (see Appendix for Proof). While the Proof involves some complicated algebra, the intuition behind this result is straightforward. For any normal good, an increase in income leads to an increase in consumption of that good. If there is a positive shock to investment and therefore income, consumption of the normal good will increase. If the normal good is associated with a tax deduction, then an increase in consumption of this good is associated with an increase in tax deductions, increasing disposable income and therefore further increasing total consumption. If the government spends directly on good i, by contrast, then a positive shock to investment and income increases consumption of good i, but the increased consumption will not be associated with any tax shelters that give a further kick to disposable income. Thus, the sensitivity of consumption (and therefore output) is greater when there are tax expenditures for a consumption good than when the government spends directly on the good without regard to the business cycle.

Furthermore, the greater the sensitivity of consumption to income for a good i, with a given amount of total consumption, the more destabilizing a tax expenditure associated with that good is. For goods whose consumption is very sensitive to income, a tax expenditure can be particularly destabilizing. In addition, if deductions are expanded to more goods while holding tax rates constant, then the economy grows increasingly unstable (see Appendix for Proof).

Theorem 1b: For any inferior good i, $\beta_i < 0$, a tax expenditure for that good stabilizes the economy relative to government spending on good i (see Appendix for Proof). Again, the intuition for this result is quite accessible. With an inferior good, increases in income reduce consumption. If there is a tax expenditure associated with this good, then an increase in income, and a decrease in consumption, is associated with a decrease in disposable income relative to the situation with no tax preferences for this good. The decrease in disposable income reduces overall consumption and mitigates the total effect of the initial positive shock to investment and income.

As a practical matter, many tax expenditures benefit goods whose consumption appears sensitive to income. For example, charitable deductions are known to be sensitive to business cycle fluctuations (Gross, 2008). In addition, other tax expenditures, such as the excludability of employer provided healthcare, are functions of employment levels, which are also very sensitive to the business cycle. Thus, there is reason to suspect that tax expenditures are generally destabilizing to the economy.

Price Effects

The destabilizing effects of tax expenditures for normal goods on the economy are exacerbated by a price effect caused by the interaction of tax expenditures and progressive income taxes. Tax expenditures increase consumption of a good by reducing the relative after-tax price of goods favored by tax expenditures (Saez, 2004). A tax expenditure places a wedge between the (after-tax) price of a good that is received by the suppliers of the good (p) and the price paid by consumers ($p - \tau_M$), where τ_M represents the average marginal tax rate faced by a consumer. With a progressive tax system, τ_M is an increasing function of output Y, $\tau_M(Y)$, where $\tau_M'(Y) > 0$. This connection between average tax rates and income creates a pro-cyclical relationship between consumption of a good and overall output, even if the good is not one that has a positive income elasticity.

Theorem 2: When there is a progressive tax system, $\tau_M(Y)$, consumption of good i rises in response to positive shocks to income even if $\beta_i = 0$.

Proof: With a positive shock to investment, ε_I, output Y rises. As a result, average marginal tax rates rise because $\tau_M'(Y) > 0$. The after-tax price faced by consumers for good i, $(p - \tau_M)$ thus goes down. This leads to a substitution effect in favor of demand for good i at any given price to suppliers, p_i,[7] and therefore an increase in output of good i, even though the demand curve has not been shifted outward by the increase in income.

Thus, in a progressive tax system, there is a price effect caused by tax expenditures that exacerbates the pro-cyclical leanings of most tax expenditures.[8] A progressive tax system implies that the average subsidy for a tax favored good rises with income. The subsidy is therefore higher in a year with a positive shock to income than in a year with a negative shock to income. This will cause individuals to consume more goods favored by tax expenditures in years with high incomes (and high subsidies). To the extent that this consumption increases aggregate demand, the tax expenditure thus introduces a further pro-cyclical impulse to the economy.

Phase-Outs

While tax subsidies are theoretically distinct from phase outs of such subsidies, many prominent tax expenditures, such as the Earned Income Tax Credit, are phased out as incomes rise (Scholz, 1994). A phaseout operates in the same fashion as a progressive income tax—rates go up as incomes rise. This makes the phase-out element of a tax expenditure an automatic stabilizer—a positive shock to everyone's income increases the number of individuals whose tax expenditure benefits are phased out, raising the average tax rate for these individuals, lowering disposable income and partially counteracting the impacts of the positive shock by moderating the secondary effect on consumption.

The net stabilizing effects of the introduction of a tax expenditure for a consumption good combined with a phase out depends upon the number of taxpayers in the phase-out range and the impact of a shock on the distribution of incomes. If a negative cyclical shock to income only effects individuals below the beginning of the phase-out range, then the presence of a phase out does not mitigate the destabilizing tendencies of tax expenditures. No individuals marginal tax rate is lowered due to the impacts of the phaseout. As a result, the preceding analysis of the pro-cyclical impacts of most tax expenditures is unchanged.

By contrast, if a negative cyclical income shock entirely affects individuals with incomes above the phase out range and brings many incomes below the phase-out range, then the impacts of the tax expenditure combined with a phase-out are identical to the impacts of an increase in progressivity of tax rates, which are unambiguously countercyclical. The net stabilizing effect of a consumption tax expenditure combined with a phase out therefore depends on how much cyclical fluctuations in income distributions resemble the first scenario versus the second. The lower the density of individual incomes at the beginning of the phase-out range, the more likely it is that a tax expenditure combined with a phase out is destabilizing.

Empirical Relevance

The previous sections examined the theoretical stabilization properties of tax expenditures and found that they are frequently destabilizing. This section attempts to answer the related question of whether these destabilizing properties are empirically important.

Based on magnitude alone, tax expenditures should have an empirically important impact on economic cycles. Tax expenditures account for approximately 7 percent of GDP (Tax Policy Center, 2008a). This figure is of comparable magnitude to the individual income tax, which collects approximately 8 percent of GDP (Tax Policy Center, 2008b), or to discretionary government spending (including

defense), which also equals approximately 8 percent of GDP (Riedl, 2006). If the automatic stabilization properties of discretionary spending and income taxes are worth emphasizing in introductory economics textbooks (e.g., Mankiw, 2006, p. 796; Frank and Bernanke, 2008, Chapt. 13; Baumol and Blinder, 2008), then surely the stabilization properties of tax expenditures are worth careful analysis.

Two recent empirical papers provide suggestive evidence for the role of tax expenditures in macroeconomic stabilization. Auerbach (2009) found that the income tax's automatic stabilization properties are at a forty-year low. Much of the decrease can be explained by decreases in marginal tax rates.[9] Increasing tax expenditures may also explain part of the decrease. As increasing portions of income become excluded from taxation, the automatic stabilization provided by income taxes inevitably decreases for any given marginal tax rate.

The Auerbach (2009) analysis does not consider the automatic stabilization properties of government spending. When tax expenditures replace government spending, they are replacing an automatic stabilizer with a destabilizer. Thus, the net destabilizing effect of tax expenditures cannot be captured by focusing on income taxes alone.

In a recent paper, Taylor (2009) decomposes the quarterly government surplus/deficit into two components. One part of the fiscal position is described as "structural," meaning that it reflects the average fiscal position of the government if the economy is growing at its "natural" rate (i.e., there are no important economic shocks). The second element of the net surplus is described as "cyclical." Cyclical surpluses or deficits are caused by shocks to the economy that move the actual growth rate of the economy away from its natural rate (see Taylor, 2009, p. 10). Because Taylor examines net surpluses, his analysis captures the effects of both changes to income taxes and changes to government spending.[10] The cyclical element of his decomposition describes how automatically sensitive the government's fiscal position is to the economy's ups and downs relative to its natural growth rate.

Taylor (2009) finds that the sensitivity of the net surplus to cyclical fluctuations has decreased, implying that the power of automatic stabilizers has waned. From 1983-1994, if GDP was 1 percent below its potential rate, then the budget gap was .35 percent higher as a percentage of GDP. From 1995-2007, a 1 percent gap between GDP and potential GDP was associated with a .29 percent increase in the budget gap.[11] The average income tax sensitivity over the two periods as measured by Auerbach (2009), however, has remained relatively constant.[12] This suggests that automatic stabilization provided by government spending has decreased during the 1990s and 2000s. The replacement of countercyclical government programs with tax expenditures may be one explanation for this decrease in automatic stabilization provided by government spending.

CONCLUSION

This paper demonstrated that tax expenditures for normal goods act as automatic destabilizers relative to the government spending that tax expenditures replace. The implications of this finding for tax expenditures depends on the salience of stabilization as a goal of tax and spending policy. During the economically stable period of the 1970s through the mid 2000s, the automatic stabilization properties of income taxes and government spending were ancillary to the goals of promoting long-run economic growth and equity among citizens. In these circumstances, tax expenditures that promote growth and optimize incentives may well have been worth the overlooked destabilization costs. However, the depth of the "Great Recession" of 2007-2009, and the overwhelming fiscal policy response, suggests that economic stabilization has assumed renewed salience. In this environment, the destabilizing aspects of tax expenditures are more troublesome, and they provide a reason for rethinking the replacement of government spending with tax expenditures. In particular, tax expenditures for goods that have high-income elasticities, which are the most destabilizing, should be reconsidered. Tax expenditures for inferior goods, by contrast, prove to stabilize the economy and may be worth expanding.

Notes

1 Examples of shocks include natural disasters, sudden changes in input prices (such as oil prices) or sudden losses or gains of confidence due to "animal spirits."

2 Disposable income equals total income minus tax payments.

3 See, e.g., Gross (2008), noting that "charitable giving fell in real terms in years in which the economy was in recession, or in years in which there was a significant stock market dislocation." There is some debate regarding whether the income elasticity of charitable giving is greater than 1 or less than 1, though all agree that the elasticity is considerably above zero. See Triest (1998), Randolph (1995), Auten et al. (2002).

[4] Definitions, 2 U.S.C.A. § 622(3) (West. 2007).

[5] Or analogously, the non-imputation of owner-occupied housing income.

[6] For textbook treatments, see Mankiw (2003, Chap. 11); Romer (2001, Chap. 5). It is important to emphasize that these models do not account for the important microeconomic effects of taxation on incentives. This paper follows this emphasis. This paper also focuses exclusively on the Keynesian Cross ("IS") aspect of the model for simplicity. The direction and qualitative nature of the results for automatic stabilizers and destabilizers is unchanged by this assumption, though the magnitudes of the effects will be smaller in the more complicated model, such as IS-LM or AD-AS.

[7] Note that this is entirely consistent with the assumption of short-term sticky prices, as tax rates go up with income.

[8] Randolph (1995) demonstrates that givers demonstrate sophisitication in moving giving from low marginal rate years to high marginal rate years, suggesting that the price effect described here is significant.

[9] As demonstrated in the first section, lower income tax rates imply a lower degree of automatic stabilization.

[10] Because it relies on aggregate data, however, Taylor's (2009) analysis is considerably less fine-grained than Auerbach's (2009), which examines how the tax code affects individuals at many levels of income.

[11] This decrease is unlikely to be statistically significant. Although Taylor's (2009) paper does not provide standard errors, a similar analysis that he conducted in 2000 estimated standard errors of around .04 (see Taylor, 2000). Thus, the difference in the two estimates is slightly greater than one standard deviation. Note that Taylor's (2009) estimate of automatic stabilizers for 1960-1982 is approximately .45.

[12] See Auerbach (2009, fig. 2, p. 5). The average sensitivity of taxes to income between 1983-1994 is approximately .261. The average sensitivity of taxes to income from 1995-2007 is approximately .256.

References

Auerbach, Alan J. Implementing the New Fiscal Policy Activism. Cambridge, MA: National Bureau of Economic Research, 2009. NBER Working Paper W14725.

Auerbach, Alan J., and Daniel Feenberg. The Significance of Federal Taxes as Automatic Stabilizers. *Journal of Economic Perspectives* 14 (Summer 2000): 37-56.

Auten, Gerald, Holger Sieg, and Charles T. Clotfelter. Charitable Giving, Income, and Taxes: An Analysis of Panel Data. *American Economic Review* 92 (March 2002): 371-382.

Baumol, William, and Alan Blinder. Economics: Principles and Policy. New York: Dryden Press, 2000.

Cohen, Darrel and Glenn Follette. The Automatic Fiscal Stabilizers: Quietly Doing Their Thing. *Economic Policy Review* 16 (April, 2000): 35-68.

Elkins, David. Horizontal Equity as a Principle of Tax Theory. *Yale Law and Policy Review* 24 (Winter 2006): 43-90.

Frank, Robert H. and Ben Bernanke. *Principles of Macroeconomics.* New York: McGraw-Hill Higher Education, 2008.

Gross, Daniel. The Coming Charity Crisis: How the Struggling Economy is Hurting Donations. *Slate.com,* June 12, 2008.

Gruber, Jonathan. *Public Finance and Public Policy.* New York: Worth Publishing, 2005.

Joint Committee on Taxation. Estimates of Federal Tax Expenditures for Fiscal Years 2000-2004. Washington, D.C., 1999.

Mankiw, N. Gregory. *Macroeconomics.* New York: Worth Publishing, 2003. *Principles of Macroeconomics.* New York: South-Western College Publishing, 2006.

Randolph, William C. Dynamic Income, Progressive Taxes, and the Timing of Charitable Contributions. *Journal of Political Economy* 103 (August 1995): 709-738.

Riedl, Brian. Discretionary Spending Trends: Past, Present, and Future. Testimony before the Budget Committee, United States House of Representatives. Washington, D.C., 2006). http://www.heritage.org/Research/budget/tst021606a.cfm.

Romer, David. *Advanced Macroeconomics.* New York: McGraw-Hill Higher Education, 2001.

Saez, Emmanuel, The Optimal Treatment of Tax Expenditures. *Journal of Public Economics* 88 (December 2004): 2657-2684.

Scholz, John Karl. The Earned Income Tax Credit: Participation, Compliance, and Antipoverty Effectiveness. *National Tax Journal* 47 (March 1994): 63-87.

Surrey, Stanley and Paul McDaniel. *Tax Expenditures.* Cambridge, MA: Harvard University Press, 1985.

Taylor, John.

Reassessing Discretionary Fiscal Policy. *Journal of Economic Perspectives* 21 (Summer 2000): 31-36.

The Lack of an Empirical Rationale for the Revival of Discretionary Fiscal Policy. *American Economic Review* 99 (May 2009): 550-555.

Tax Policy Center.

Have Tax Expenditures Changed Over Time. In *The Tax Policy Briefing Book: A Citizen's Guide for the 2008 Election and Beyond.* Washington, D.C.: Tax Policy Center, 2008a. http://www.taxpolicycenter.org/briefing-book/background/expenditures/change.cfm.

The Numbers: What Are the Federal Government's Sources of Revenue? In *The Tax Policy Briefing Book: A Citizen's Guide for the 2008 Election and Beyond.* Washington, D.C.: Tax Policy Center, 2008b. http://www.taxpolicycenter.org/briefing-book/background/numbers/revenue.cfm.

Triest, Robert K. Econometric Issues in Estimating the Behavioral Response to Taxation: A Nontechnical Introduction. *National Tax Journal* 51 (December 1998): 761-772.

APPENDIX

Proofs of Theorems 1a and 1b

Begin with the consumption functions for goods benefiting from a tax expenditure (C_D) and those goods not benefiting from a tax expenditure (C_{ND}).

(A1) $C_D = \alpha_D + \beta_D\left(Y - \tau Y + \tau C_D\right)$

(A2) $C_{ND} = \alpha_{ND} + \beta_{ND}\left(Y - \tau Y + \tau C_D\right).$

If there are only two goods in the economy, then $\beta_D + \beta_{ND} = \beta$.

Solving for $C_D = \frac{1}{1-\beta_D\tau}\left(\alpha_D + \beta_D Y - \beta_D\tau Y\right)$,

and plugging into equation 1 yields

$$Y = C_D + C_{ND} + I + G =$$

$$\frac{1}{1-\beta_D\tau}\left(\alpha_D + \beta_D Y - \beta_D\tau Y\right) + \alpha_{ND} + \beta_{ND}\left(Y - \tau Y + \frac{\tau}{1-\beta_D\tau}\left[\alpha_D + \beta_D Y - \beta_D\tau Y\right]\right) + \bar{I} + \varepsilon_I + \bar{G}$$

$$\frac{\partial Y^D}{\partial \varepsilon_I} = \frac{1}{1-\beta_D\tau}\beta_D Y - \frac{1}{1-\beta_D\tau}\beta_D\tau Y + \beta_{ND}Y - \beta_{ND}\tau Y + \frac{\beta_{ND}\tau\beta_D}{1-\beta_D\tau}Y - \frac{\beta_{ND}\tau^2\beta_D}{1-\beta_D\tau}Y - Y$$

$$\frac{\partial Y^D}{\partial \varepsilon_I} = \frac{1}{\left[1 - \frac{\beta_D}{1-\beta_D\tau} + \frac{\beta_D\tau}{1-\beta_D\tau} - \beta_{ND} + \beta_{ND}\tau - \frac{\beta_{ND}\tau\beta_D}{1-\beta_D\tau} + \frac{\beta_{ND}\tau^2\beta_D}{1-\beta_D\tau}\right]}$$

$$= \frac{1}{\left[\frac{1-\beta_D\tau}{1-\beta_D\tau} - \frac{\beta_D}{1-\beta_D\tau} + \frac{\beta_D\tau}{1-\beta_D\tau} - \frac{1-\beta_D\tau}{1-\beta_D\tau}\beta_{ND} + \frac{1-\beta_D\tau}{1-\beta_D\tau}\beta_{ND}\tau - \frac{\beta_{ND}\tau\beta_D}{1-\beta_D\tau} + \frac{\beta_{ND}\tau^2\beta_D}{1-\beta_D\tau}\right]}$$

$$= \frac{1}{\left[\frac{1-\beta_D\tau}{1-\beta_D\tau} - \frac{\beta_D}{1-\beta_D\tau} + \frac{\beta_D\tau}{1-\beta_D\tau} - \frac{\beta_{ND} - \beta_{ND}\beta_D\tau}{1-\beta_D\tau} + \frac{\beta_{ND}\tau - \beta_D\tau\beta_{ND}\tau}{1-\beta_D\tau} - \frac{\beta_{ND}\tau\beta_D}{1-\beta_D\tau} + \frac{\beta_{ND}\tau^2\beta_D}{1-\beta_D\tau}\right]}$$

$$= \frac{1-\beta_D\tau}{1-\beta_D-\beta_{ND}+\beta_{ND}\tau}.$$

Equation 3 specifies that, without deductions, $\partial Y/\partial\varepsilon_I = 1/(1 - \beta + \beta\tau)$. This will remain true if government spending on good *D* is increased by τC_D so that the budget deficit when $\varepsilon_I = 0$ is the same with tax deductions on good *D* or with government spending on good *D*. The economy with deductions is therefore less stable with respect to shocks to investment iff

$$\frac{\partial Y^D}{\partial \varepsilon_I} > \frac{\partial Y}{\partial \varepsilon_I}$$

or

$$\frac{1-\beta_D\tau}{1-\beta_D-\beta_{ND}+\beta_{ND}\tau} > \frac{1}{\left(1-\beta+\beta\tau\right)}.$$

Creating a common denominator, this condition is true if $\beta\beta_D\tau - \beta\beta_D\tau^2$, which is positive since $0 < \tau < 1$. Because $\beta > 0$, this condition is true if $\beta_D > 0$. This proves Theorem 1a. If $\beta_D > 0$, then $\partial Y^D/\partial\varepsilon_I < \partial Y/\partial\varepsilon_I$ proving Theorem 1b. Finally, note that $\beta\beta_D\tau - \beta\beta_D\tau^2$ is increasing with β_D, so that the higher the sensitivity to income, the greater the instability of the economy with the deduction relative to the economy without deduction. β_D can be high for two reasons. It may be high because good *D* is very sensitive to income compared to some other good that makes a similar percentage of consumption. β_D may also be high because it makes up a large percentage of the economy. In either case, the economy is more unstable as β_D rises.