Uncovering Heterogeneity in Income Tax Perceptions*

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Abstract

This paper uses a finite mixture model to uncover heterogeneous income tax rate perceptions, establishing four new results. First, almost half of respondents do not distinguish between marginal and average tax rates. Second, roughly 30 percent of respondents know the statutory marginal tax rates schedule (and answer questions accordingly). Third, among respondents who think all income is taxed at the same rate, roughly 40 percent think all of their income is taxed at the statutory marginal tax rate. Finally, respondents with higher cognitive ability are more likely to report statutory marginal tax rates, but only among respondents who prepare their own income tax returns.

1 Introduction

Understanding how people perceive tax rates is critical to evaluating the impact of tax rate changes. If people think all income is taxed at the average tax rate, then changes in marginal tax rates will have smaller effects on behavior than if they understood the distinction between marginal and average rates. Similarly, if people think all income is taxed at the statutory marginal tax rate (and behave accordingly), then changes that impact the marginal tax rates or the income thresholds associated with these rates will have a larger impact on behavior.

Yet little is known about the behavioral or structural mechanisms underlying individuals' perceptions of income tax rates. Almost a half century ago, James Buchanan hypothesized that

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> *...the institution of progression, per se, tends to create an excess feeling of tax burden on the part of the taxpayer. The effect here stems from the divergence between the average and the marginal rate of tax, and the observed tendency of persons to think in terms of marginal rates. This illusion, if present, is supported by discussions of the rate structure in the popular press and in political debates.* (James Buchanan, 1967; paragraph 4.10.37)

More recent empirical evidence suggests otherwise. de Bartelome (1995) analyzes choices in an experimental setting to see whether they are more consistent with subjects using marginal or average tax rates and concludes that people are more likely to use average rates. In related work, Ito (2014) uses observed electricity consumption to uncover consumers' perceived price of the non-linear price schedule. He finds evidence that consumers respond to average electricity prices rather than marginal or expected marginal prices.

Given conflicting perspectives on whether it is more intuitive to use average rates or marginal rates, it seems reasonable that some respondents focus exclusively on the average price, others on the marginal price, and that some correctly distinguish between the two. But using realized outcomes to infer beliefs makes it difficult to identify and characterize heterogeneity. There is the identification problem of separating beliefs from utility structure, unless one or the others are measured directly.

The goal of this paper is to characterize systematic heterogeneity documented in Gideon (2014). Data from the Cognitive Economics (CogEcon) Study provides a unique opportunity to uncover such latent heterogeneity, as it includes panel data on self-reported MTR and self-reported ATR. This paper develops a mixture model allowing respondents to belong to one of the following four types.

- Type A: Distinguish between ATR and MTR; report statutory MTR.
- Type B: Distinguish between ATR and MTR; do not report statutory MTR.
- Type C: Do not distinguish between ATR and MTR; report statutory MTR for both.
- Type D: Do not distinguish between ATR and MTR: do not report statutory MTR for both.

This categorization incorporates two interesting dimensions of heterogeneity. First, Type A and Type B respondents distinguish between marginal and average rates, where Type C and Type D respondents do not.[1] Second, Type A and Type C respondents understand (and report) statutory

[1]Following the terminology from Liebman and Zeckhauser (2004), the Type C and Type D respondents are "schmedulers." These two types correspond to Liebman and Zeckhauser's (2004) distinguishing between "ironing" and "spotlighting." In the context of a progressive tax schedule, "ironing" is focusing on the average tax rate and making marginal decisions as if the MTR is the true ATR. In contrast, "spotlighting" refers to responding to a local price rather than the entire schedule. This corresponds to thinking all income is taxed at the statutory marginal tax rate and making decisions accordingly.

marginal tax rates (0, 10, 15, 25, 28, 33 or 35), while Type B and Type D do not. The clusters reflect distinct "mental models," or the way they perceived marginal and average tax rates.

I use a finite mixture model to semi-parametrically estimate (i.) the fraction of respondents who understand the statutory marginal tax rate schedule, and (ii.) the fraction of respondents, among those who think all income is taxed at the same rate, who think all income is taxed at their statutory marginal tax rate. The mixture model exploits distributional information from respondents who provide different numbers for MTR and ATR to estimate the fraction of respondents, among those who answered $MTR = ATR$, who think all income is taxed at their ATR versus their MTR. Using estimates from the mixture model, I classify respondents based on whether they understand (and report) statutory marginal tax rates.

I find strong evidence of heterogeneous mental models of tax rates. First, based on the raw data across two waves, half of respondents think all income is taxed at the same rate. Second, among respondents who think all income is taxed at the same rate, there is substantial heterogeneity in whether they think all their income is taxed at their average or statutory marginal tax rate. And, overall, roughly 30 percent of respondents know the statutory marginal tax rates schedule (and answer questions accordingly).

In the second part of the paper, I analyze determinants of tax perceptions by examining individual characteristics that are correlated with class membership. In particular, I analyze how knowledge of statutory marginal tax rates is related to cognitive ability, general financial sophistication and the use of paid tax preparers. My main finding is that cognitive ability is strongly associated with knowledge of statutory marginal tax rates, but only among those who file their own tax return (rather than use a paid preparer). The incentive to learn about tax rates varies based on the extent to which someone can actively respond to such incentives. Similarly, tax rules are particularly complex and one's ability to learn might depend on observable characteristics.

This paper contributes to the growing number of finite mixture and latent class analyses in economics. Latent class models have been shown to provide a more flexible model of health care utilization (e.g., Deb and Travedi (1997, 2002)), replacing the standard two-part model of zero versus positive utilization with a two-class model of low and high utilization. Kapteyn and Ypma (2007) use a finite mixture model to distinguish between different sources of deviations between survey and administrative data. From a more behavioral perspective, Bruhin et al (2010) find evidence of a mixture of individuals who weight probabilities as expected value maximizers and those more consistent with prospect theory.

2 Data

The data and sample are the same as in Gideon (2014). Data come from the 2011 and 2013 waves of the Cognitive Economics (CogEcon) study. CogEcon 2011 introduces new questions asking about federal income tax rates, some of which were repeated in CogEcon 2013. The CogEcon sample consists of households over 50 years old who were chosen using a random sample selection design. Partners were also included in the sample, regardless of their age. My estimation sample includes the 348 respondents who gave valid responses to all four tax rate questions and reported total income above $5,000 in both waves.[2] See Gideon (2014) for an extended discussion of the survey instrument and the differences between CogEcon 2011 and CogEcon 2013.

2.1 Measuring tax rates using income

Self-reported income data are used to compute Adjusted Gross Income (AGI) and taxable income (TI). Filing status and taxable income determine statutory marginal tax rates and tax liability, and the computed average tax rate equals tax liability divided by adjusted gross income (AGI). I assume the true marginal tax rate is the statutory rate, which is consistent with how the question was worded.

Statutory marginal tax rates for wage and salary income were 10%, 15%, 25%, 28%, 33% and 35%, the levels set in the Jobs and Growth Tax Relief Reconciliation Act of 2003 (JGTRRA). Table **??** presents the taxable income thresholds associated with these marginal tax rates for tax years 2010 and 2012, broken down by filing status.

Unfortunately, I do not have data on respondents' filing status and I must make assumptions based on marital status. Single respondents are assumed to file as single (rather than head of household), and married respondents are assumed to file jointly. Nevertheless, if married respondents file separately then I will underestimate their true MTR and, therefore, overestimate the difference between subjective and true MTR; the opposite will be true for singles, for whom I will overestimate their MTR and underestimate their bias. The income variables come from self-reported information about different sources of income. I use the NBER's TAXSIM tax rate calculator to transform this vector of income variables into AGI.[3] Taxable income equals AGI minus exemptions and deductions. Information about dependent exemptions was collected in the survey and I assume all taxpayers

[2] This includes 302 households, and 46 households have two respondents. In all analyses I include both respondents and cluster standard errors at the household level.

[3] See Feenberg and Coutts (1993) for a discussion of TAXSIM.

claim the standard deduction. Details about constructing AGI and TI are in Appendix B.1.

2.2 Known income tax rate functions

The specification of the mixture model uses the known tax rate functions described in this section. Individual i's true statutory marginal tax rate ($m^{\star}_{i,w}$) in wave w is a function of true taxable income ($I^{\star}_{i,w}$), written as $m^{\star}_{i,w} = M_w\left(I^{\star}_{i,w}\right)$. Taxable income ($I^{\star}_{i,w}$) is the greater of zero and adjusted gross income ($AGI^{\star}_{i,w}$) minus deductions ($D^{\star}_{i,w}$), which is either the standard deduction or itemized deductions, and exemptions ($E^{\star}_{i,w}$)

$$I^{\star}_{i,w} = \max\left\{AGI^{\star}_{i,w} - D^{\star}_{i,w} - E^{\star}_{i,w}; 0\right\}. \tag{1}$$

The marginal tax rate function $M_w(\cdot)$ is defined by tax law

$$M_w\left(I^{\star}_{i,w}\right) = \tau_j \iff \overline{I^{j-1}_w} \leq I^{\star}_{i,w} < \overline{I_w}^j. \tag{2}$$

and maps true taxable income into one of seven statutory tax rates, $\tau_j \in \{0; 10; 15; 25; 28; 33; 35\}$. The statutory marginal tax rate is τ_j when taxable income in wave w is between thresholds $\overline{I^{j-1}_w}$ and $\overline{I_w}^j$. The top threshold for bracket $j-1$ is the bottom threshold for bracket j. These income thresholds depend on filing status (married or single), but subscripts for filing status are supressed to simplify notation. While the statutory rates are the same in both waves, the tax functions are not, as there are changes to the taxable income thresholds.

Taxpayer i's true tax liability in wave w, $T^{\star}_{i,w} = T_w\left(I^{\star}_{i,w}\right)$, is a continuous, piece-wise linear deterministic function of taxable income

$$T_w\left(I^{\star}_{i,w}\right) = \tau_j \cdot \left(I^{\star}_{i,w} - \overline{I^{j-1}_w}\right) + C^j_w \text{ when } \overline{I^{j-1}_w} \leq I^{\star}_{i,w} < \overline{I_w}^j \tag{3}$$

That is, someone with taxable income between $\overline{I^{j-1}_w}$ and $\overline{I_w}^j$ is taxed at rate τ_j on all taxable income above $\overline{I^{j-1}_w}$. Everyone with marginal tax rate τ_j (and, technically, the same filing status) pays tax C^j_w on all income less than threshold $\overline{I^{j-1}}$, where $C^j_w \equiv \tau_1 \cdot \overline{I^1_w} + \tau_2 \cdot \left(\overline{I^2_w} - \overline{I^1_w}\right) + ... + \tau_{j-1} \cdot \left(\overline{I^{j-1}_w} - \overline{I^{j-2}_w}\right)$.

See Gideon (2014) for an extended discussion of this approach to measuring income tax rates using the CogEcon data. In particular, by ignoring tax credits and assuming all income is taxed as wage or salary income, the tax liability computation is an upper bound on true tax liability at a given taxable income. This is because tax credits reduce tax liability and tax-preferred investments are taxed at lower rates than wages and salary income.

3 Mixture model

3.1 Descriptive evidence of heterogeneous types

Two patterns in the response data suggest that people answer the tax rate questions in distinct ways and that a fundamentally more flexible model is needed to better understand the substantial heterogeneity in tax perceptions.

First, many respondents reported the same number for MTR and ATR. If deviations of the reported rates from the true rates were due to random noise, reporting exactly the same number for both MTR and ATR is a zero probability event. In Table 1, approximately half of respondents reported $MTR = ATR$ for 2010 (50.9%) and a similar fraction for 2012 (49.1%), while 31 percent did so in both waves. There were also 31 percent of respondents who reported $MTR \neq ATR$ in both waves, providing evidence consistent with them knowing they are taxed on the margin differently from their average rate.[4] Having two years of data sheds light on the limitations to analyzing a cross-section of reported tax rates. Approximately 39 percent of respondents report $MTR = ATR$ in one wave but not the other. While the marginal rate must strictly be greater than the average (when non-zero), they are sometimes relatively close and respondents could provide rounded estimates of these rates.

Systematically reporting the same number for both ATR and MTR provides strong evidence that the respondent believes all their income is taxed at the same rate, but it is uninformative about the mental model underlying this belief. The interpretation based on previous studies is that people are reporting their ATR for both. This interpretation is also consistent with the question ordering, with the question about ATR before the one about MTR. Respondents may answer their average tax rate and then reason that additional income would be taxed at the same rate. However, for people who know about the statutory rate schedule, it is also plausible that when asked about their tax rates they first think about their statutory tax bracket. If they understand the progressivity of the tax schedule they will report ATR that is less then their statutory MTR. But they may also think all their income is taxed at that statutory MTR.

The second important dimension of heterogeneity is that many respondents reported numbers

[4] I do not distinguish between respondents who report $MTR < ATR$ versus $MTR > ATR$. The fact that the former relationship is not possible for the true rates suggests that the latter provides stronger evidence that respondents know the tax structure. Incorporating this distinction into the mixture model is left for future work.

that were statutory marginal tax rates, while many others did not. In Table 2, approximately 65% of respondents reported MTR that was one of the seven statutory rates for 2010 and $MTR = ATR$ for 2010 (50.9%) and a similar fraction for 2012 (49.1%), while 46% did so in both waves. Assuming that people who think about statutory rates will always report a statutory rate, albeit potentially incorrect, then at most 46% know and answer according to the marginal tax rate schedule. This suggests that some people know about this tax rate schedule and answer based on where they think they are in the tax schedule. It also provides strong evidence that not everyone knows the statutory marginal tax rate schedule.

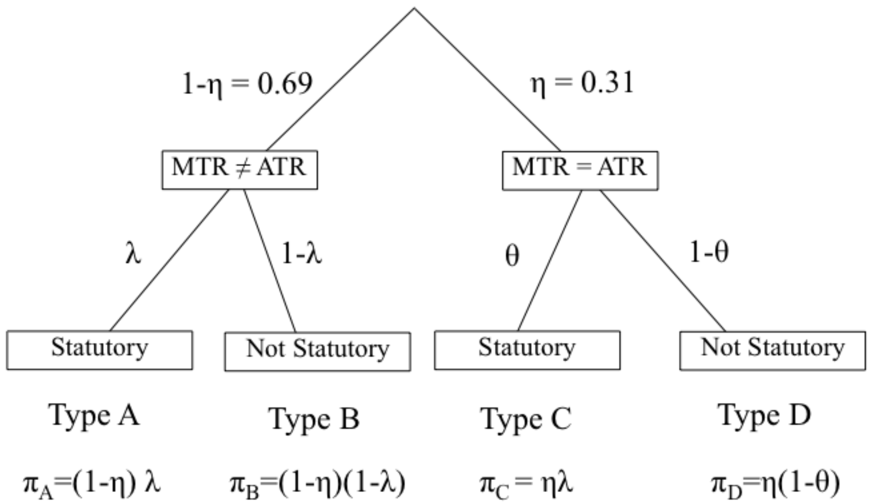
Table 3 shows the cross tabulation of these two categorizations of tax rate responses. The rows reflect whether the rates are the same in both waves, or not, and the columns reflect whether survey MTR was a statutory MTR in both waves, or not. Conditional on reporting $MTR \neq ATR$ in at least one wave, 42.5% reported statutory MTRs in both waves (0.293/0.690). Conditional on reporting $MTR = ATR$ in both waves, 53.7% reported statutory MTRs in both waves (0.167/0.310). According to the conventional wisdom based on previous findings, all of the respondents who report the same number in both waves are calculating their ATR and answering the same rate as the MTR. However, over half of these are statutory MTRs. Assuming that people in fact knew their correct rates, but might only think in terms of MTR or ATR, this would mean there is substantial heterogeneity among respondents who think all of their income is taxed at the same rate. These patterns involving the relationship between MTR and ATR and the statutory MTR versus non-statutory MTR motivate the mixture model developed in the following section.

3.2 Specification of mixture model

Each individual i has a survey measure of log income $y_{i,w}$, of marginal tax rate $m_{i,w}$, and of average tax rate $a_{i,w}$ across two waves of data ($w = 1, 2$). Variables with subscript $w = 1$ are for tax year 2010, using data from CogEcon 2011, and $w = 2$ means tax year 2012, using data from CogEcon 2013. These variables are potentially noisy measures of the true values of log income $y^{\star}_{i,w}$, marginal tax rate $m^{\star}_{i,w}$ and average tax rate $a^{\star}_{i,w}$, which are not observed. The identification strategy distinguishes between responses that happen to be at a statutory MTR versus those which come from respondents mapping their own income into the tax brackets, thereby intentially answering in terms of the statutory rates.

The following tree diagram in Figure 1 (below) provides a visual representation of the mixture model. First, respondents either distinguish between MTR and ATR, or do not. Then, respondents

Figure 1: Tree Representation of Mixture Model



report statutory marginal tax rates, or do not report statutory marginal tax rates.

Among those who distinguish between MTR and ATR, fraction λ report statutory marginal tax rates, whereas $1 - \lambda$ do not report statutory rates. Similarly, among those who do not distinguish between MTR and ATR, fraction θ think all is taxed at their marginal tax rate. The fractions θ and λ are allowed to differ and are expected to do so. The fractions π_A, π_B, π_C and π_D are the population shares of Type A, Type B, Type C and Type D respondents, respectively.

Responses from Types A and C come from the same data generating process (*Report Statutory MTR*) and Types B and D share a different data generating process (*Do Not Report Statutory MTR*). While these are assumed to come from bivariate normal distributions Type C respondents only report their perception of their MTR and Type D only report their perception of ATR. Therefore, responses for Types C and D come from the marginal distributions associated with the bivariate normal distributions generating the data. The parameters

3.2.1 Data generating process 1: *Not Reporting Statutory MTR*

Type B and Type D respondents are assumed not to know the statutory marginal tax rate schedule and do not select from the schedule. Instead, they report a noisy and potentially biased measure of their true rate. Taxpayer i's reported marginal tax rate in wave w ($m_{i,w}$) equals the true MTR plus systematic error and stochastic noise

$$m_{i,w} = m^{\star}_{i,w} + \varepsilon^{m}_{i,w} \tag{4}$$

where true MTR ($m^{\star}_{i,w}$) is defined in the previous section. While $m_{i,w}$ might equal one of the statutory marginal tax rates (S $= \{0; 10; 15; 25; 28; 33; 35\}$), this is assumed to be the result of noisily reporting the rate rather than incorrectly mapping one's income into the statutory rate schedule. For example, many of the statutory marginal tax rates are multiples of 5, so respondents who select among rounded numbers might provide a statutory rate without intending to do so.

Taxpayer i's reported average tax rate in wave w equals the true ATR plus systematic error and stochastic noise

$$a_{i,w} = a^{\star}_{i,w} + \varepsilon^{a}_{i,w} \tag{5}$$

where true ATR is true tax liability (from previous section) divided by true adjusted gross income, $a^{\star}_{i,w} = \frac{T^{\star}_{i,w}}{AGI^{\star}_{i,w}}$. The errors $\varepsilon^{a}_{i,w}$ and $\varepsilon^{m}_{i,w}$ capture heterogeneity in survey reports, conditional on the true tax rates. If respondents had precise beliefs about their tax rates, then the errors $\varepsilon^{a}_{i,w}$ and $\varepsilon^{m}_{i,w}$ capture heterogeneity in misperceptions and random survey noise. If respondents do not have precise beliefs, then the reported values can be interpreted as an unbiased estimate of the mean subjective rates. Heterogeneity in survey measures includes systematic misperception, variation in reporting beliefs due to unresolved uncertainty on behalf of the respondent and random survey noise.

Conditional on true income, observed marginal and average tax rates, summarized as $\mathbf{r_i} = (a_{i,1}, a_{i,2}, m_{i,1}, m_{i,2})'$, have a multivariate normal distribution

$$\mathbf{r_i} \mid \mathbf{y}^{\star}_i \sim N\left(b_{\mathbf{r},ns}, \Sigma_{\mathbf{r},ns}\right)$$

where $b_{\mathbf{r},ns} = (b_a, b_a, b_m, b_m)'$ is the vector of mean tax rate errors and

$$\Sigma_{\mathbf{r},ns} = \begin{pmatrix} \sigma_a^2 & \rho_a \sigma_a^2 & \rho_{am}\sigma_a\sigma_m & 0 \\ \rho_a\sigma_a^2 & \sigma_a^2 & 0 & \rho_{am}\sigma_a\sigma_m \\ \rho_{am}\sigma_a\sigma_m & 0 & \sigma_m^2 & \rho_m\sigma_m^2 \\ 0 & \rho_{am}\sigma_a\sigma_m & \rho_m\sigma_m^2 & \sigma_m^2 \end{pmatrix}$$

I assume the mean and variance are the same across waves. The mean b_a reflects systematic bias in perceptions of average tax rates, and b_m does the same for reported marginal tax rates. The parameters in this variance-covariance matrix are defined as follows. I let σ_a represent the standard deviation of the ATR error (ε^{a}_{w}) and σ_m represents the standard deviation of the MTR error (ε^{m}_{w}). Parameter $\rho_a = Corr\left(\varepsilon^{a}_{1}, \varepsilon^{a}_{2}\right)$ is the correlation of ATR errors across waves, $\rho_m = Corr\left(\varepsilon^{m}_{1}, \varepsilon^{m}_{2}\right)$

is the correlation of MTR errors across waves, and $\rho_{am} = Corr\left(\varepsilon_1^m, \varepsilon_1^a\right) = Corr\left(\varepsilon_2^m, \varepsilon_2^a\right)$ is the correlation of ATR errors and MTR errors within the same wave, and the correlation of ATR and MTR errors across waves is set to zero: $Corr\left(\varepsilon_2^m, \varepsilon_1^a\right) = 0$. The values of ρ_a, ρ_m, and ρ_{am} are between -1 and 1.

3.2.2 Data generating process 2: *Report Statutory MTR*

In contrast, Type A and Type C respondents are assumed to know the statutory marginal tax rate schedule (S $= \{0; 10; 15; 25; 28; 33; 35\}$) and select from this set when answering the survey. They answer the question about their marginal tax rate by mapping their subjective taxable income $I_{i,w}^S$ in wave w into the known income tax brackets.[5] Subjective taxable income ($I_{i,w}^S$) is a potentially error-ridden measure of true taxable income ($I_{i,w}^\star$). The errors are multiplicative, such that

$$I_{i,w}^S = I_{i,w}^\star \cdot \exp\left(\varepsilon_{i,w}^I\right) \tag{6}$$

where $\exp\left(\varepsilon_{i,w}^I\right)$ is the multiplicative error in subjective taxable income. And, with known tax rate schedule, they report statutory rate τ_j when $I_{i,w}^S$ is between known thresholds $\underline{I_w}^j$ and $\overline{I_w}^j$, or

$$m_{i,w} = \tau_j \text{ if } \underline{I_w}^j < I_{i,w}^S < \overline{I_w}^j. \tag{7}$$

More concisely, $m_{i,w} = M\left(I_{i,w}^S\right)$, where $M(\cdot)$ is the step-wise function mapping taxable income to tax rates.

While the average tax rates are again written in terms of additive errors, $a_{i,w} = a_{i,w}^\star + \varepsilon_{i,w}^a$, I allow the parameters on the ATR errors to be different across the two data generating processes for marginal tax rates. The distribution of observed tax rates, conditional on true income (and, hence, true tax rates), is again jointly normal

$$\mathbf{r_i} \mid \mathbf{y}_i^\star \sim N\left(b_{\mathbf{r},st}, \Sigma_{\mathbf{r},st}\right)$$

where $b_{\mathbf{r},st} = \left(b_{a,st}, b_{a,st}, b_I, b_I\right)'$ is the vector of mean tax rate errors and

[5] Subjective taxable income can diverge from their actual taxable income for several reasons and this could equivalently be modeled with known taxable income but unknown tax bracket income thresholds.

$$
\Sigma_{\mathbf{r},st} = \begin{pmatrix} \sigma^2_{a,st} & \rho_{a,st}\sigma^2_{a,st} & \rho_{aI}\sigma_{a,st}\sigma_I & 0 \\ \rho_{a,st}\sigma^2_{a,st} & \sigma^2_{a,st} & 0 & \rho_{aI}\sigma_{a,st}\sigma_I \\ \rho_{aI}\sigma_{a,st}\sigma_I & 0 & \sigma^2_I & \rho_I\sigma^2_I \\ 0 & \rho_{aI}\sigma_{a,st}\sigma_I & \rho_I\sigma^2_I & \sigma^2_I \end{pmatrix}
$$

The mean $b_{a,st}$ reflects systematic bias in perceptions of average tax rates, and b_I is the mean percentage deviation of subjective taxable income from true taxable income. The parameters in this variance-covariance matrix are defined as follows. I let $\sigma_{a,st}$ represent the standard deviation of the ATR error (ε^a_w) and σ_I represents the standard deviation of the taxable income error ($\varepsilon^I_{i,w}$). Parameter $\rho_{a,st} = Corr\left(\varepsilon^a_1, \varepsilon^a_2\right)$ is the correlation of ATR errors across waves, $\rho_I = Corr\left(\varepsilon^I_1, \varepsilon^I_2\right)$ is the correlation of taxable income errors deviations from true taxable income, and $\rho_{aI} = Corr\left(\varepsilon^I_1, \varepsilon^a_1\right) = Corr\left(\varepsilon^I_2, \varepsilon^a_2\right)$ is the correlation of ATR errors and taxable income errors within the same wave, and the correlation of ATR and subjective taxable income errors across waves is set to zero: $Corr\left(\varepsilon^I_2, \varepsilon^a_1\right) = 0$. The values of $\rho_{a,st}$, ρ_I, and ρ_{aI} are between -1 and 1.

Among those who report statutory MTR, the probability of subjective rate τ_j in wave 1 and τ_k in wave 2 is defined as

$$
\varphi_i^{j,k} = \Pr\left(\mathbf{m}_i^S = (\tau_j, \tau_k)' \mid \mathbf{m}_i^\star\right)
$$

Reported statutory marginal tax rates can be written in terms of bounds on the taxable income error, conditional on true taxable income,

$$
\underline{I_w}^j < I^S_{i,w} < \overline{I_w}^j \Longleftrightarrow \log\left(\underline{I_w}^j\right) - \log\left(I^\star_{i,w}\right) \leq \varepsilon^I_{i,w} < \log\left(\overline{I_w}^j\right) - \log\left(I^\star_{i,w}\right) \tag{8}
$$

Therefore, the probability $\varphi_i^{j,k}$ can be written as

$$
\varphi_i^{j,k} = \overrightarrow{\Phi}\left(z^I_{j,H},\, z^I_{k,H},\, \rho_I\right) + \overrightarrow{\Phi}\left(z^I_{j,L},\, z^I_{k,L},\, \rho_I\right) - \overrightarrow{\Phi}\left(z^I_{j,H},\, z^I_{k,L},\, \rho_I\right) - \overrightarrow{\Phi}\left(z^I_{j,L},\, z^I_{k,H},\, \rho_I\right) \tag{9}
$$

where $z^I_{j,H} = \frac{\log\left(\overline{I}^j\right) - Y^\star - D^\star}{\sigma_{eI}}$ and $z^I_{j,L} = \frac{\log\left(\underline{I}^j\right) - Y^\star - D^\star}{\sigma_{eI}}$ for all j and k that are not the bottom or top bracket. The first and fourth terms give the probability of being in category j given that income satisfies the condition on how large category k income is. Then the second and third terms subtract out the probability that we have category j but are less than the lower bound for being in category k.

3.3 Likelihood function

The goal is to estimate the fraction of respondents reporting statutory marginal tax rates. In doing so, I calibrate the standard deviation of income measurement error at $\sigma_e = 0.3$, which is toward the higher end of the estimates in Chapter 1. This calibration accounts for income measurement error while allowing me to estimate a more flexible model of MTR and ATR perceptions.

I observe tax rate responses $\mathbf{r} = (a_1, a_2, m_1, m_2)$ for each respondent i, which is postulated as a draw from a population which is an additive mixture of $C = 4$ distinct types or subpopulations in proportions π_t, such that

$$g\left(\mathbf{r}_i \mid \Psi, \mathbf{y}_i^\star\right) = \sum_{c\in\{A,B,C,D\}} \pi_c g_c\left(\mathbf{r}_i \mid \mathbf{y}_i^\star, \Psi_c\right),\ 0 \leq \pi_c \leq 1, \sum_{c\in\{A,B,C,D\}} \pi_c = 1.$$

The c^{th} mixing component has density $g_c\left(\mathbf{r}_i \mid \mathbf{y}_i^\star, \Psi_c\right)$ which is characterized by the set of parameters Ψ_c. I do not know a priori to which group an individual belongs, so the proportions π_c are interpreted as probabilities of group membership. For each individual, the likelihood is a weighted average of the likelihood of being in each mixing component. In another words, summing over all four components yields the individual's contribution to the likelihood function L. The log likelihood of the finite mixture model is given by

$$\ln\, L\left(\Psi; \mathbf{r}_i\right) = \sum_{i=1}^{N} \ln \sum_{c\in\{A,B,C,D\}} \pi_c g_c\left(\mathbf{r}_i \mid \mathbf{y}_i^\star, \Psi_k\right)$$

where $\Psi = (b_{a,st}, b_I, b_a, b_m, \sigma_{a,st}, \sigma_I, \sigma_a, \sigma_m, \rho_{a,st}, \rho_I, \rho_a, \rho_m, \rho_{aI}, \rho_{am}, \theta, \lambda)'$ is the vector of parameters of the mixture model and Ψ_c is the vector of parameters for component c. As described before, the weights are determined by the mixing proportions

$$\begin{aligned}
\pi_A &= (1-\eta)\cdot\lambda \\
\pi_B &= (1-\eta)\cdot(1-\lambda) \\
\pi_C &= \eta\cdot\theta \\
\pi_D &= \eta\cdot(1-\theta)
\end{aligned}$$

where $\eta = 0.31$ is observed directly from the survey responses.

The density function associated with each of the four components are

$$
\begin{aligned}
g_A &= f_S\left(\mathbf{a}_i, \mathbf{m}_i\right) \\
g_B &= f_{NS}\left(\mathbf{a}_i, \mathbf{m}_i\right) \\
g_C &= f_S\left(\mathbf{m}_i\right) \\
g_D &= f_{NS}\left(\mathbf{a}_i\right)
\end{aligned}
$$

Reporting a statutory MTR is a necessary condition for thinking that all income is taxed at their marginal tax rate. This assumption means that people who think in terms of statutory rates will always answer using a statutory rate and that any measurement error will be in the latent variable deterimining their choice of statutory rate.

3.4 Maximum Simulated Likelihood

The major complication in evaluating the likelihood function arises from the fact that true income is not observed. Estimating the parameters of the model by maximum likelihood involves integrating over the distribution of these unobserved income errors. This problem is solved by simulating the likelihood function. The simulated log-likelihood function is then given by

$$
\mathrm{SLL}\left(\Psi\right) = \sum_{i=1}^{N} \ln\ \check{L}_i\left(\Psi\right) \tag{10}
$$

The contribution of each individual i is $\check{L}_i\left(\Psi\right)$, which is a simulated approximation to $L_i\left(\Psi\right)$, derived as

$$
\check{L}_i\left(\Psi\right) = \frac{1}{K}\sum_{k=1}^{K} L_i^k\left(\Psi\right) \tag{11}
$$

where the average is over the likelihood evaluated at each simulation draw

$$
L_i^k\left(\theta\right) = \Pr\left(\mathbf{r}_i \mid \mathbf{y}_i, e_{i(k)}\right) \cdot f\left(\mathbf{y}_i\right) \tag{12}
$$

and K is the number of pseudorandom draws of the vector of errors $e_{i(k)}$. The algorithm involves simulating a distribution of income errors for each respondent. The individual's likelihood contribution is computed for each set of income errors, and density of the implied tax errors are averaged over the K values to obtain the simulated likelihood contribution.

3.5 Discussing the assumptions underlying identification of the mixture model

Identification of the mixture model requires strong assumptions about the mental models people use when answering questions about marginal and average tax rates. First, respondents who think all income is taxed at the same rate will give the same number to both questions. While this is reasonable because the questions are immediately following one another, it ignores random errors that could come from mistyping an answer, or errors the come from the data processing and cleaning process. Second, reporting a statutory rate for one's own MTR in both waves is a necessary condition for being characterized as knowing and reporting marginal tax rates, but it is insufficient.

First, respondents are partially classified based on whether they distinguish between ATR and MTR, or not, based on their reported rates across the two waves. Partial classification into A/B versus C/D is based on the survey responses in CogEcon 2011 and CogEcon 2013.

When people report $MTR = ATR$ in both waves, I have information on perceived ATR or perceived MTR, but not both. The purpose of this model is to estimate the fraction of these respondents who are reporting their perception of their ATR or their perception of their MTR. Statistically, this means determining whether the implied ATR and MTR errors associated with reporting MTR=ATR are more likely to be from the distribution of MTR errors or ATR errors. This information comes from respondents who report $MTR \neq ATR$ in at least one wave.

I assume reported rates are perceived MTR only if a statutory marginal tax rate is given for both. Someone who reports MTR=ATR=20 will be categorized as giving their ATR, Someone who reports MTR=ATR=15 may be categorized as ATR or MTR. The fraction of MTR respondents is therefore a lower bound on the fraction who think all income is taxed at this marginal rate. To determine the fraction who answering in terms of statutory rates I need to distinguish between people who are answering in terms of statutory rates and those who are rounding non-statutory perceived rates. This part of the model allows me to identify the fraction of respondents who report in terms of statutory marginal tax rates.

3.6 Estimation

Estimation of $\Psi = (b_{a,st}, b_I, b_a, b_m, \sigma_{a,st}, \sigma_I, \sigma_a, \sigma_m, \rho_{a,st}, \rho_I, \rho_a, \rho_m, \rho_{aI}, \rho_{am}, \theta, \lambda)'$ is done using the method of maximum simulated likelihood, implemented in Stata, taking the estimates of the income error distribution as given. In practice, I use 50 draws per individual to simulate the likelihood. Quasi-random Halton sequence draws, rather than random draws, are used to simulate the likelihood because of the documented superior performance of quasi-random Halton draws relative to random draws in the simulation of integrals (e.g., Train 1999, Bhat 2001). Halton sequences are used to construct draws over the two-dimensional income measurement error. The individual's likelihood contribution is computed for each set of income errors, and density of the implied tax errors are averaged over the K values to obtain the simulated likelihood contribution. I use Stata's modified Newton-Ralphson algorithm to maximize the log likelihood function. It converges under Stata's rigorous criteria for declaring convergence.[6] A detailed description of the simulation algorithm and likelihood evaluation are in Appendix **??**.[7]

Estimation of mixture models is often challenging. Direct maximization of the log likelihood function may encounter several problems, even if it is, in principle, feasible. The highly nonlinear form of the log likelihood causes the optimization algorithm to be slow or be even incapable of finding the maximum. At the same time, the likelihood of a finite mixture model is often multimodal and it cannot be guaranteed that standard optimization routines will converge to the global maximum rather than to one of the local maxima.

I reduce the computational burden substantially by calibrating the standard deviation of income measurement error at $\sigma_e = 0.3$, which is toward the higher end of the estimates in Chapter 1. This calibration helps me avoid numerical issues that arise when estimating this parameter using simulated maximum likelihood, as discussed at length in Chapter 1.

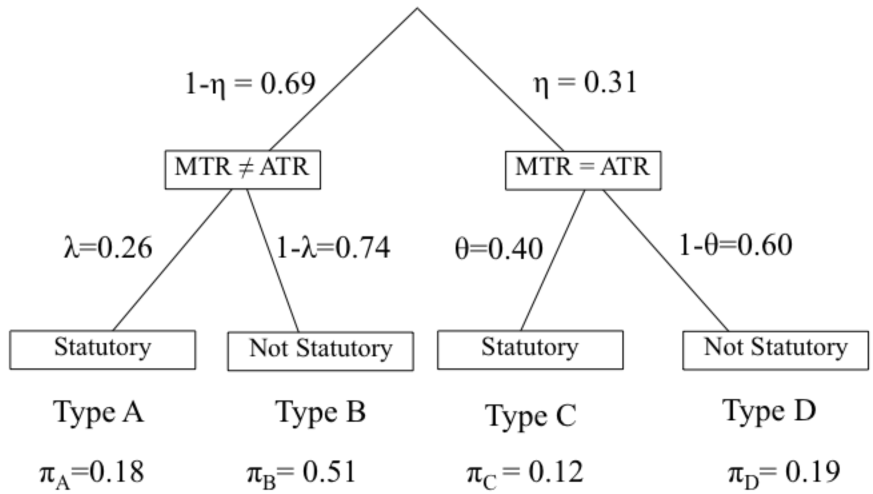
The estimation algorithm uses several transformations to ensure that standard deviations are positive, correlation coefficients are between -1 and 1 and the mixing proportions are between 0 and 1. The latter transformations are new to this analysis, whereas the others are the same as the algorithm for estimating the single component model. The estimation algorithm parameterizes the mixing proportions θ and λ are parameterized as logistic functions to constrain them to lie between 0 and 1. After the algorithm converges, estimates of $\widehat{\theta}$ and $\widehat{\lambda}$ are recovered by transformation.

Finally, I select starting values for the parameters to avoid converging to local maxima. In particular, I use starting values for the mixing parameters to avoid convergence to a local maximum that is a degenerate case in which all respondents who report numbers that are statutory MTR are

[6] Stata's default optimization routines have three requirements for declaring convergence. First, the tolerance for changes in the coefficient vector from one iteration to the next or the tolerance for changes in the likelihood from one iteration to the next must be sufficiently small. Second, the second criterion is having a sufficiently small gradient relative to the Hessian (`nrtolerance`()). This is formally based on $g'\mathbf{H}^{-1}g$, where the gradient g and Hessian matrix $\mathbf{H}$ are calculated at the parameter vector $\widehat{\Psi}$. Finally, the Hessian must be concave.

[7] That appendix is written as if the standard deviation on income measurement error is estimated jointly, but the algorithm is the same when this parameter is instead calibrated. The important difference is that calibrating income measurement error allows me to use Stata's more rigorous convergence criteria, which is important when estimating a mixture model that is prone to having multiple maxima.

Figure 2: Tree Representation of Mixture Model Estimates



classified as giving statutory MTR. Starting values for the distributional parameters are based on estimates in Chapter 1.

4 Results

Table 4 presents estimated parameters and standard errors for the baseline mixture model. Block bootstrapped standard errors are computed based on 200 replications, with sampling blocks defined across households.

These mixing proportions $\widehat{\lambda}$ and $\widehat{\theta}$ presented in the top panel of Table 4 provide strong evidence of heterogeneous mental models of tax rates. This estimates are also presented in Figure 2, below, which displays them in the tree diagram introduced in Figure 1.

The parameter $\widehat{\lambda} = 0.26$ is the estimated fraction of the population, among those who distinguish between MTR and ATR, who report statutory marginal tax rates. The parameter $\widehat{\theta} = 0.40$ is the estimated fraction of the population, among those who do not distinguish between MTR and ATR, who think all of their income is taxed at their statutory MTR. This means that altogether $0.31 \times 0.40 + 0.69 \times 0.26 = 0.30$ report statutory marginal tax rates. This should be interpreted as the fraction who know the statutory marginal tax rate schedule and report MTR accordingly. This is substantially smaller than 0.460, the fraction who reported statutory MTR in both waves (see Table 2). Comparing these fractions highlights the role of the model-based approach to clustering. Because statutory MTR are often rounded numbers (0, 10, 15, 25, 35), the statistical model is needed to distinguish between someone who is randomly guessing their rate and answers using rounded numbers (as is often the case in survey measures, more generally). The estimated proportions for knowing statutory MTR are likely a lower bound. By construction, reporting errors are constrained to be of a particular form. Errors in filling out the survey or in processing the data would make a respondent categorically ineligible for being classified as the statutory MTR type.

Parameters on the MTR and ATR error distributions are in the lower panel of Table 4, with parameters governing the two data generating processes for those who report statutory MTR and those who do not report statutory rates on the left and right sections, respectively.

Respondents who report statutory MTR also provide more accurate measures of ATR. This is apparent in the parameters on the MTR and ATR error distributions. The mean of the ATR bias (b_a) is the mean difference between the survey and latent true rate and reflects systematic bias in perceptions of average tax rates. The mean bias in ATR is around 3.3, compared to 6.2 for those who do not know statutory MTR. The estimated standard deviation (σ_a) is also smaller for those who report statutory MTR. It does not make sense to compare the MTR distributions across the two types, as they are fundamentally different error processes and have different scale.

Most striking is the difference in the cross-wave correlations among those who report statutory MTR compared to respondents who do not report statutory MTR. The implied taxable income errors are strongly correlated across waves ($\widehat{\rho}_I = 0.620$) for the former, while the correlation of the MTR errors are indistinguishable from zero ($\widehat{\rho}_m = 0.037$) for the latter. A similar pattern holds for the correlation of the ATR errors, with estimated correlations of $\widehat{\rho}_{a,st} = 0.464$ and $\widehat{\rho}_a = 0.229$.

These estimates should be interpreted in light of the different error structures, where taxable income error is latent (even conditional on knowing true income), whereas MTR errors is continuously distributed and observed (once I know true income and reported MTR). This might account for the stronger correlation of ATR errors across wave but smaller correlation of the MTR and ATR errors within the same wave.

One explanation is that the errors in subjective taxable income capture private information about filing behavior whereas the MTR errors for the non-statutory respondents are effectively random, after accounting for the relationship with ATR errors. This could be the case if respondents answer both income and tax rates perfectly, but they have private information about their deductions. This intuition is best conveyed through a stylized example. Consider a respondent with total income $Y^\star = 100,000$, which is reported without error. I assume they claim the standard deduction of $SD = 20,000$, but the household in fact itemizes deductions and has $D^\star = 30,000$. Exemptions are perfectly measured, $E^{\star} = 10,000$, which means true taxable income is $I^{\star} = 60,000$. The computed taxable income would be $I^{\star} = 70,000$. The tax system is such that taxable income up to $20,000$ is taxed at 10% and taxable income above $20,000$ is taxed at 20%. Based on my tax calculator, I would compute $m^{\star} = 20$ and $a^{\star} = \frac{(0.1)\times 20{,}000+(0.2)\times 50{,}000}{100{,}000} = 12\%$. and the respondent, who is actually giving the correct rates, is reporting $a^{S} = \frac{(0.1)\times 20{,}000+(0.2)\times 40{,}000}{100{,}000} = 10\%$. There is no additional information about subjective taxable income, besides what comes from reported average tax rate. The systematic inability to account for itemized deductions shows up in the ATR errors over time and the correlation of the MTR and ATR error within wave, but not with errors in subjective taxable income across waves.

To check the sensitivity of the estimates, I extend the model to better account for large errors by modeling contaminated responses. The "contaminated" distribution has the same mean as the error distribution but the standard deviation of both MTR and ATR errors is scaled up by an amount k. Intuitively, the goal is to categorize the responses which are plausibly the perceived ATR or perceived MTR. Both the ATR and MTR responses are modeled as a mixture of "normal" responses and the "extreme" responses. The model with contamination is derived in Appendix C.3.

Table 5 presents estimated parameters and standard errors for the baseline mixture model with contamination. The fraction of statutory type respondents increased because it is now interpreted as the fraction of statutory MTR conditional on not having contaminated data. As expected, accounting for contamination results in smaller estimates of the tax error standard deviation among the Type B respondents.

While the prior (unconditional) probability of class membership is constant across observations, I use Bayes Theorem and the finite mixture parameter estimates to calculate the posterior probability of being in each latent class. The posterior probability that individual i is in class q is computed as

$$\pi_{iq}\left(\Omega_i, \widehat{\Psi}\right) = \frac{\pi_q \cdot g_q\left(\Omega_i \mid \widehat{\Psi_q}\right)}{\sum_{c\in\{A,B,C,D\}} \pi_c g_c\left(\Omega_i \mid \widehat{\Psi_c}\right)}$$

Individuals are assigned group membership based on the maximum of these posterior probabilities. This characterization depends on the magnitude of their errors relative to the distribution of the ATR and MTR errors themselves.

Table 6 displays the estimated relative group sizes of the behavioral types for the main model. Taken together, 20.4% are classified as Type A, 12.1% as Type C, and 19.0% as Type D. The Type C respondents overestimate the disincentive associated with (particular) marginal decisions, as changing tax brackets is associated with discontinuous and large changes in tax liability, and Type D respondents underestimate the disincentive associated with marginal decisions. Finally, 48.6% as Type B. This is the remaining group who cannot be classified as knowledgeable or having systematic errors. These fractions classified as each type are similar to, but do not equal, the estimated mixing proportions.

For a classification of tax rate perceptions to be of value when analyzing tax-related behavior, all individuals should be clearly associated with one component. The high quality of classification can be inferred from the distributions of the individuals' posterior probabilities of group membership. Figure **??** shows a histogram of the posterior probabilities of assignment to the four classes and Figure 3 does the same for the five classes in the model with contamination. As the distributions show, the individuals' posterior probabilities are either close to 1 or close to 0 for almost all individuals, indicating an extremely clean segregation of subjects to types. This result substantiates that there are distinct types of perceptions in the population and that the statistical model provides a sound basis of discriminating between them.

It is also interesting to observe the relationship between individuals' reported income and tax rates and how they get classified by the model. Figure 4 presents a scatterplot showing classification of types by reported log income and MTR (among those reporting statutory rates in both waves). The plots for each observation are categorized by the ex-post classification by type. Respondents who report marginal tax rates consistent with their reported income get classified as Type A or Type C, depending on whether they distinguish between ATR and MTR. When reported MTR and income diverge, there is more heterogeneity in classification, which depends on reported ATR and the observations in the other wave.

Looking at a few specific cases helps illustrate the mechanisms driving the classification. This table is recreated from Chapter 1, but focused on classification of types rather than imputed tax rates. It displays observed income, MTR and ATR in 2010 and 2012. The "type" is the classification based on the mixture model. The "raw type" is the classification when taking the survey responses at face value.

	Observed (2010)			Observed (2012)			Classification	
R's ID	Income	MTR	ATR	Income	MTR	ATR	Type	Raw Type
R1: 7013920010	100000	3	15	121660	15	15	B	B
R2: 7013920020	100000	7	7	121600	15	15	D	D
R3: 7004980020	100000	10	10	112930.5	2	22	B	B
R4: 7020620010	100000	15	28	115600	28	28	A	A
R5: 7005380020	100000	18	18	91440	19	18	B	B
R6: 7007370010	100000	25	9	100000	5	1.5	B	B
R7: 7013590010	100000	35	30	25000	15	15	B	A

Respondent R7 is classified as Type A based on the raw rates, because both 35 and 15 are statutory marginal tax rates and $MTR \neq ATR$ in 2010. However, R7 gets classified as Type B as a result of the model-based classification. This means the relationship between the reported income and rates is more similar to the distribution for those who do not reporte statutory MTR. As illustrated in Table 6, the fraction of Type A respondents falls from 29.3% to 19.0% when classifying types using the mixture model rather than the raw rates.

The main take-away is there is substantial heterogeneity. One-fifth of respondents, among those who answered these questions, definitively know the structure of the tax system. At the same time, almost a third of respondents report $MTR = ATR$ in both CogEcon 2011 and CogEcon 2013, with an estimated 60% reporting perceived ATR for both and 40% reporting perceived statutory MTR for both. This is consistent with de Bartelome's finding that people are more likely to use average tax rates instead of marginal rates. However, a more precise conclusion is that some people use ATR while others use MTR.

Whether people's behavior responds to perceptions of tax rates or the true rates is an empirical question beyond the scope of this paper. To the extent to which perceptions matter, understanding how people perceive tax rates is critical to evaluating the anticipated impact of tax rate changes. For example, changes in statutory marginal tax rates will likely have the largest effect on Type C respondents and smallest effect on Type D respondents. The former will respond because they think all of their income gets taxed at that rate, while the latter will respond only to the extent to which changes to statutory MTR affect perceived ATR. More broadly, the Type A classification is an indicator of tax knowledge, and it would be interesting to examine the relationship between tax knowledge and sensitivity to tax changes.

4.1 Model Fit

Model selection in the context of finite mixtures remains difficult and unresolved. Standard likelihood ratio tests are inappropriate because mixture models do not satisfy the regularity conditions (McLachlan and Peel, 2000; p. 185-6). The difficulty comes from the parameter boundary hypothesis problem, wherein the null hypothesis is specified by the true value being on the boundary of the parameter space. Without a standard way to assess model fit, the model should be assessed in light of its intended use. There are two main purposes for using finite mixture models. One is to have a semi-parametric framework to model data that comes from unknown distributions. The second is to use the model for model-based clustering. In both cases an important issue is the number and form of the mixing components. This paper mainly focuses on classification of individuals into groups based on their reported tax rates. Yet the first purpose is also important. While the mixture components have a clear interpretation, reported tax rates inherently come from unknown distributions.

Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC) appear to work well in the former cases (McLachlan and Peel, 2000; p. 175). These common penalized information criteria trade off model fit with parsimony by penalizing model complexity (Cameron and Travedi, 2005; p. 278). AIC selects the model that minimizes

$$AIC = -2 \log \widehat{L} + 2d$$

where $\log \widehat{L}$ is the maximized log likelihood and d is the number of parameters of the model. BIC selects the model that minimizes

$$BIC = -2 \log \widehat{L} + d \log N$$

where N is the sample size. The BIC reduces the tendancy of AIC to overfit models by penalizing model complexity more heavily than does AIC.

Table 7 compares the one-component model with the full mixture model and the full mixture model with contamination. Both the AIC and BIC point toward the mixture model as a substantial improvement over the one-component model. The mixture model accounts for latent heterogeneity that is unaccounted for in the one-component model. When intergroup differences are large, the finite mixture provides a much better fit than the one-component model.

Entropy criteria, based on the posterior probabilities of group membership, can be used to evaluate the quality of classification. Celeux and Soromenho (1996) proposed the normalized entropy criterion (NEC) provides another way to summarize this information about the quality of classification. The NEC is an entropy criteria based on the posterior probabilities of group membership. The entropy of classification is defined as

$$E\left(c\right)=\sum_{c=1}^{C}\sum_{i=1}^{N}t_{ic}\log\left(t_{ic}\right)$$

where t_{ic} represents the posterior probability that person i arises from component c. The NEC is then defined in terms of the normalized entropy

$$NEC=\frac{E\left(c\right)}{\log\widehat{L}_c-\log\widehat{L}_0}$$

where $\log\widehat{L}_c$ is the maximized log likelihood of the mixture model with c components, and $\log\widehat{L}_0$ is the maximized log likelihood of the single-component model. NEC values are smaller when there is precise classification of individuals. If all individuals can be clearly assigned to one of the different behavioral groups, the posterior probabilities are close to 0 and 1, and NEC≈0. The NEC always is close to 0, so there are hardly any mixed types with ambiguous group affiliation.

Other criterion have been proposed to account for the fact that the NEC does not incorporate model fit. The classification likelihood information criterion (CLC), proposed by Biernacki and Govaert (1997), combines the model log-likelihood with the estimated entropy $E\left(c\right)$ to penalize for model complexity. The number of components is chosen to minimize

$$CLC=-2\log\widehat{L}+2E\left(c\right)$$

The Integrated Classification Likelihood Criterion (ICL), as proposed by Biernacki et al. (1998), is the BIC with an additional penalty for mean entropy:

$$ICL=-2\log\widehat{L}+d\log N+2E\left(c\right)$$

All four criteria point toward using the mixture model (with or without accounting for contamination) instead of the single component model. The model that accounts for contamination is preferred to the baseline mixture model based on all criteria but NEC. This means the two-component model of statutory and non-statutory MTR respondents is preferable if the central issue is a parsimonious representation of tax rate perceptions rather than having the best model fit. The model with contamination provides a more detailed description of the non-statutory respondents.

5 Heterogeneity within and between classes

The baseline mixture model implicitly assumes that every respondent has an equal ex ante likelihood of being in the respective classes. Since class membership reveals information about knowledge of the tax system, class membership could be related to other observable variables. I explore heterogeneity in class membership using two approaches. First, I use a multinomial logistic model to model class membership.

5.1 Analyzing determinants of ex-post class membership

In this section I analyze the relationship between tax perceptions and cognitive ability, financial knowledge and using professional tax preparers. The variable for tax perceptions is predicted class membership discussed in Section 4. Classes are derived from the posterior probabilities from the mixture model with contamination and I remove 9 respondents who were classified as having contaminated tax rates data.

I am most interested in how cognitive ability and financial sophistication are correlated with tax perceptions. I expect that people who are knowledgeable about financial matters, overall, would be more likely to know statutory tax rates and to know that tax rate progressivity implies that $MTR \neq ATR$. I include log of income to account for a possible mechanical relationship between level of income and whether someone gets classified as reporting statutory MTR, or not. This income measure is the average of log of reported income in the two waves. In order to interpret these groups as having distinct perceptions, I need to ensure that there are not variables that are correlated with ability and the use of tax preparation that could be driving the classification into types. For this reason, I include a dummy variable for using a tax return while completing the survey.[8] It seems plausible that responses are influenced by information used to answer the questions. I expect that

[8] The following question is asked at the end of each survey: "H1: What sources of information did you use to assist you in answering the questions about your finances in this questionnaire? *Please check all that apply.*" Tax returns was listed as an option. The dummy variable equals one if the respondent used tax returns in both waves. However, it is important to reiterate that this question refers to the questionnaire, overall, which has over one hundred questions; using a tax return at some point in the questionnaire does not necessarily mean they used tax returns to answer the tax rate questions.

someone who answered the income questions using their tax return is more likely to check their tax return to calculate their average tax rate.

Table 8 shows results from ordinary least squares regressions of class membership on the explanatory variables described above. The main result is that the impact of cognitive ability on knowing statutory rates is larger for people who prepare their own tax returns than for people who use hire professional assistance. Surprisingly, among respondents who used paid tax preparers, whether someone reports statutory marginal tax rates is unrelated to cognitive ability and financial sophistication. Among respondents who report $MTR = ATR$, higher financial sophistication has no bearing on whether the respondent reported ATR or their statutory MTR.

These results are robust to model specification and the choice of covariates. The same qualitative results hold when using logistic and probit models, as well as in a multinomial logistic model of all four classes. Including indicator variables for level of education reduces the estimated coefficients on number series and financial sophistication scores, as is expected given the correlation of education, but does not qualitatively change the results.

Treating tax return preparation as exogenous limits what sorts of conclusions can be drawn from these analyses. In particular, I cannot distinguish between two explanations for this pattern. People who do not understand the tax system might have higher demand for third party tax assistance. Another explanation is learning by doing, in which the amount that is learned depends on cognitive ability. People with higher cognitive ability might learn more from preparing their own taxes than someone with lower cognitive ability. As a result, cognitive ability is correlated with reporting marginal tax rates only among the population who do not use tax preparers.

5.2 Mixture model with covariates

Now I examine the role of covariates in determining the mixture proportions and the systematic heterogeneity within the mental model. I model systematic heterogeneity by specifying the mean tax rate errors as linear indices. This is defined as $b_x^m = \mathrm{x}'\beta^m$ for marginal tax rates, $b_x^a = \mathrm{x}'\beta^a$ for average tax rates, and $\mu_{y^\star} = \mathrm{x}'\beta^y$ for true (latent) income. Estimates of parameters in β^m and β^a tell us how tax rate perceptions vary, on average, with other observable characteristics. These parameters can differ across the two data generating processes. The mixture model with covariates converges incredibly slowly. To reduce the computational costs of performing the maximum likelihood estimation, I iterate between the Newton-Raphson algorithm and the less computationally burdensome DFP algorithm.

Table 9 presents estimated parameters and standard errors when including covariates. The probability of reporting statutory marginal tax rates is positively associated with financial sophistication and negatively associated with using a paid tax preparer. This suggests that accounting for the mental model does help interpret the results on financial sophistication in the one component model. The relationship between cognitive ability (number series score) and reporting statutory MTR is more difficult to understand. There is a positive relationship among those who distinguish between MTR and ATR and a negative relationship among those who do not. This suggests that smarter people are more likely to think all the income is taxed at their average tax rate than at their statutory MTR.

Using a paid tax preparer is positively associated with tax rate errors, but only among people who do not report statutory MTR. If someone is informed (and hence uses statutory MTR) then using a tax preparer has no impact. Another way to say this is that tax preparation may not inherently induce people to report higher tax rates. But using a tax preparer is correlated with being uninformed and those who are uninformed are more likely to report higher rates. Financial sophistication has a strong and negative association with the ATR errors, but only among respondents who report statutory MTR.

One caveat is that the estimation algorithm uses a logistic transformation to estimate the model. The marginal effects of the mixing proportions must be transformed from the estimated coefficients in order to interpret their magnitude. The qualitative conclusions should still hold. Nevertheless, I exercise caution when drawing conclusions from these estimates, as the standard errors do not account for the dependence across households or the noise that results from using a simulated likelihood function. Bootstrapped standard errors would likely be larger, conveying greater uncertainty about the estimated coefficients. These marginal effects calculations and clustered standard errors are left for the next version of the paper.

6 Conclusion

This paper uses a finite mixture model to identify and characterize heterogeneous "mental models" of tax rates. Based on the raw data across two waves, half of respondents think all income is taxed at the same rate. Using estimates from the mixture model, roughly 30 percent of respondents know the statutory marginal tax rates schedule (and answer questions accordingly). And, among respondents who report the same number for MTR and ATR in both waves, close to 40 percent report a statutory marginal tax rate. Cognitive ability is strongly associated with knowledge of statutory marginal tax rates, but only among those who file their own tax return (rather than use a paid preparer).

There are several limitations to this analysis. First, I must rely on whether people reported the same amount for MTR and ATR. If someone was confused about the questions they might report different numbers for MTR and ATR even though they think all income is taxed the same. There is also survey noise and someone knows that marginal and average rates are not equal, but simplifies their survey responses by giving the same number.

The sample size limits my ability to empirically identify both the mixing proportions and the distributions by the different types. Having two years of data provides a strong justification for partially classifying observations by whether they reported $MTR = ATR$ in both waves or $MTR \neq ATR$ in at least one wave. With more observations, it would be interesting to make these mixing proportions stochastic rather than deterministic and allow the mixing proportions to be different across waves. Respondents might have learned about the tax system in-between waves, or different information might be salient in one wave relative to the next. If people who are employed are less likely to think in terms of the marginal tax rates, then changes in employment status could affect the salience of information about taxes.

My substantive contribution is the estimate of the fraction of respondents with different mental models about income tax rates. However, this research is also related to a broader attempt to measure and analyze perceptions of complicated incentive structures. The methodological contribution is the semi-parametric mixture model approach to measuring and accounting for heterogeneity. This semi-parametric mixture model approach to analyzing tax rate perceptions is likely applicable in such other settings, as well.

Table 1: MTR & ATR across waves based on raw responses (fractions)

	2012		
2010	MTR $\neq$ ATR	MTR=ATR	Total
MTR $\neq$ ATR	0.310	0.181	0.491
MTR=ATR	0.198	0.310	0.509
Total	0.509	0.491	1

Notes: The rows represent the MTR and ATR responses about tax year 2010 and the columns reflect the MTR and ATR responses about tax year 2012. Each cell presents the fraction of respondents associated with its row and column. There are 348 observations.

Table 2: Statutory & non-statutory MTR across waves based on raw responses (fractions)

	2012		
2010	Statutory	Non-statutory	Total
Statutory	0.460	0.190	0.649
Non-statutory	0.167	0.184	0.351
Total	0.626	0.374	1

Notes: The rows represent the MTR responses about tax year 2010 and the columns reflect the MTR responses about tax year 2012. Each cell presents the fraction of respondents associated with its row and column. There are 348 observations.

Table 3: Categorization based on raw responses (fractions)

MTR and ATR in both waves	Statutory MTR in both waves	Not statatutory MTR in both waves	Total
MTR $\neq$ ATR	0.293	0.397	0.690
MTR=ATR	0.167	0.144	0.310
Total	0.460	0.540	1

Notes: The rows are categories of responses for MTR and ATR. The top row is for respondents who do not give the same number for both rates in both waves and the bottom row is for respondents who give MTR=ATR in both waves. The columns reflect whether the reported MTR is a statutory rate in both waves, or not. Each cell presents the fraction of respondents associated with its row and column. There are 348 observations.

Table 4: Maximum Likelihood Estimates of mixture model (no contamination)

Fraction StatMTR type:								
among MTR≠ATR	$\widehat{\lambda}$	0.264						
		(0.034)						
among MTR=ATR	$\widehat{\theta}$	0.402						
		(0.052)						
	Report Statutory				**Does Not Report Statutory**			
	ATR Error		TI Error		ATR error		MTR error	
Means	$\widehat{b}_{a,st}$	3.225	$\widehat{b}_I$	-0.143	$\widehat{b}_a$	6.733	$\widehat{b}_m$	1.054
		(1.305)		(0.118)		(0.597)		(0.766)
Standard deviations	$\widehat{\sigma}_{a,st}$	9.220	$\widehat{\sigma}_I$	1.100	$\widehat{\sigma}_a$	10.529	$\widehat{\sigma}_m$	12.646
		(1.041)		(0.136)		(0.867)		(0.770)
Correlations	$\widehat{\rho}_{a,st}$	0.464	$\widehat{\rho}_I$	0.620	$\widehat{\rho}_a$	0.229	$\widehat{\rho}_m$	0.037
		(0.152)		(0.130)		(0.074)		(0.066)
	$\widehat{\rho}_{aI}$	0.164			$\widehat{\rho}_{am}$	0.434		
		(0.103)				(0.067)		
Log likelihood		-5436.4523						
Parameters		16						
Observations		348						

Notes: This table presents maximum likelihood estimates of the mixture model parameters from the baseline model without contamination. Respondents who report statutory marginal tax rates come from one distribution, respondents who do not report statutory MTR come from another distribution. ATR error refers to the difference between reported and true ATR; MTR error refers to the difference between reported and true MTR; and the TI error refers to the multiplicative error in the subjective taxable income. I assume income measurement error has standard deviation 0.3 and for each respondent I use 50 Halton draws to integrate over the income measurement error when computing the log-likelihood function. The optimization routine uses a modified Newton-Raphson algorithm. Bootstrapped standard errors based on 200 replications are listed in parentheses below parameter estimates. Replications are generated after clustering by household to account for unobserved within-household correlation.

Table 5: Maximum Likelihood Estimates of mixture model (with contamination)

Fraction StatMTR type:								
among MTR$\neq$ATR	$\widehat{\lambda}$	0.281						
		(0.038)						
among MTR=ATR	$\widehat{\theta}$	0.402						
		(0.057)						
Fraction contaminated	$\widehat{\gamma}$	0.031						
		(0.013)						
Contamination scaling factor	$\widehat{k}$	2.834						
		(0.566)						
		Report Statutory				**Does Not Report Statutory**		
		ATR Error		TI Error		ATR error		MTR error
Means	$\widehat{b}_{a,st}$	3.290	$\widehat{b}_I$	-0.147	$\widehat{b}_a$	6.156	$\widehat{b}_m$	0.550
		(1.394)		(0.270)		(0.515)		(0.656)
Standard deviations	$\widehat{\sigma}_{a,st}$	9.289	$\widehat{\sigma}_I$	1.129	$\widehat{\sigma}_a$	8.892	$\widehat{\sigma}_m$	11.293
		(1.003)		(0.562)		(0.371)		(0.596)
Correlations	$\widehat{\rho}_{a,st}$	0.439	$\widehat{\rho}_I$	0.591	$\widehat{\rho}_a$	0.211	$\widehat{\rho}_m$	0.037
		(0.178)		(0.125)		(0.065)		(0.050)
	$\widehat{\rho}_{aI}$	0.138			$\widehat{\rho}_{am}$	0.571		
		(0.093)				(0.057)		
Log likelihood		-5368.021						
Parameters		16						
Observations		348						

Notes: This table presents maximum likelihood estimates of the mixture model parameters from the baseline model with contamination. Respondents who report statutory marginal tax rates come from one distribution, respondents who do not report statutory MTR come from another distribution. I also incorporate contaminated responses, which are assumed to come from a distribution that does not report statutory MTR but with the standard deviation scaled by a parameter k that is also estimated in the model. ATR error refers to the difference between reported and true ATR; MTR error refers to the difference between reported and true MTR; and the TI error refers to the multiplicative error in the subjective taxable income. I assume income measurement error has standard deviation 0.3 and for each respondent I use 50 Halton draws to integrate over the income measurement error when computing the log-likelihood function. The optimization routine uses a modified Newton-Raphson algorithm. Bootstrapped standard errors based on 200 replications are listed in parentheses below parameter estimates. Replications are generated after clustering by household to account for unobserved within-household correlation.

Table 6: Classification of tax rate perceptions: relative group size (fractions)

Classification	Raw	No Contamination	Contamination
Type A	0.293	0.190	0.198
Type B	0.397	0.500	0.466
Type C	0.167	0.126	0.126
Type D	0.144	0.184	0.184
Contaminated			0.260
All	1	1	1

Notes: This table shows the distribution of respondents across the four types, first based on the raw data and then from classifying individuals based on the maximum of the posterior probabilities of group membership. Type A respondents are those who distinguish between ATR and MTR and report statutory marginal tax rates. Type B respondents distinguish between ATR and MTR and do not report statutory marginal tax rates. Type C respondents do not distinguish between ATR and MTR and report statutory marginal tax rates for both. Type D respondents do not distinguish between ATR and MTR and report average tax rates for both. There are 348 observations.

Table 7: Model selection criteria

	AIC	BIC	NEC	CLC	ICL
One component model	12,943	12,970	n.a.	12,929	12,970
Mixture model	10,905	10,976	**0.0169**	10,908	11,001
Mixture model with contamination	**10,772**	**10,841**	0.0240	**10,789**	**10,894**

Notes: This table presents various penalized information criterion to use for model selection. See paper for details for the five criteria. Bold means the model is chosen based on that criterion. Estimates from the one component model are not presented in the paper.

Table 8: OLS Estimates of Classification Indicator on covariates

Dependent variable: / Sample restriction: **Variables**	Statutory Full model (1)	Statutory MTR≠ATR (2)	Statutory MTR=ATR (3)	MTR=ATR Statutory (4)
Number Series x (Tax Preparer =Yes)	0.000	0.021	-0.047	-0.064
	(0.044)	(0.048)	(0.091)	(0.097)
Number Series x (Tax Preparer = No)	0.102	0.074	0.133	0.057
	(0.047)	(0.054)	(0.089)	(0.075)
Financial Sophist x (Tax Preparer =Yes)	0.048	-0.003	0.152	0.080
	(0.043)	(0.048)	(0.105)	(0.079)
Financial Sophist x (Tax Preparer = No)	0.041	0.071	-0.072	-0.029
	(0.044)	(0.050)	(0.068)	(0.077)
Used paid tax preparer	-0.055	-0.033	-0.086	0.092
	(0.057)	(0.069)	(0.093)	(0.112)
log(income)	0.018	0.067	-0.072	-0.048
	(0.054)	(0.062)	(0.101)	(0.079)
log(wealth)	0.000	0.007	-0.020	-0.117
	(0.032)	(0.043)	(0.056)	(0.053)
Used tax return	-0.027	0.023	-0.190	-0.164
	(0.062)	(0.073)	(0.131)	(0.096)
Have tax-adv retire accounts?	-0.124	-0.107	0.081	0.013
	(0.076)	(0.101)	(0.142)	(0.149)
Working?	-0.074	0.032	-0.246	-0.279
	(0.056)	(0.065)	(0.115)	(0.102)
Constant	0.286	-0.487	1.573	2.560
	(0.475)	(0.537)	(1.028)	(0.952)
R-squared	0.0468	0.0704	0.1749	0.2753
Observations	332	227	105	110

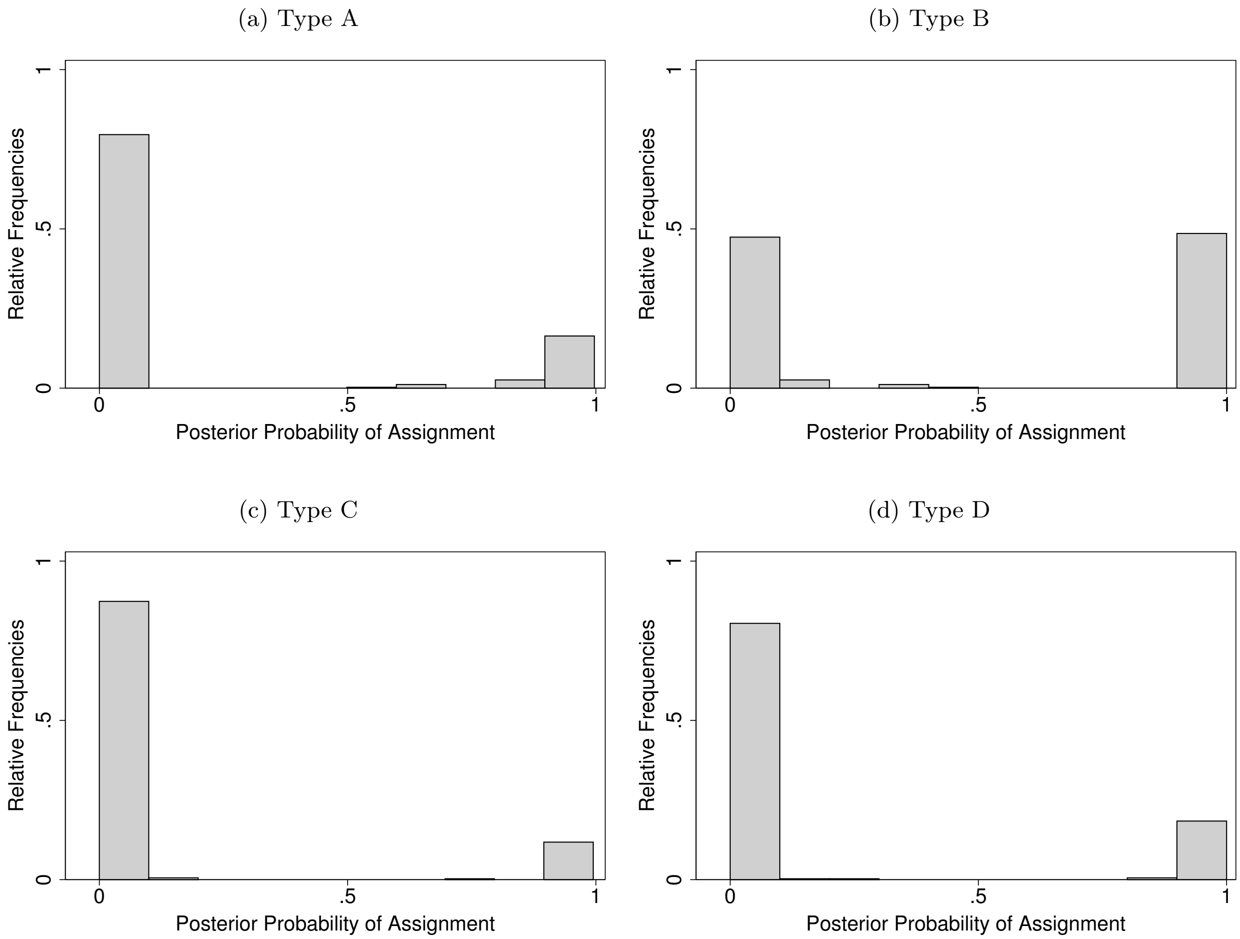
Notes: This table presents estimated coefficients from OLS regressions of classification (0 or 1) on explanatory variables of interest. The dependent variable in Columns (1), (2) and (3) is equal to one if the respondent was classified as reporting a statutory MTR (Type A or Type C). To construct these indicators, estimates from the baseline model with contamination are used to predict class membership. Respondents are classified according to the most probable class membership. Column (1) uses the full sample after removing respondents classified as having contaminated data. Columns (2) restricts the sample to respondents who reported MTR = ATR in both waves, and column (3) is respondents who reported MTR≠ATR in at least one wave. Column (4) restricts the sample to respondents who are classified as the StatMTR type and regress an indicator for whether MTR= ATR in both waves, or not. Cognitive ability (number series score) is interacted with an indicator for using a tax preparer to file one's income tax return. The coefficients should be interpreted as the effect of one unit change in the explanatory variable on the percentage chance of having the indicator equal to one. Asymptotically robust standard errors, clustered at the household level, are reported in parentheses below each parameter estimate.

Table 9: Maximum Likelihood Estimates of mixture model with covariates

	Mixing proportions		Statutory		Not Statutory	
Parameters	MTR≠ATR	MTR=ATR	ATR error	TI Error	ATR error	MTR error
Used paid tax preparer	-0.332	-0.627	4.144	0.110	2.417	2.203
	(0.333)	(0.435)	(1.397)	(0.227)	(0.996)	(1.245)
Number Series score	0.315	-0.172	0.628	-0.063	-1.310	-0.703
	(0.201)	(0.314)	(0.992)	(0.158)	(0.651)	(0.800)
Financial sophist score	0.244	0.174	-1.252	0.022	-0.309	1.733
	(0.184)	(0.251)	(0.803)	(0.129)	(0.599)	(0.742)
log(wealth)			-1.602	-0.039	-0.153	-0.761
			(0.711)	(0.114)	(0.437)	(0.628)
Age/10 (in 2011)	-0.007	0.042	-0.182	-0.007	-0.061	0.131
	(0.022)	(0.024)	(0.069)	(0.011)	(0.062)	(0.087)
Married (=1 if yes in 2011)	0.160	-0.042	0.876	1.037	0.729	-0.420
	(0.405)	(0.457)	(1.642)	(0.274)	(1.128)	(1.474)
Education : 13-16 years			-2.926	0.643	1.334	-0.690
			(3.088)	(0.391)	(1.345)	(1.616)
Education: >16 years			-5.670	1.108	0.733	-3.991
			(3.497)	(0.449)	(1.552)	(1.915)
Constant	-0.785	-2.976	37.218	-0.686	10.059	2.420
	(1.533)	(1.700)	(9.479)	(1.540)	(6.999)	(9.678)
Standard deviation			6.336	1.030	9.312	11.305
			(0.572)	(0.100)	(0.335)	(0.461)
Correlation across waves			-0.050	0.356	0.193	-0.030
			(0.126)	(0.126)	(0.053)	(0.057)
Correlation within wave			0.516		0.560	
			(0.121)		(0.040)	
Fraction contaminated	0.025					
	(0.010)					
Contamination scaling	2.785					
	(0.384)					
Log likelihood	-5275.6548					
Parameters						
Observations	344					

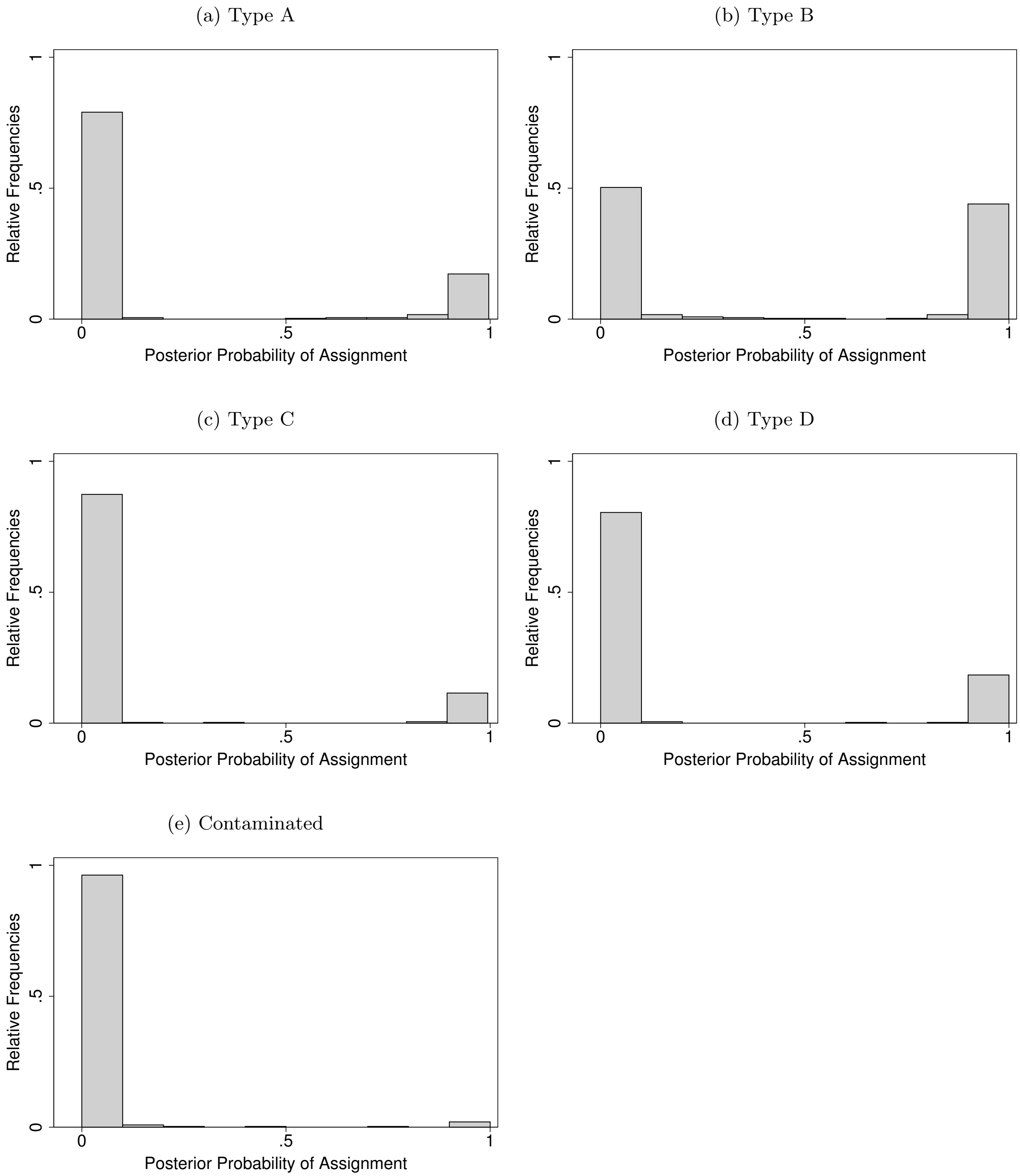
Notes: This table presents maximum likelihood estimates of the baseline mixture model (with contamination) when tax rate errors can vary systematically with observed covariates. Statutory refers to the distribution associated with reporting statutory marginal tax rates; Not Statutory refers to the distribution associated with respondents who do not report statutory MTR. The number series, verbal analogies and financial sophistication scores are all standardized. Number series and verbal analogies are measures of fluid intelligence and come from the face-to-face CogUSA cognitive assessments. Financial sophistication score is the average of the CogEcon financial sophistication scores from CogEcon 2008 and CogEcon 2009 (when both were available). Male, Married, and Used paid tax preparer are all dummy variables. The coefficients should be interpreted in terms of a one standard deviation change in the scores.

Figure 3: Distribution of posterior probability of assignment to groups (no contaminationl)



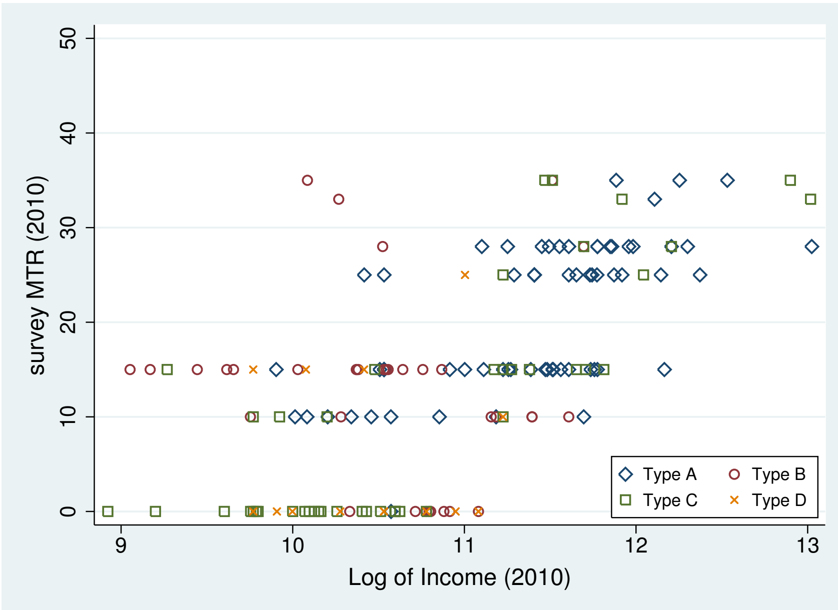
Notes: Type A respondents are those who distinguish between ATR and MTR and report statutory marginal tax rates. Type B respondents distinguish between ATR and MTR and do not report statutory marginal tax rates. Type C respondents do not distinguish between ATR and MTR and report statutory marginal tax rates for both. Type D respondents do not distinguish between ATR and MTR and report average tax rates for both.

Figure 4: Distribution of posterior probability of assignment to groups (with contamination)



Notes: These charts are the same as in the figure on the previous page for Types A, B, C and D. Contaminated tax rates refers to reporting extreme tax rates that come from the fat tail of the distribution and are hence considered contaminated (because these rates are not possible).

Figure 5: Scatterplot showing classification of types by reported income and MTR (among those reporting statutory rates in both waves)



Notes: This figure is a scatterplot of observed log income and marginal tax rate in 2010. Observations are distinguished by their predicted type, from using the mixture model without contamination. Type A respondents are those who distinguish between ATR and MTR and report statutory marginal tax rates. Type B respondents distinguish between ATR and MTR and do not report statutory marginal tax rates. Type C respondents do not distinguish between ATR and MTR and report statutory marginal tax rates for both. Type D respondents do not distinguish between ATR and MTR and report average tax rates for both.

References

[1] Biernacki, C. and G. Govaert. 1997. "Using the classification likelihood to choose the number of clusters." Computing Science and Statistics 29: 451-457.

[2] Biernacki, C., G. Celeux and G. Govaert. 1998. "Assessing a mixture model for clustering with the integrated classification likelihood. Technical Report No. 3521. Rhone-Alpes:ANRIA. (NOTE: citation found in McLachan and Peel, 2000)

[3] Buchanan, James. 1967. Public Finance in Democratic Process: Fiscal Institutions and Individual Choice.

[4] Bruhin, A., Fehr-Duda, H. and Epper, T. (2010), Risk and Rationality: Uncovering Heterogeneity in Probability Distortion. Econometrica, 78: 1375–1412. doi: 10.3982/ECTA7139

[5] Cameron, A. Colin and Pravin K. Trivedi, 2005. "Microeconometrics," Cambridge Books, Cambridge Univeristy Press, number 9781107014169.

[6] Celeux, G. and G. Soromenho. 1996. "An Entropy criterion for assessing the number of clusters in a mixture model." Classification Journal 13, 195-212.

[7] de Bartelome, Charles A.M. 1995. "Which tax rate do people use: Average or marginal?" Journal of Public Economics, 56(1): 79-96.

[8] Deb, P. and P.K. Trivedi 1997. "Demand for Medical Care by the Elderly: a Finite Mixture Approach," Journal of Applied Econometrics 12, 313-336.

[9] Deb, P. and P.K. Trivedi. 2002. "The Structure of Demand for Health Care: Latent Class versus Two-part Models," Journal of Health Economics, 21, 601-625.

[10] Gideon, Michael S. 2014. Survey Measurement of Income Tax Rates. Mimeo.

[11] Gideon, Michael, Seth Koch and Brooke Helppie McFall. 2013. "Cognitive Economics Study: CogEcon 2011 Documentation." December 2013. Available at http://cogecon.isr.umich.edu/survey/docs/datadoc2011.pdf

[12] Ito, Koichiro. 2014. "Do Consumers Respond to Marginal or Average Price? Evidence from Nonlinear Electricity Pricing." American Economic Review, 104(2): 537-63.

[13] Helppie, Brooke, Kandice A. Kapinos and Robert J. Willis. "Occupational Learning, Financial Knowledge, and the Accumulation of Retirement Wealth." Michigan Retirement Research Center Research Paper No. 2010-23.

[14] Kapteyn, Arie and Jelmer Y. Ypma. 2007. "Measurement Error and Misclassification: A Comparison of Survey and Administrative Data," Journal of Labor Economics, Vol. 25, No. 3 (July 2007), pp. 513-551.

[15] Liebman and Zeckhauser. 2004. Schmeduling. Unpublished mimeo.

[16] Lindsay, B. J. 1995. "Mixture Models: Theory, Geometry, and Applications" NSF-CBMS Regional Conference Series in Probability and Statistics, Vol. 5, IMS-ASA

[17] McLachlan, Geoffrey and David Peel. 2000. Finite Mixture Models. Wiley Series in Probability and Statistics.

A Survey Instruments

A.1 Tax Rates in 2011

Instructions for the following questions:

These questions focus on current and future federal income tax rates, both in general and for you personally.

*The **marginal tax rate** is the tax rate on the last dollars earned. For example, if a household's income tax bracket has a marginal tax rate of 15%, then a household owes an extra $15 of taxes when it earns an extra $100.*

Answer each question with a percentage between 0 and 100. Please provide your best estimate of the marginal tax rate even if you are not sure.

*These questions are about **federal income taxes only**; please do not include state or local taxes, or payroll taxes for Social Security and Medicare.*

Q1. *For a household in the **highest tax bracket** in **2010**:*

The marginal tax rate on *wage and salary income* was _____% and the marginal tax rate on *dividend income* was _____%.

Q2. *Tax rates may change in the future. I think that for a household in the **highest tax bracket** in **2014**:*

The marginal tax rate on *wage and salary income* will be _____% and the marginal tax rate on *dividend income* will be _____%.

We now want to ask you about your household's federal taxes. Please use the same definitions of federal income tax and marginal tax rate as on the previous page.

Q3. *Please think about your household's income in 2010 and the amount of federal income tax you paid, if any.*

Approximately what percentage of your household income did you pay in federal income taxes in 2010? _____%

Q4. *Now we want to ask about **your households' marginal income tax rate**. Please think about your household's federal income tax bracket and the tax rate on your last dollars of earnings.*
In 2010, my household's marginal tax rate was _____%.

Q5. *Suppose that in **2014** your household receives the same income you had in 2010. However, the federal income tax schedule might change.*
I expect that my household's marginal tax rate **in 2014** would be _____%.

A.2 Tax Rates in 2013

Instructions for the following questions:
*These next two questions focus on your federal income tax rates. These questions are about **federal income taxes only**; please do not include state or local taxes, or payroll taxes for Social Security and Medicare.*
Answer each question with a percentage between 0 and 100. Please provide your best estimate of the marginal tax rate even if you are not sure.

Q1. *Please think about your household's income in 2012 and the amount of federal income tax you paid, if any.*
Approximately what percentage of your household income did you pay in federal income taxes in 2012? _____%

Q2. *Now we want to ask about **your households' marginal income tax rate**. The marginal tax rate is the tax rate on the last dollars earned. Please think about your household's federal income tax bracket and the tax rate on your **last** dollars of earnings.*
In 2012, my household's marginal tax rate was _____%.

B Data construction and measurement

B.1 Constructing baseline taxable income measure

The objective is to measure taxable income for each taxable unit in 2010 and 2012 using self-reported information about household income from CogEcon 2011 and CogEcon 2013, respectively.

B.1.1 Measuring gross income: Total and components

CogEcon measures household-level gross income for each respondent as follows. First, respondents are asked for the total combined income of all members of their family (living in same household) during the past 12 months (Question C2). This includes wages or salary, net income from business, farm or rent, pensions, dividends, interest, Social Security payments, and any other money or income. Then respondents are asked about four particular sources of income: (i.) wages and salary, (ii.) employer-provided pensions, (iii.) Social Security and (iv.) distributions from retirement accounts. The questions were identical across the two waves of the study, although some question numbers and the reference year changed. Respondents who do not provide an exact value are then asked to select a range of values from the list of ranges presented in the question.

For income categories (i.), (ii.) and (iii.), there were separate questions about the respondent's own income and other questions in which the respondent is supposed to answer on behalf of his or her partner. Distributions from retirement accounts are only asked about in terms of the household's distributions overall, not distinguishing between distributions from accounts held by one partner or the other.

The survey asked respondents whether they currently received Social Security or Railroad Retirement benefit payments. If yes, they were asked to provide the age at which benefits were first received and the amount received each month. The same set of questions was asked about the respondents' spouse or partner, when applicable.

Respondents were then asked whether they currently received payments from an employer- or union-provided (defined benefit) pension plan. If yes, they were asked to provide the amount received each month. Again, the same set of questions was asked about the respondents' spouse or partner, when applicable.

Next, they were asked about total earnings before taxes from all jobs during the previous calendar year. Question C33 in CogEcon 2011 asked for the respondent's total earnings before taxes from all jobs during 2010; question C28 in CogEcon 2013 asked the same thing but about earnings during

2012. Again, the same set of questions was asked about the respondents' spouse or partner, when applicable. Question C42 in CogEcon 2011 asked for the spouse's/partner's total earnings before taxes from all jobs during 2010; question C36 in CogEcon 2013 asked the same thing but about earnings during 2012.

Later in the survey, respondents are asked whether they (or their spouse/partner) withdrew money or received payments from tax-advantaged retirement accounts in the previous calendar year. This includes 401(k) plans, 403(b) plans, Keoghs, traditional IRAs and Roth IRAs. If yes, they are asked for the amount withdrawn, before taxes and other deductions.

I construct two measures of household-level gross income. The first equals the self-reported sum of all income:

$$Inc_{total} = C2_{val}$$

The second measure equals the sum of the component parts of income (that is, based on the questions discussed above):

$$Inc_{sum} = wage + pension + SocSec + retdist$$

There are a few issues of measurement that must be accounted for when comparing the income components and the self-reported total household income.

First, I made the following edits to the income data. For reported total income (Inc_{total}), question C2, I make the following edits. If the yearly income was reported as less than \$200, I assume it was written in thousands (e.g., 75 rather than 75,000) and I multiply the reported income by 1000. It was apparent that these respondents wrote numbers in thousands, which could have been unintentional shorthand for their full answer. Looking at individual cases—comparing reported income across waves and reported wealth amounts across waves—corroborates this interpretation of the data.

Next, if they clearly gave their monthly income (e.g., they reported the number that they gave as their monthly Social Security retirement benefit) then I multiply the amount by 12 to convert it into yearly. If the reported monthly Social Security benefit is greater than \$5,000 per month, I assume this was meant as the yearly amount (since it is more than any household could receive in a month) and divide it by twelve. If the monthly pension is reported as greater than \$20,000 per month, I assume this was meant as the yearly amount and divide it by twelve. Finally, I divide the reported amount by 100 for a particular respondent who provided other information that led to the conclusion that what the respondent reported as \$10000000 was intended as \$100000.[9]

If the reported monthly Social Security benefit is less than \$10 per month and includes decimal places, then I assume the value was keyed in with a period rather than a comma and multiply it by 1000. If the reported monthly Social Security benefit is less than \$50 per month, I assume this was meant as the yearly amount and report monthly value after multiplying by 1000 and divide it by 12. If the yearly wage/salary value is reported as less than \$200, I assume this was written in thousands and multiple by one thousand.

Second, Social Security retirement benefits and pensions are reported as a monthly value but I do not know how many months the respondent (or spouse) received these sources of income. I impute Social Security and pension income during 2010 and 2012 using the reported monthly value and an imputed number of months they were received.

I calculate the Social Security retirement benefits received during the specified tax year (either 2010 or 2012) in the following steps. I impute the number of months using self-reported age when benefits were first received, along with information about birth month and year (collected by CogUSA in prior waves). The birth month is missing for a handful of respondents. In such cases I assume it was in July. I assume benefits were claimed immediately when the respondent turned the specified age and then received beginning in the subsequent month. For example, if someone claimed benefits at age 62 and was 63 in January 2010, assume they received benefits for all twelve months of 2010. If they turned 62 in May 2010, then assume they received benefits for seven months, June thru December. If the respondent received benefits at all during 2010 then I assume they received benefits for all 12 months of 2012. I then multiply the number of months times the reported monthly amount to get the imputed social security benefits received during the specified year. I follow the same procedure for the spouse's Social Security benefits. If the respondent claims to be married or partnered (with a financial future) but the person is not in the sample, I assume that the spouse is the same age as the respondent. The respondent and spouse's Social Security retirement benefits are added together to get the value of all household Social Security retirement benefits.

The benefits received during the specified tax year (either 2010 or 2012) were calculated in the following steps. If the respondent worked during 2010 (2012) but was retired by the time of the

[9] One respondent (sampid=7017080020) reported \$100,000 in 2011 and then \$10,000,000 in 2013. This R is over 80 years old and generally provided reasonable numbers; e.g., their SS benefits were \$855 a month in one year and \$850 a month in the other. Aggregated parts of income summed to \$34,400 in 2011 and \$53,700 in 2013, suggesting that the "other income" would, if anything, be larger in 2011 than in 2013, which would mean the total income reported in 2011 should be larger than in 2013. Hence, I divided the reported income by \$100.

CogEcon 2011 (CogEcon 2013) survey, I impute the number of months that the pension was received during 2010 (2012). I use self-reported retirement age, current employment status, and number of weeks working in 2010 and 2012 to impute the number of months the respondent received pension payments in 2010 and 2012. I assume the respondent received the pension for non-work weeks. I assign zero months for respondents who worked all of 2010 (or 2012) but then retired by the time they completed the survey during 2011 (or 2013). I assume they received for 12 months if they were over 65 years old in January 2010 (or January 2012) with the exception of one respondent below 65 but is clearly receiving pension for the whole year.[10] The imputed number of months is multiplied times the reported monthly amount to get the imputed pension received during the specified year. I follow the same steps to calculate the spouse's pension amount. The respondent and spouse's pensions are added together to get the value of all household pensions.

My preferred measure of gross income combines information from these two preceding measures. When computing tax rates for respondents we want to use the components of income in order to account for the fact that not all income sources are taxed in the same way. However, the income components are not exhaustive and therefore we would underestimate total income if we took these measures at face value.

$$GrossIncome = Inc_{sum} + \widehat{OthInc}$$

where

$$\widehat{OthInc} = Inc_{tot} - Inc_{sum}$$

I assume $\widehat{OthInc} = 0$ if $Inc_{tot} < Inc_{sum}$ or if the respondent has less than fifty thousand dollars in financial assets outdside of tax-advantaged retirement accounts.

Respondents who are partnered and planning a financial future together were asked to give information about their own and their partner's income. However, for tax purposes I want only their own income and not that of their partner. In such cases I use only the respondents amounts for (i) thru (iii), half of the retirement distributions (iv) and half of this residual.

Finally, I use total income when the components are missing.

[10]The respondent with sampid=7003600010 was 60 years old and claimed to be retired at 55 but was still working 52 weeks in 2010. However, as he also reported collecting a pension in an earlier wave of the survey, I determine that he was likely to have been receiving said pension for the entire year.

B.1.2 Measuring Adjusted Gross Income (AGI)

I use NBER's Taxsim tax calculator to determine adjusted gross income for each respondent.[11] This program allows me to distill taxable Social Security benefits from the total amount. The survey did not collect information about above-the-line deductions, so the difference between gross income and AGI will be the same as going from gross income to total income.

$$AGI = \text{Total Income} - \text{Above Line Deductions}$$

Total Income is gross income minus tax exempt income. This includes tax exempt interest, qualified dividends, the part of Social Security benefits, pensions and annuities that are not taxable.

Above-the-line deductions include IRA contributions, student loan interest, self-employed health insurance contributions, etc. For traditional plans these withdrawals are subject to taxation and therefore should be included in our measure of taxable income. I do not know whether these withdrawals were from traditional or Roth-designated accounts. I assume that distributions come from traditional accounts. The rules concerning distributions from traditional versus Roth designated accounts would make it more likely that distributions are from traditional accounts. This is corroborated by the fact that over half of respondents claimed they took out the required minimum distribution.

Tax treatment of Social Security retirement benefits:

- Depends on total income and marital status
- Generally, if Social Security benefits were only income, benefits are not taxable.
- If the household received income from other sources, benefits will not be taxed unless modified adjusted gross income is more than the base amount for the household's filing status.
- Quick computation:
  - Add one-half of the total Social Security benefits the household received to all other income, including any tax exempt interest and other exclusions from income.

$$\text{Combined Income} = AGI + \text{Nontaxable Interest } + \text{Half of SS benefit}$$

[11] The citation for the NBER Taxsim calculator is Feenberg and Coutts (1993).

– Then, compare this total to the base amount for the household's filing status. If the total is more than the household's base amount, some of the benefits may be taxable.

– If Combined Income < \$32,000 for married filing jointly, none of benefits taxable

– If Combined Income < \$25,000 for single, none of benefits taxable

B.1.3 Measuring Taxable Income (TI)

Taxable Income (TI) is the amount of income that is actually subject to federal income taxation. It is adjusted gross income minus all deductions and exemptions:

$$\text{Taxable Income} = AGI - Deductions - Exemptions$$

The amount of exemptions comes from marital status and the number of dependents that reported in the CogEcon survey. In tax year 2010 the personal exemption was \$3,650 for each qualified dependent. For tax year 2012 this amount increased to \$3,800 per dependent exemption.

Deductions: standard deduction for all I do not know whether respondents itemized deductions and for simplicity assume everyone claims the standard deduction. The standard deduction for households who filed as single was \$5,700 in 2010 and \$5,950 in 2012. The standard deduction for taxpayers who were married filing jointly was \$11,400 in 2010 and \$11,900 in 2012. Taxpayers who turned 65 on or before January 2nd of the subsequent year (either 2011 or 2013) were eligible for an additional deduction. The standard deduction increases by \$1,400 in 2010 and \$1,450 in 2012 for single filers, and increases by \$1,100 in 2010 and \$1,150 in 2012 for each elderly member for those who are married filing jointly. I use the information on birth date for the respondent and his or her spouse (described above) to determine the number of members over age 65.

B.1.4 Limitations

Tax liability also gets adjusted by tax credits, and refundable credits can generate a net transfer to the household, as is typical with the Earned Income Tax Credit (EITC). Tax credits are largely ignored in my analyses. First, we do not have information about which credits people filed for. Second, the sample consists of older and higher income taxpayers, many of whom no longer have children living with them, so will not be eligible for many of the largest tax credits (e.g., the EITC).

It is important to recall that the individual components and reported total household income are reported over different time horizons. There is no obvious way to improve upon this ad hoc approach.

Another drawback of using total income 1 is that it is given for the "past 12 months," which do not coincide with the tax years that I focus on. CogEcon 2011 was fielded at the end of 2011, so the total income is for 2011 rather than 2010.

Because surveys were fielded in late 2011 (and 2013), C2 provides 2011 (and 2013) income rather than 2010 (and 2012) income. It is important to note that this question asks about income in the past 12 months but for the purpose of the tax rate measurement we want income in the preceding calendar year.

B.2 Description of variables

- Used paid tax preparer (in 2011): Indicator variable equal to one if the respondent used a paid tax preparer the last time she filed a tax return. Assigned yes if any of the following were selected as answers to question C28: Commercial tax preparation company (like H&R Block); Financial planner or advisor; Accountant; Lawyer. The exact wording and answer options are the following:

  The last time you or your spouse filed a tax return, did you receive assistance or use tax software? If yes, please check all that apply. If no, please check "Did not receive assistance or use software." If you are not married, please answer only for yourself. [Family member, friend or colleague; Tax software (like Turbotax); Commercial tax preparation company (like H&R Block); Financial planner or advisor; Accountant; Lawyer; Did not receive assistance or use software]

- Cognitive ability is capture by two variables: (i.) standardized number series score, (ii.) standardized verbal analogies score. These are both measures of fluid intelligence and come from the face-to-face CogUSA cognitive assessments.

- Financial sophistication is the average of the CogEcon financial sophistication scores computed from CogEcon 2008 and CogEcon 2009.[12] The score is standardized after taking the average score from CogEcon 2008 and CogEcon 2009 (when both were available).

- State income tax: Indicator variable for living in a state that has a state income tax

[12] See CogEcon documentation for details about their construction.

- Financial occupation dummy variables: For the occupation deals with finance/investment, defined as working in occupations that do rates of return, cost-benefit analysis and investment decisions. Budgeting occupstions are defined as occupations with resource management or budgeting content. See McFall et al (2011) for details about these occupational measures.

- Log(wealth): This is the log of total household wealth. Total household wealth is the average from wealth measured in CogEcon 2011 and CogEcon 2013. It includes financial, housing and miscellaneous assets and debts. Of the sample of 348, there are 11 respondents for whom we have no information about their wealth. It is implausible that they had zero wealth in both CogEcon 2011 and CogEcon 2013. Because zeros are problematic when using logs, I assign the mean of log wealth (12.57) to these 11 respondents with zero wealth in both CogEcon 2011 and CogEcon 2013.

- Male: Indicator variable equal to one for men.

- Nonwhite: dummy variable equal to one if race is non-white or histpanic, based on the CogUSA race variable.

- Education is a categorical variable: less than 12 years, 12-16 years and over 16 years; the category for less than 12 years is excluded.

- Age/10: Age on date the survey was completed. See CogEcon documentation for more information.

- Working: indicator for being employed at all during 2010 (or 2012).

C Derivations

C.1 Baseline one-component measurement model

Each individual i has a survey measure of log income $y_{i,w}$, marginal tax rate $m_{i,w}$, and average tax rate $a_{i,w}$ across two waves of data ($w = 1, 2$). Wave 1 refers to data collected in CogEcon 2011, with reference to tax year 2010. Wave 2 refers to data collected in CogEcon 2013, with reference to tax year 2012. These variables are potentially noisy measures of the true values of log income $y^\star_{i,w}$, marginal tax rate $m^\star_{i,w}$ and average tax rate $a^\star_{i,w}$, which are not observed. The relationship between the true and observed variables is specified according to the three-equation measurement model developed in Gideon (2014):

$$y_{i,w} = y^\star_{i,w} + e_{i,w}$$

$$m_{i,w} = m^\star_{i,w} + \varepsilon^m_{i,w}$$

$$a_{i,w} = a^\star_{i,w} + \varepsilon^a_{i,w}$$

where $e_{i,w}$ is assumed to be mean-zero random noise, whereas tax rate errors $\varepsilon^m_{i,w}$ and $\varepsilon^a_{i,w}$ could be biased. Conditioning on true marginal and average tax rates, variation in the survey measures reflect systematic heterogeneity and random survey noise.

The distribution of observed income is the same as before.

$$\begin{bmatrix} y_{i,1} \\ y_{i,2} \end{bmatrix} \sim N \left(\begin{matrix} \mu_{y^\star} \\ \mu_{y^\star} \end{matrix}, \begin{bmatrix} \sigma^2_{y^\star} + \sigma^2_e & \rho_{y^\star} \cdot \sigma^2_{y^\star} \\ \rho_{y^\star} \cdot \sigma^2_{y^\star} & \sigma^2_{y^\star} + \sigma^2_e \end{bmatrix} \right)$$

where $\mu_{y^\star}$ is the mean of true log income, $\sigma_{y^\star}$ is the standard deviation of true log income, σ_e is the standard deviation of income measurement error and $\rho_{y^\star}$ is the correlation of true income across waves. For the observed tax rates, define$\mathbf{r_i} = (\mathbf{r}_{i,1}, \mathbf{r}_{i,2})'$ with $\mathbf{r}_{i,w} = (a_{i,w}, m_{i,w})'$ responses in waves $w = 1, 2$. The distribution of observed tax rates, conditional on true income (and, hence, true tax rates), is again jointly normal

$$\mathbf{r_i} \mid \mathbf{y}^\star_i \sim N\left(b_{\mathbf{r}}, \Sigma_{\mathbf{r}}\right)$$

with $b_{\mathbf{r}} = (b_a, b_m, b_a, b_m)'$ and

$$\Sigma_{\mathbf{r}} = \begin{pmatrix} \sigma_a^2 & \rho_{am}\sigma_a\sigma_m & \rho_a\sigma_a^2 & 0 \\ \rho_{am}\sigma_a\sigma_m & \sigma_m^2 & 0 & \rho_m\sigma_m^2 \\ \rho_a\sigma_a^2 & 0 & \sigma_a^2 & \rho_{am}\sigma_a\sigma_m \\ 0 & \rho_m\sigma_m^2 & \rho_{am}\sigma_a\sigma_m & \sigma_m^2 \end{pmatrix}$$

For this exercise I assume the distributional parameters on true income and the tax rate errors are the same across waves.

I model systematic heterogeneity by specifying the mean log income and mean tax rate errors as linear indices. This is defined as $b_x^m = \mathbf{x}_i \cdot \beta^m$ for marginal tax rates, $b_x^a = \mathbf{x}_i \cdot \beta^a$ for average tax rates, and $\mu_{y^\star} = \mathbf{x}_i \cdot \beta^y$ for true (latent) income. Estimates of parameters in β^m and β^a tell us how tax rate perceptions vary, on average, with other observable characteristics.

The likelihood function is derived the same way as in Gideon (2014), but is now conditional on covariates $\mathbf{x}_i$. Let $\mathbf{\Omega}$ denote the set of observed data, which is a vector $\Omega_i = (\mathbf{y}_i, \mathbf{r}_i, \mathbf{x}_i)'$ of reported income and tax rates for individuals $i = 1, \ldots, N$. The likelihood $L(\theta \mid \mathbf{\Omega})$ that the distributional parameters for the specified model are θ given data $\mathbf{\Omega}$ is proportional to the probability $\Pr(\Omega_i \mid \theta)$ of observing $\mathbf{\Omega}$ given the specified model and parameters θ:

$$\begin{aligned} L(\theta \mid \mathbf{\Omega}) &\propto \Pr(\mathbf{\Omega} \mid \theta) \\ &= \prod_{i=1}^{N} \Pr(\mathbf{r}_i \mid \mathbf{y}_i, \mathbf{x}_i, \theta) \cdot \Pr(\mathbf{y}_i \mid \mathbf{x}_i, \theta) \end{aligned}$$

The term $\Pr(\mathbf{y}_i \mid \mathbf{x}_i, \theta)$ is simply the density $f(\mathbf{y}_i \mid \mathbf{x}_i, \theta)$. To calculate $\Pr(\mathbf{r}_i \mid \mathbf{y}_i, \mathbf{x}_i)$, I need to integrate over the two-dimensional income measurement error distribution because latent true income is not observed in the data. Each realization of income measurement error vector $\mathbf{e}$ (conditional on observed income $\mathbf{y}_i$) pins down true income $\mathbf{y}_i^\star$, which determines true average tax rates $\mathbf{a}_i^\star$ and marginal tax rates $\mathbf{m}_i^\star$.

The major complication in evaluating the likelihood function arises from the fact that true income is not observed. Estimating the parameters of the model by maximum likelihood involves integrating over the distribution of these unobserved income errors. This problem is solved by simulating the likelihood function. The simulated log-likelihood function is then given by

$$\mathrm{SLL}(\theta) = \sum_{i=1}^{N} \ln \check{L}_i(\theta)$$

The contribution of each individual i is $\check{L}_i(\theta)$, which is a simulated approximation to $L_i(\theta)$, derived as

$$\check{L}_i(\theta) = \frac{1}{K}\sum_{k=1}^{K} L_i^k(\theta)$$

where the average is over the likelihood evaluated at each simulation draw

$$L_i^k(\theta) = \Pr\left(\mathbf{r}_i \mid \mathbf{y}_i, e_{i(k)}\right) \cdot f(\mathbf{y}_i)$$

and K is the number of pseudorandom draws of the vector of errors $e_{i(k)}$. The algorithm involves simulating a distribution of income errors for each respondent. The individual's likelihood contribution is computed for each set of income errors, and density of the implied tax errors are averaged over the K values to obtain the simulated likelihood contribution. A detailed description of the simulation algorithm and likelihood evaluation are in Gideon (2014).

C.2 Likelihood function in baseline mixture model

In the mixture model there are two distinct data generating processes. The first process is the same as in the one-component model. The second component is also bivariate normally distributed but is derived differently. The probability of reporting a statutory tax rate τ_j is the probability that subjective taxable income $I_{i,w}^S$ in wave w is between thresholds $\overline{I_w^{j-1}}$ and $\overline{I_w}^j$.

C.2.1 Multiplicative error on taxable income

Respondents know the income thresholds associated with the tax rates but have subjective taxable income which is an error-ridden measure of true taxable income, $I_{i,w}^S = I_{i,w}^\star \cdot \exp\left(\varepsilon_{i,w}^I\right)$.

$$\begin{aligned} \underline{I}^{j,w} \leq \quad & I_{i,w}^S && < \overline{I}^{j,w} \\ \log\left(\underline{I}^{j,w}\right) - \log\left(I_{i,w}^\star\right) \leq \quad & \varepsilon_{i,w}^I && < \log\left(\overline{I}^{j,w}\right) - \log\left(I_{i,w}^\star\right) \\ \mathrm{I}_{i,j,w}^L \leq \quad & \varepsilon_{i,w}^I && < \mathrm{I}_{i,j,w}^H \end{aligned}$$

where

$$\mathrm{I}_{i,j,w}^H = \log\left(\overline{I}^{j,w}\right) - \log\left(I_{i,w}^\star\right)$$

$$\mathrm{I}_{i,j,w}^L = \log\left(\underline{I}^{j,w}\right) - \log\left(I_{i,w}^\star\right)$$

Assuming that the errors have mean μ_{eI} and correlation ρ_{eI}, then

$$\begin{aligned}\Pr\left(m_1=\tau_j, m_2=\tau_k \mid y_1^\star, y_2^\star\right) &= \Pr\left(\varepsilon_{i,1}^I, \varepsilon_{i,2}^I \mid y_1^\star, y_2^\star\right)\\ &= \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)+\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\\ &- \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)-\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\end{aligned}$$

for all j and k that are not the bottom or top bracket. The first and fourth terms give the probability of being in category j given that income satisfies the condition on how large category k income is. Then the second and third terms subtract out the probability that we have category j but are less than the lower bound for being in category k.

The first and last tax brackets require a slight adjustment, such that

$$\begin{aligned}\varphi_i^{1,k} &= \Pr\left(m_1=0, m_2=\tau_k \mid y_1, y_2\right)\\ &= \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,1,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)-\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,1,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\end{aligned}$$

$$\begin{aligned}\varphi_i^{j,1} &= \Pr\left(m_1=\tau_j, m_2=0 \mid y_1, y_2\right)\\ &= \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,1,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)-\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,1,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\end{aligned}$$

$$\begin{aligned}\varphi_i^{7,k} &= \Pr\left(m_1=35, m_2=\tau_k \mid y_1, y_2\right)\\ &= \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,7,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)+\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,7,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\\ &- \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,7,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)-\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,7,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,k,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\end{aligned}$$

$$\begin{aligned}\varphi_i^{j,7} &= \Pr\left(m_1=\tau_j, m_2=35 \mid y_1, y_2\right)\\ &= \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,7,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)+\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,7,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\\ &- \overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^H-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,7,2}^L-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)-\overrightarrow{\Phi}\left(\frac{\mathrm{I}_{i,j,1}^L-\mu_{eI}}{\sigma_{eI}}, \frac{\mathrm{I}_{i,7,2}^H-\mu_{eI}}{\sigma_{eI}}, \rho_{eI}\right)\end{aligned}$$

C.3 Mixture with contamination

Some respondents report tax rates that are extremely large and reflect either reporting errors or complete ignorance about their taxes. One standard way to account of "contamination" of this sort is to assume observations come from a mixture of the contaminated and uncontaminated populations. The tax rates $\mathbf{r}_i$ are contaminated with probability γ and not contaminated with probability $1-\gamma$. This means contamination is at the individual level rather than at the level of each reported tax rate. Intuitively, this is meant to account for observations that do not provide any meaningful information about perceptions. The log likelihood of the finite mixture model is given by

$$\ln L\left(\Psi;\mathbf{r}_i\right)=\sum_{i=1}^{N}\ln\sum_{c=1}^{C}\pi_c g_c\left(\mathbf{r}_i\mid\mathbf{y}_i^{\star},\theta_j\right)$$

where $\Psi=\left(\mu_{y^\star},b_a,b_m,\sigma_{y^\star},\sigma_e,\sigma_a,\sigma_m,\rho_a,\rho_m,\rho_{am}\right)'$ is the vector of distributional parameters of the mixture model, with weights π_c and density functions g_c. The weights are

$$\begin{aligned}
\pi_1 &= (1-\gamma)\cdot(1-\eta_i)\cdot(1-\lambda)\\
\pi_2 &= (1-\gamma)\cdot(1-\eta_i)\cdot\lambda\\
\pi_3 &= (1-\gamma)\cdot\eta_i\cdot(1-\theta)\\
\pi_4 &= (1-\gamma)\cdot\eta_i\cdot\theta\\
\pi_5 &= \gamma
\end{aligned}$$

and distributions $g_1=f_{NS}\left(\mathbf{a}_i,\mathbf{m}_i\right)$, $g_2=f_S\left(\mathbf{a}_i,\mathbf{m}_i\right)$, $g_3=f_{NS}\left(\mathbf{a}_i\right)$, $g_4=f_S\left(\mathbf{m}_i\right)$ and $g_5=f_C\left(\mathbf{a}_i,\mathbf{m}_i\right)$. Conditional on not being labeled as having contaminated data, observations are restricted to being a member of groups 1 or 2 if $\mathbf{a}_i^S\neq\mathbf{m}_i^S$ and groups 3 or 4 if $\mathbf{a}_i^S=\mathbf{m}_i^S$.

Another way to represent this is with the likelihood function

$$\begin{aligned}
f\left(\Omega_i\mid\mathbf{y}_i^\star\right) &= (1-\gamma)\left[\lambda\cdot f_1\left(\mathbf{a}_i,\mathbf{m}_i\right)+(1-\lambda)\cdot f_2\left(\mathbf{a}_i,\mathbf{m}_i\right)\right]^{1\left(\mathbf{a}_\mathrm{i}^\mathrm{S}\neq\mathbf{m}_\mathrm{i}^\mathrm{S}\right)}\\
&+ (1-\gamma)\left[\theta\cdot f\left(\mathbf{a}_i\right)+(1-\theta)\cdot\left[\lambda\cdot f_1\left(\mathbf{m}_i\right)+(1-\lambda)\cdot f_2\left(\mathbf{m}_i\right)\right]\right]^{1\left(\mathbf{a}_\mathrm{i}^\mathrm{S}=\mathbf{m}_\mathrm{i}^\mathrm{S}\right)}\\
&+ \gamma\cdot f_C\left(\mathbf{a}_i\right)\cdot f_C\left(\mathbf{m}_i\right)
\end{aligned}$$

where $1\left(\bullet\right)$ is an indicator function which equals one when the expression $\bullet$ is satisfied and is zero otherwise.