Abstract

This paper derives the conditions under which the elasticity of capital with respect to the net of corporate tax rate is positive. In doing so, this model nests the traditional result originating from the works of Harberger [1962] and Jorgenson [1967] and the neutrality result obtained by Stiglitz [1973] and Sandmo [1974]. The key assumption is how the marginal dollar of investment is financed. If the marginal dollar is financed at a cost equal to debt financing the neutrality result obtains. If, instead, the marginal dollar is financed through equity, which is not tax deductible, then the traditional result obtains. We test this implication of the model using administrative tax records for the population of US corporations and the control group bunching method in Patel, Seegert, and Smith (2014).

Keywords : Effective Incidence; Corporate Tax.

1 Corporate Taxation And Taxable Income

This paper derives the conditions under which the elasticity of capital with respect to the net of corporate tax rate is positive. In doing so, this model nests the traditional result originating from the works of Harberger [1962] and Jorgenson [1967] that \( r = (1 - \tau_c)(f_K - \delta) \) and the neutrality result \( r = f_K - \delta \) obtained by Stiglitz [1973] and Sandmo [1974]. The key assumption is how the marginal dollar of investment is financed. If the marginal dollar is financed at a cost equal to debt financing the neutrality result obtains. If, instead, the marginal dollar is financed through equity, which is not tax deductible, then the traditional result obtains. The model below leaves the marginal financing decision general depending on the value of \( \alpha \) and demonstrates its affect on the elasticity of capital with respect to the net of corporate tax rate.

The model presented below also allows for firms to deduct their net operating losses from previous time periods. This formulation motivates the empirical design in the paper, in which firms with net operating losses act as if they have firm specific kink points. This feature of the tax code allows for our novel empirical design which is able to use firms with different levels of net operating losses as control groups in estimating counterfactual distributions.

Accounting profits are given by revenues net of depreciation and interest costs,

\[
\Pi = F(K) - \delta K - rB.\quad (1)
\]

Dividend payments can be written as accounting profits, plus net inflows from bonds and equity minus investment, net of corporate income taxes incorporating a tax schedule with (i) two separate tax rates conditional on the kink point \( \kappa \) and (ii) the ability to deduct net operating losses from previous periods \( \nu \),

\[
D = (1 - \tau_{c,1})[F(K) - \delta K - rB - \nu] + b + E - I - \tau_{c,2}[\Pi - \nu - \kappa]1(\Pi > \kappa) + (1 - \theta_d)[\tau_{c,1}\nu + \tau_{c,2}(\nu + \kappa)]1(\Pi > \kappa). \quad (2)
\]

After the dividends are distributed by the firm they are taxed upon individuals receiving them at the dividend tax rate, \( \theta \), where \( \phi_d \) is defined as \( 1 - \theta \),

\[
\phi_d D = (1 - \tau_{c,1})(1 - \theta)[F(K) - \delta K - rB] + (1 - \theta)[b + E - I] + \theta_d[\tau_{c,1}\nu + \tau_{c,2}(\nu + \kappa)]1(\Pi > \kappa). \quad (3)
\]

The effective kink point the firm faces is given by \( (\nu + \kappa) \) which means that firms have different kink points depending on their stock of net operating losses. The empirical strategy exploits this unique feature of the corporate tax code.

Firms maximize the market value of its shares, which is just the present value of the net
max \, B,E,I \int_0^\infty e^{-\frac{\phi_d}{\phi_g} \phi_d e^{rt}} \left[ \frac{\phi_d}{\phi_g} D_t - E_t \right] \, dt \quad (4)

by choosing inflows of debt, \( b \), new issues of shares, \( E \), and investment, \( I \), taking as given the time path of the interest rate, \( r \), and one minus the dividend tax and capital gains tax, \( \phi_d \) and \( \phi_g \) respectively. The capital stock and the level of debt are the state variables in this optimization with equations of motion given by,

\[
\dot{K} = I \quad \text{and} \quad \dot{B} = b
\]

and initial conditions \( K_0 > 0 \) and \( B_0 < P_K K_0 \). The optimization is constrained by three additional financial constraints. First, we constrain the value of new issues of shares to be positive, abstracting from share repurchases, \( E > 0 \). Second, we constraint dividend distributions to be positive, \( D > 0 \). Third, we restrict the proportion of investment that can be debt financed,

\[
b \leq \alpha I
\]

where \( \alpha \) constrains the amount of net investment that can be financed through new debt. The necessary conditions include the first-order conditions with respect to the control variables ( \( b, E, \) and \( I \)), the co-state equations,

\[
-\frac{\partial \mathcal{H}}{\partial K} = \lambda_K - \lambda_K \frac{\phi_d}{\phi_g} r,
\]

and the transversality conditions,

\[
\lim_{t \to \infty} \left[ \dot{K} + \lambda_K - \frac{\phi_d}{\phi_g} r \right] < 0
\]

where \( \lambda_K \) and \( \lambda_B \) are the shadow prices or co-state variables of the capital stock and stock of debt. The necessary conditions include the first-order conditions with respect to the control variables ( \( b, E, \) and \( I \)), the co-state equations,
\[ \lim_{t \to \infty} [\dot{B} + \hat{\lambda}_B - \frac{\phi_d}{\phi_g} r] < 0. \] (11)

The first order conditions with respect to debt and equity determine the firm’s preferences for financing investment with, equity, debt, or retained earnings. The marginal advantage of using new shares rather than retained earnings to finance investment is given by the first order condition with respect to equity issuances,

\[ \frac{\partial \mathcal{H}}{\partial E} = \frac{\phi_d}{\phi_g} - 1. \] (12)

If this value is positive issuing new shares will be preferred to using retained earnings and if negative retained earnings will be preferred. Similarly, the marginal advantage of using debt rather than retained earnings to finance investment is given by the first order condition with respect to new debt issuances,

\[ \frac{\partial \mathcal{H}}{\partial b} = \frac{\phi_d}{\phi_g} + \hat{\lambda}_B. \] (13)

To interpret this result economically the value for the shadow price of debt needs to be calculated using the differential,

\[ \hat{\lambda}_B = \frac{dV}{dB} = \int_{t}^{\infty} e^{rt - \frac{\phi_d}{\phi_g} r(s) ds} \left( -r(s) \frac{\phi_d \phi_c}{\phi_g} ds \right) \]

\[ = - \phi_c \] (14)

which exists as long as \( \lim_{t \to \infty} r(t) > \bar{r} \) for some strictly positive and arbitrarily small constant \( \bar{r} \). Using this value in the first order condition with respect to debt issuances equation (13) becomes,

\[ \frac{\partial \mathcal{H}}{\partial b} = \frac{\phi_d}{\phi_g} - \phi_c. \] (15)

The first order condition with respect to investment,

\[ \frac{\partial \mathcal{H}}{\partial I} = \lambda_K - \frac{\phi_d}{\phi_g} \] (16)

provides the partial change in value due to a marginal increase in investment in the firm. The benefit of additional capital within the firm is given by the shadow price of capital while the opportunity cost is given by additional dividends net of taxes.
The user cost of capital is calculated using the necessary conditions above and the general necessary condition,
\[
\frac{d\mathcal{H}}{dI} = \frac{\partial \mathcal{H}}{\partial I} + \frac{\partial \mathcal{H}}{\partial b} \frac{\partial b}{\partial I} + \frac{\partial \mathcal{H}}{\partial E} \frac{\partial E}{\partial I} = 0
\]  
(17)
for an optimum of the firm. The effect of taxation on investment and the equilibrium level of capital employed is then determined by deviations of the user cost of capital from a model without taxation; \(F'(K) = r + \delta\). The shadow price of capital can be derived using the first order conditions in equations (12), (15), and (16) and the debt constraint, assumed to bind, given in equation (6). With some rearranging the shadow price of capital can be represented as,
\[
\lambda_K = \phi_c \alpha + 1 - \alpha.
\]  
(18)

When additional investment is completely debt financed, \(\alpha = 1\), the shadow price of capital is equal to one minus the corporate income tax rate. In contrast, when additional investment is completely financed through equity, \(\alpha = 0\), the shadow price of capital equals one. In both extreme cases, and for all intermediate cases, the shadow price of capital is stable over time implying \(\dot{\lambda}_K = 0\). The user cost of capital is calculated using the co-state equation for capital, the value of the shadow price of capital in equation (18), and its implication that

\[
\begin{align*}
\frac{d\mathcal{H}}{dI} &= \frac{\partial \mathcal{H}}{\partial I} + \frac{\partial \mathcal{H}}{\partial b} \frac{\partial b}{\partial I} + \frac{\partial \mathcal{H}}{\partial E} \frac{\partial E}{\partial I} \\
&= \lambda - \phi_d \frac{\phi_d}{\phi_g} + \alpha (\phi_d - \phi_c) + (1 - \alpha) (\phi_d - 1) \\
\lambda_K &= \phi_d (1 - \alpha + \phi_c \alpha \frac{\phi_d}{\phi_g} - (1 - \alpha) + (1 - \alpha) \frac{\phi_d}{\phi_g}) \\
\lambda_K &= \phi_d (\phi_c \alpha + (1 - \alpha) \frac{\phi_d}{\phi_g}) \\
\lambda_K &= \phi_c \alpha + (1 - \alpha)
\end{align*}
\]
\[ \dot{\lambda}_K = 0. \] With some rearranging the user cost of capital can be expressed as,

\[ F'(K) = r \left[ \frac{\phi_c \alpha + 1 - \alpha}{\phi_c} \right] + \delta. \tag{19} \]

The user cost of capital in this model with taxation demonstrates the importance of the financial choices of the firm. For example, if investment is completely debt financed, \( \alpha = 1 \), then the user cost of capital is the same as in a model without taxation. This implies there is no distortion to the user cost of capital when investment is completely debt financed. In contrast, if investment is completely equity financed, \( \alpha = 0 \), then the user cost of capital is distorted by the corporate income tax and it becomes more distorted the larger the corporate tax rate.

This discussion demonstrates that this model nests both the traditional result originating from the works of Harberger [1962], Jorgenson [1967], and others that \( r = (1 - \tau_c)(f_K - \delta) \) and the neutrality result \( r = f_K - \delta \) obtained by Stiglitz [1973], Sandmo [1974], and others. The crucial difference is whether investment, on the margin, is financed at a cost equal to debt financing or not. When the marginal investment is financed with debt which is tax deductible \( \alpha = 1 \) and the corporate tax is investment neutral. In contrast, when investment is financed through equity which has a larger, and non-tax deductible, cost then the corporate tax does distort investment.

The elasticity of capital with respect to one minus the corporate income tax is derived by totally differentiating the user cost of capital condition in equation (19).

\[ F''(K) dK = \frac{-r(1 - \alpha)}{\phi_c^2} d\phi_c \]

\[ \frac{dK}{d\phi_c} = \frac{r(1 - \alpha)}{\phi_c^2(-F''(K))} \]  

\[ \varepsilon_{K,\phi_c} = \frac{r(1 - \alpha)}{\phi_c(-F''(K) K)} \]  

The elasticity of capital with respect to one minus the corporate income tax is decreasing

\[ \frac{\partial \mathcal{H}}{\partial K} = \dot{\lambda}_K - \lambda_K \frac{\phi_d}{\phi_q} r \]

\[ \frac{\phi_d \phi_d}{\phi_d} (F'(K) - \delta) = \frac{\phi_d}{\phi_q} r(\phi_c \alpha + 1 - \alpha) \]

\[ F'(K) = r \left[ \frac{\phi_c \alpha + 1 - \alpha}{\phi_c} \right] + \delta \]
with the proportion of investment that is debt financed.

The elasticity of taxable corporate income with respect to one minus the corporate income tax rate is derived by totally differentiating taxable corporate income and using the relationship derived in equation (20) and the constraint $B = \alpha K$.

$$\begin{align*}
\frac{dY}{dK} &= (F'(K) - \delta)dK - rdB \\
\frac{dY}{dK} &= r\frac{1 - \alpha}{\phi_c} \\
\frac{dY}{dK} \frac{dK}{d\phi_c} &= r^2 \frac{(1 - \alpha)^2}{\phi_c^2(-F''(K))} \\
\varepsilon_{Y,\phi_c} &= \frac{r^2 (1 - \alpha)^2}{\phi_c^3 Y(-F''(K))}
\end{align*}$$

(23)

The elasticity of taxable corporate income with respect to one minus the corporate income tax rate is decreasing in both the proportion of investment that is debt financed and one minus the corporate income tax rate. When investment is completely debt financed the elasticity is zero. There is a large literature on the Modigliani-Miller theorem which provides many reasons, despite the tax preferential to finance investment through debt, for the fact that investments empirically are not fully debt financed. These include bankruptcy costs, moral-hazard problems, and limited loss-offsets DeAngelo and Masulis [1980], for a survey of this literature see Modigliani [1982]. The model is flexible enough to capture these reasons by choosing the appropriate level of $\alpha$.

While the elasticity of taxable corporate income with respect to the net of corporate income tax rate need not be zero the elasticity with respect to the net of the dividend tax rate does. This is consistent with the work by King [1974] and Auerbach [1979] demonstrating...
the neutrality of the dividend tax and empirical work by Yagan [2013] that finds no effect of the dividend tax rate change in 2003 on investment decisions.

1.1 The Relationship Between Bunching and the Elasticity of Corporate Income

This section demonstrates the elasticity found in the previous section can be estimated using bunching in the tax schedule, similar to Saez [2010] with a few substantial extensions to the corporate elasticity of taxable income. Similar to Saez [2010] a small kink at $\kappa$ will induce bunching for firms whose taxable income in the absence of the kink would be in the segment $[\kappa, \kappa+\delta Y]$ as depicted in Figure 1. Here revenues and costs have been written as a function of before-tax income $Y$, where firms are heterogeneous in their revenue functions but not their cost functions. The cost function includes all non-tax deductible costs and taxes, causing a kink in the cost function at $\kappa$ where a higher tax rate applies.

In Figure 1 firm 1 optimally chooses a before-tax income below the kink and therefore is unaffected by the kink. Firm 2 optimally chooses a before-tax income exactly at the kink. Firm 3 chooses before-tax income $\delta Y$ in the absence of the kink and $\kappa$ after the kink. Firm 3 is the marginal firm that bunches at the kink because at the kink its marginal revenue is equal to the marginal cost of the post-kink cost function, $\tilde{c}(Y)$.

Therefore, all firms with revenue functions “between” firm 2 and firm 3 bunch at the kink. Formally, these firms are characterized by two conditions; (i) $R'(\kappa) > c'(\kappa)$ and (ii) $R'(\kappa + \epsilon) < \tilde{c}'(\kappa + \epsilon)$ assuming the revenue and cost functions are sufficiently smooth and the revenue function is concave.

Firm 4 optimally chooses a before-tax income above the kink with and without the kink, but is affected by the kink optimally selecting less before-tax income with the kink, depicted by $K_4$ and $\tilde{K}_4$. The empirical strategy exploits the fact that firms below the kink, firm 1 in this example, are unaffected by the kink while firms above the kink, firm 4 in this example, are affected by the kink.
The amount of bunching is equal to the number of firms with before-tax income in the absence of the kink in the segment \([\kappa, \kappa + dY]\) which is equal to the density multiplied by the change in before-tax income, \(h(Y)dY\). The amount of bunching can then be shown to be proportional to the elasticity \(e\) by rewriting the definition as

\[
\frac{dY}{Y} = e \frac{d(1 - \tau_c)}{(1 - \tau_c)}.
\]  

(24)

This relationship demonstrates bunching at a kink point increases with the elasticity. In the case where the elasticity is zero there will be no bunching at the kink, something that is easily refuted in the case of corporate taxable income. The amount of bunching also depends on the change in tax rates relative to its base. Thus the same amount of bunching will be induced from a tax rate change from 0 to 10 percent as from a tax rate change from 90 to 91 percent.

The previous discussion implicitly assumes a small change in tax rates at a kink point but the result that the elasticity can be estimated using bunching at a kink point can be extended to large changes. To do this consider a parametric example where each firm has a production function,

\[
F_i(K) = \frac{1 + e A_i K^{\tau_c}}{e},
\]  

(25)

where \(A_i\) is a firm specific production factor distributed with density \(h(A)\) in the population. Taxable income is simplified to equal production, where the output price is normalized to
one, minus the portion of financing costs that are tax deductible,

\[ Y = F(K) - \alpha r K, \]  

(26)

where given the discussion in the previous section \( \alpha \) can be thought of as the debt to equity ratio. Each firm chooses its capital stock as to maximize,

\[(1 - \tau_c)[F(K) - \alpha r K] - (1 - \alpha)rK, \]  

(27)

which leads to the first order condition: \((1 - \tau)[F'(K) - \alpha r] - (1 - \alpha)r = 0\), which can be written as

\[ K = \left[\frac{A(1 - \tau_c)}{(1 - \alpha)r + \alpha r(1 - \tau_c)}\right]^{1+e}. \]  

(28)

The elasticity of taxable income with respect to the net of tax rate \( 1 - \tau_c \) can be calculated using this equilibrium behavior,

\[ \varepsilon_{Y,(1-\tau_c)} = \frac{\partial Y}{\partial(1-\tau_c)} \frac{1 - \tau_c}{Y} \]

\[ = \frac{[F'(K) - \alpha r \frac{\partial K}{\partial(1-\tau_c)}]}{Y} (1 - \tau_c) \]

\[ = \frac{\partial K}{\partial(1-\tau_c)} \left[ \frac{1 - \tau_c F'(K)K + \alpha rK}{K} \right] \]

\[ = \varepsilon_{K,(1-\tau_c)} \frac{e}{1+e} \frac{F(K) + \alpha rK}{F(K) + \alpha rK} \]

where \( \varepsilon_{K,(1-\tau_c)} = (1 - \alpha)(1 + e)/(1 - \alpha \tau_c) \). When financing costs are not tax deductible, \( \alpha = 0 \), the elasticity of taxable income with respect to the net of tax rate reduces to \( e \). In the other extreme when financing costs are totally tax deductible \( \alpha = 1 \) the elasticity of taxable income with respect to the net of tax rate is zero.

Now introduce a (convex) kink into the corporate tax schedule such that the marginal tax rate is \( t_0 \) for taxable income less than \( \kappa \) and \( t_1 \) above \( \kappa \), \( t_1 > t_0 \). In this example all firms with before-tax income in the absence of the kink in the segment \([\kappa, \kappa + dY]\), characterized by heterogenous production factors \( A_i \), bunch at the kink after the kink is introduced. This produces the relationship,

\[ \frac{\Delta Y}{Y} = \left( \frac{\theta_{t_0}}{\theta_{t_1}} \right)^e \frac{(1 + e)A - \alpha er\theta_{t_0}}{(1 + e)A - \alpha er\theta_{t_1}} - 1 \]

\[ = \left( \frac{1 - t_0}{1 - t_1} \right)^e - 1 \quad \text{When} \ \alpha = 0 \]  

(29)
where $\theta_{t_j} = A(1 - t_j)/[(1 - \alpha)r + \alpha r(1 - t_j)]$ which equals $A(1 - t_j)/r$ when $\alpha = 0$. This relationship generalizes the relationship in equation (24) to allow for large changes in tax rates at the kink.
References


