Optimal Debt Use by Local Governments with Volatile Revenues and Stable Infrastructure Needs

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Abstract

Debt plays an important role in local public finance, especially in capital investment. With decreasing reliance over the last half century on property tax revenue (and alongside adoption of other revenue sources), the revenue volatility of local governments has increased. Thus, it is important to balance revenues and outlays across the economic cycle instead of balancing the local revenues and expenditures on an annual basis in order to help maintain a locality's desired financial standing in terms of credit rating, debt capacity, service provision, and employment stability.

Local infrastructure is a significant part of public services that include roads/streets, parks, libraries and so on. The long-term needs for these facilities trend up with population growth, urbanization, and suburbanization but remain relatively stable across booms and busts of the economic cycle. Therefore, debt in tandem with pay-go has long been a regular and equitable means to finance the construction of such facilities as capital projects. To better handle the cyclical fluctuations of recurring tax revenues, scholars have suggested an optimal path (model) of debt use (Barro, 1979; Hou, 2013) in financing infrastructure as a countercyclical fiscal tool (i.e., more debt issuance during bust years and more debt retirement during boom years). Previous research has studied and provided empirical evidence on countercyclical use of debt at the national (Barro, 1979) and subnational (Hou, 2013) levels; but the local level has not been studied. This paper fills a niche and will be among the first to thoroughly test the countercyclical model of debt use by local governments.

Built upon and extending Hou's (2013) framework, this paper develops a theoretical framework and conducts empirical tests of its practicality and efficacy. This paper uses US Census data for aggregated analysis of US local governments; it also uses a balanced panel data set that includes all counties and municipalities in the state of Georgia from 1985 to 2011 and examines the operational mechanisms of debt issues over the booms and busts of the three full economic cycles in our sample period. This study contributes to the literature by providing a thorough understanding of whether or not and how local governments can use debt optimally to mitigate revenue fluctuations to satisfy local infrastructure needs.

1. Introduction

Debt plays an important role in local public finance, especially in capital investment. Different from the federal government, state constitutions often prohibit state and local governments from borrowing for operating budgets. State and local governments borrow money by selling bonds, and there are usually tax exemptions for bond interest as purchasing incentives to investors. There are three primary purposes of borrowing for state and local governments (Fisher, 2007, p. 231): (1) to finance public capital projects; (2) to support and subsidize economic development; and (3) to provide cash flow for short-term spending. Among the three, spending for capital projects is the major reason for debt financing. Regarding the second purpose, local governments often take advantage of the relatively low interest rates of their long-term bonds to reloan the yields to public authorities or local economic development corporations.

Local infrastructures projects are a significant part of public services that include roads/streets, parks, libraries, and so on. The long-term needs for these facilities trend up with population growth, urbanization, and suburbanization but remain relatively stable across booms and busts of the economic cycle. Therefore, debt in tandem with pay-go has long been a regular and equitable means of financing the construction of such facilities as capital projects. To better handle the cyclical fluctuations of recurring tax revenues, scholars have suggested an optimal path (model) of debt use (Barro, 1979; Hou, 2013) in financing infrastructures as a countercyclical fiscal tool – more debt issuance during bust years and more debt retirement during boom years. Past research has examined and provided empirical evidence on the countercyclical use of debt at the national (Barro, 1979) and subnational (Hou, 2013) levels; however, the local level has not been studied. This paper fills a niche and is among the first to thoroughly test the countercyclical model of debt use by local governments.

Built upon and extending Hou's (2013) framework, this article asks the following questions: Considering local infrastructure investment, do local governments use debt counter-cyclically? If some local governments use countercyclical debt, how much can it help in mitigating a recession? And what is the difference between the localities that use debt as a countercyclical tool and those that do not? Eventually, an optimal pattern of debt use will be calculated based on empirical results. This paper uses an aggregated U.S. local data set from 1960 to 2008 and a balanced panel data set that includes all counties and municipalities in the state of Georgia from 1985 to 2011 to examine the operational mechanisms of debt issuance over the booms and busts of three full economic cycles. We examine all the localities in one state (Georgia) that are subject to the same laws and rules regulating debt use, which is different from across-state studies containing social-economic and statutory variations. It is interesting that within one state, there are large variations in the share of debt in local expenditures and debt burdens measured as total debt outstanding at the end of the fiscal year as a percentage of personal income or total local revenue. When encountering a recession, how do these localities respond, and how do local fiscal policies, especially debt issuance, vary with their revenue portfolio and political structure during periods of revenue shortfall? This study will contribute to the literature by moving toward a thorough understanding of whether or not and how local governments can use debt optimally to mitigate revenue fluctuations in financing local infrastructure needs.

The rest of this paper is organized as follows. The next section reviews the literature on debt financing and pay-go, rationales and determinants of debt financing, and countercyclical use of debt. The third section provides a theoretical framework to illustrate how debt financing of local infrastructure investment can work practically and efficiently as a countercyclical tool to help stabilize expenditure fluctuations over the economic cycles. The fourth section introduces the data sets and methodologies used in the paper. The fifth section discusses the empirical results, and the sixth and final section concludes the paper.

2. Literature Review

The Ricardian Equivalence Theorem (Barro, 1974, 1976; Buchanan, 1967, 1976; Feldstein, 1976) suggests that outcomes are independent of whether a public program is financed by current taxes or debt issuance. Since the very early history of the United States, there have been alternate periods of dominance between pay-go and debt financing during different (Wang & Hou, 2009). The literature is rich on several perspectives of debt financing. In this section, we review studies of debt financing versus taxes, the rationales and determinants, and the use of debt as a countercyclical tool at the national and subnational levels.

2.1 Debt Financing and Pay-(As-You)-Go

Contrary to debt financing (pay-use), pay-go was designed to help control debt increases and to reduce excessive deficits, with the purpose of prioritizing expenses and exercising fiscal restraint (control revenue deductions and spending increases). After the government realized that cash transactions were too restrictive and inadequate for capital investment, state and local governments borrowed heavily in the early 19th century in the absence of regulation; however, defaults occurred in several states and cities. The purpose of pay-go at that time was to avoid debt by not spending until they had money in hand (New York Comptroller, 1843). Balanced budget requirements (BBRs) and debt limitations were also subsequently added in reaction to these defaults (Hou & Smith, 2006; Kiewiet & Szakaly, 1996).

After the Civil War, a new round of borrowing started to support the reconstruction and large-scale internal improvements, until the Progressive Movement reinstalled the balanced budget norm (Wang & Hou, 2009). Based on time-series data, Barro (1979) finds positive effects of the temporary increases in government spending on the federal debt issue, especially during war and postwar periods. Recently, pay-go financing has been considered a necessary supplement to debt financing and has been taken for granted "as the flip side of long-term debt financing" (Wang, Hou, & Duncome, 2007). For a countercyclical fiscal policy, Alvin Hansen was recognized as the first to propose the complementary use of pay-go and debt financing in the 1940s (Hou, 2004). Basically, debt financing increases during periods of necessity or emergent funding requirements, such as large capital projects and wars, while pay-go appears more often in boom years.

2.2 Rationales and Determinants of Debt Financing

Wang and Hou (2009) point out several rationales for debt financing. One of their arguments claims that the borrowed money will benefit future taxpayers if they are invested in human resources through funding education, job retraining, and health programs. Their income will benefit more substantially from these investments, and thus, the debt can be easily paid by them.[1] Another rationale is that debt could finance programs benefiting people who would never receive funding without these debts. Additionally, governmental spending with debt will promote economic growth based on the Keynesian theory, which will eventually benefit individuals as taxpayers and consumers. Last, long-term debt can help increase intergenerational equity, because later generations will benefit from capital projects that have a long useable life

[1] Also, people with higher educational attainment tend to purchase more goods and services from the private sector, which implies that they will pay more taxes embedded in these purchases.

and, thus, should be responsible for part of the accumulated debt.[2] Another reason is consumption smoothing, a widely accepted practice and economic principle, as shown in diminishing marginal utility. The addition to utility in good times from an extra unit is smaller than the loss of utility from an existing unit. Debt financing in recession years may increase the overall utility by transferring from good times to bust years.

However, public debt has been considered a threat to the solvency of governments during much of U.S. history due to the default and even bankruptcy of local governments (Wang & Hou, 2009). The intergenerational equity issue is also embedded in debt financing. For long-term debt, future generations need to pay off the debt and interest, while they are excluded from the decision making on debt issuance and capital projects. It has been widely held that state and local governments should in principle balance their budgets annually, though the balanced budget norm has often been violated due to financing large capital projects and wars. The results indicate that pay-go is "associated with lower volatility in capital spending in the long run, but may increase short-run variability" (Wang & Hou, 2009). The unison of the two mechanisms is recommended by Wang and Hou (2009) as a symmetric countercyclical fiscal policy – more pay-go in boom years and more debt in recession years.

Regarding the question of the determinants of state debt financing, debt at the state level is a function of economic conditions that reflect both the need to borrow and the capacity of states to repay debt (Clingermayer & Wood, 1995). Also, political factors, such as culture, partisan competition, and electoral cycles, are relevant elements of state debt. While tax and expenditure limitations on state governments have little or no impact on the increasing reliance on revenue sources from general taxes (Joyce & Mullins, 1991), this implies that increases in debt financing are more likely due to project demands than to revenue shortage. However, the

[2] This rationale is consistent with the "benefit to pay" principle in taxation.

situation in local governments may be different, which deserves more investigation. The changes of state and local debt burden in the 1980s were attributed to increases in demand for capital projects and to the preferences of each state. Poorer states were found to have a larger debt burden than richer states when all other factors are taken into account (Bahl & Duncombe, 1993). Based on these results, Bahl and Duncombe (1993) raised the question of whether or not poorer states might be avoiding present fiscal constraints by offloading high-debt burdens onto future generations. They found that tax and expenditure limitations tend to be associated with higher debt burden. Moreover, state limits on local debt may be beneficial for local governments (Epple & Spatt, 1986).

2.3 Debt as a Countercyclical Tool

For the countercyclical use of debt, Barro (1979) introduces a theoretical model concerning the determination of public debt issue. The public debt theory promotes an optimal time path of debt issue at the national level, which means increasing temporary government debt in a bust year to raise current spending and retiring debt in boom years. According to Barro's theory, debt issue is almost independent of the outstanding debt-income ratio or the level of government spending, but is a countercyclical response to the *change* in current income. Hou (2013) substantively extends Barro's (1979) propositions to the state level and concludes that states could have been better prepared for economic downturns if they had issued debt as a countercyclical tool (borrowing more in recession years and retiring these debts in boom years).

The long-term needs for local infrastructures usually remain relatively stable across booms and busts of the economic cycle, and thus, infrastructure investment can stabilize the economy by itself. Taylor, Baily, and Fischer (1982) illustrate how countercyclical infrastructure investment can work, and Hou (2011) further designs a framework indicating how to use countercyclical federal transfers for infrastructure investment, service maintenance, and business tax relief for states to save in boom years and make it through recession years. As mentioned above, debt is mainly issued for capital projects for state and local governments, and capital construction can bring the largest multiplier effect to the economy based on empirical evidence. Therefore, with the internal connection between infrastructure investment and debt issue, countercyclical debt use can be a natural tool to combat economic downturns. However, the countercyclical use of debt and its possible effects at the local level have not been studied. This paper is among the first to thoroughly test how, and to what extent, countercyclical debt use by local governments can help to improve the stability in revenue and service provision during recession years.

3. Theoretical Framework

The countercyclical use of debt at the local level is different from that at the national or subnational levels in several ways. Hou (2013) raises issues for the subnational government that are different from those for the national government. These include exogenous variables like population, inter-state migration, and interest rate. For the local level, however, some unique assumptions also hold. First, in one state (Georgia in this paper), all localities are subject to the same laws and rules regulating debt use and debt limits. Second, the population size is much more volatile than that at the state level, since it is much easier to move from one county to another (sometimes moving between incorporated and unincorporated areas within the same county). Third, local governments do not have any influence on the interest rate. However, the cost of debt can vary much more than inter-state differences, since local debt endures a much larger risk than state or federal debt. The cost depends heavily on local fiscal conditions and revenue capacity. Based on the above, we assume an economic cycle that includes a 2-year recession and a 6-year economic boom.[3] In the rest of this section, building upon Barro (1979) and Hou (2013), we design a theoretical model covering an entire economic cycle and illustrate how localities can use countercyclical debt to smooth public expenditure to increase citizen welfare.

First assume that in normal years household income (Y_t) and local government revenue (R_t, mainly tax revenue) both grow at a constant rate ρ and that government program outlays grow at a different rate γ and $\gamma \leq \rho$:[4]

$$Y_t = (1+\rho)Y_{t-1}$$

$$R_t = (1+\rho)R_{t-1}$$

$$O_t + C_t = (1+\gamma)(O_{t-1} + C_{t-1})$$

where O_t is operating expenditure and C_t is capital expenditure. Note that financial variables are real terms, as in Barro (1979) and Hou (2013). Restricted by balanced budgeting constraint, the annual budget equation for a local government is:

$$O_t + C_t = R_t - r^* D_{t-1} + d_t$$

where D is countercyclical debt stock, r^* is real interest rate on local debt, and d_t is net amount of debt issued ($d_t = D_t - D_{t-1}$), which is equal to debt issue minus debt retirement annually. Here, we assume ND_t, designated as *normal debt* (to distinguish it from countercyclical debt), remains and increases at the same rate as government revenue, the changes of D_t are counted in

[3] By Hamilton (1993) and Gordon (2010), the average duration of a recession in the United States from the end of WWII to the 1980s is 1-2 years and a boom lasts 4-6 years.
[4] Revenue (R_t) includes own-source revenue, intergovernmental transfers, and normal debt.

changes of R_t, and we restrict d_t to *countercyclical debt* only. Thus, we put our focus on countercyclical debt use, which is extra from the current trend of debt stock.

If local governments implement a countercyclical debt use policy, the net debt issued d_t has different values over economic cycles:

$$d_t \begin{cases} > 0, & in\ bust\ years\ \ (t = 1, \dots, t_1) \\ < 0, & in\ boom\ years\ \ (t = t_1 + 1, \dots, T) \end{cases}$$

Due to recession shocks, governmental revenue falls below the trend by a fraction μ (< 0):

$$R_t = (1 + \mu)(1 + \rho)R_{t-1}.$$

According to the countercyclical debt use policy, local governments increase capital outlay funded by debt during a recession to smooth expenditure and retire this debt during boom years. Therefore, for a T-year economic cycle, the budget equations are:

Bust: $O_t + (1 + \varphi)C_t = (1 + \mu)R_t - rD_{t-1} + d_t \quad (d_t > 0)$

Boom: $O_t + C_t = R_t - rD_{t-1} + d_t \quad (d_t < 0)$.

We assume that operating budgets remain unchanged with the trend during recessions and φ is the fraction of increased capital outlay (mainly infrastructure investment). Summarizing all the years in the economic cycle with the discount rate at r, we get:

$$\sum_{t=1}^{T}(O_t + C_t) + \sum_{t=1}^{T_1}\varphi C_t = \sum_{t=1}^{T}(R_t - rD_{t-1}) + \sum_{t=1}^{T_1}\mu R_t + \sum_{t=1}^{T} d_t.$$

where t_1 is the duration of recession years. In this model, local government will pay off all the increased debt in the following boom years; thus, $\sum_{t=1}^{T} d_t \cong 0$.[5] Recall that $\varphi > 0$, $\mu < 0$, and if, as assumed in Barro (1979), $\gamma \cong \rho$, the budget constraint cannot hold unless local governments also make certain cuts in government expenditures. In section 3.1, we first assume

[5] In this scenario, we assume that the discount rate is negligible for the short term. The other condition that makes this equation tenable is that the interest rate on debt is exactly equal to the discount rate.

a scenario with no discount rate, the revenue and expenditures are constant except during bust years, and the interest of extra countercyclical debt is negligible. Then, we relax these restrictive assumptions and generalize the results to a more complicated scenario that approximates to reality.

3.1 Scenario (1)

Given the average duration of boom and bust, we assume that a recession lasts two years, followed by a 6-year expansion (i.e., $t_1 = 2$ and $T = 8$), and local revenue recovers back to the original level in the third year in the cycle ($R_3 = R_1$). With diminishing marginal utility, the best way to smooth outlays over the cycle is to equalize the expenditure for each year, i.e., divide the revenue shortage into all years. Thereby localities smooth borrowing and debt payoff:

$$d_1 = d_2 = \tfrac{1}{2}D > 0 \quad \text{and} \quad d_3 = \cdots = d_8 = -\tfrac{1}{6}D < 0.$$

Assume that over a short period, the growth and discount rates of revenue and expenditure are both negligible or that the two are equal, which makes the real terms constant. For convenience, at this stage, we also do not consider the extra interest expenditure from the countercyclical debt use.[6] In this way, the countercyclical debt is used to equally distribute the revenue shortfall into different years (i.e., to complement *part* of the expenditure cuts).

$$(1+\mu)R + \frac{1}{2}D = R - \frac{1}{6}D$$

i.e.,

$$D = -1.5\mu R \ (\mu < 0) \qquad (1)$$

An example may help illustrate this point. If a two-year recession causes a 10% revenue shortfall ($\mu = -10\%$), the optimal countercyclical debt should be $D = -1.5\mu R = 15\%R$, issuing new

[6] We will relax these restrictive assumptions in the next step.

debt of $d = 7.5\%$ of the revenue for each recession year. Table 1 provides different options indicating how this amount of new debt can be optimal.

[Table 1 about here]

To generalize the above discussion, denote $G_t = O_t + C_t + rD_{t-1}$, and assume a Cobb-Douglas utility function for households: $U(C, G) = c_1 ln(C) + c_2 ln(G)$[7], where Y is private consumption, G is government expenditure funded mainly with property tax, and c_1, c_2 are parameters. In a partial equilibrium of government, without considering the discount rate, the growth rate of revenue and expenditure, and the interest of extra countercyclical debt, to maximize utility, maximize $\sum_{t=1}^{T} c_2 \ln G_t$, subject to $G_t \leq R_t + d_t$ and $\sum_{t=1}^{T} d_t = 0$, i.e.,

$$\sum_{t=1}^{T} G_t \leq \sum_{t=1}^{T} R_t = \sum_{t=1}^{T_1} (1+\mu) R_0 + \sum_{t=T_1+1}^{T} R_t = TR_1 + \mu T_1 R_1$$

where T_1 is the duration (year) of a recession and μ (< 0) is the fraction of revenue shortfall because of economic downturns. The first-order conditions are the following:

$$G_t = G_{t+1} = R_1 + \mu \frac{T_1}{T} R_0.$$

Thus, under the countercyclical debt policy, in bust years ($t = 1, \dots, t_1$), we have:

$$d_t = G_1 - (1+\mu) R_0 = \frac{T_1 - T}{T} \mu R_0 > 0;$$

in boom years ($t = t_1 + 1, \dots, T$), we have:

$$d_t = G_1 - R_0 = \frac{T_1}{T} \mu R_0 < 0.$$

The total amount of countercyclical debt is

[7] Another option of the utility function would be $U(C) = \frac{C^{1-\theta} - 1}{1-\theta}$.

$$D = \sum_{t=1}^{T_1} d_t = \frac{T_1 - T}{T} \mu T_1 R_0.$$

By substituting the values of the parameters into these equations, we can easily obtain the results in the examples illustrated above.

3.2 Scenario (2)

Now, we relax the restrictive assumptions and assume that the discount factor is $\frac{1}{1+r}$ and that the revenue and expenditure grow at a constant rate ρ.[8] In this scenario, the partial equilibrium of government becomes: maximize $\sum_{t=1}^{T} c_2 \frac{1}{(1+r)^{t-1}} \ln G_t$, subject to

$$G_t \leq R_t + d_t - r^* \sum_{i=1}^{t-1} d_i \quad \text{and} \quad \sum_{t=1}^{T} d_t = 0$$

where r^*, used to distinguish it from the discounter factor r, is the interest rate for the countercyclical debt.[9] Combining the two subjective conditions, we get

$$\sum_{t=1}^{T} G_t \leq \sum_{t=1}^{T} R_t - r^* \sum_{t=1}^{T} \sum_{i=1}^{t-1} d_i.$$

Thus, the first order conditions are:

$$G_{t+1} = \frac{1}{1+r} G_t.$$ [10]

Since

$$R_t = (1+\rho) R_{t-1} \text{ in boom years and}$$

[8] In this subsection, we do not consider the interest of extra countercyclical debt, which we will do in Subsection 3.3 with precise calculation.

[9] r^* should vary in different years. In this framework, we assume the interest rate r^* is the same across all years, to facilitate explanation.

[10] The intertemporal Euler equation implies that $U'(c_t) = (1+r^*)\beta U'(c_{t+1})$, where r^* is interest rate and β is discount parameter. A special case is $(1+r^*)\beta = 1$. Thus, compared to our scenario, $\frac{1}{1+r}$ should be equal to $(1+r^*)\beta$, and $r = 0$ is a benchmark which we applied most often in the simulation. The "discount factor" r used here is actually a combination of interest rate and discount parameter. For more details, refer to the Neoclassical Consumption Model, http://www.stanford.edu/~chadj/Consumption2009-11-25.pdf (retrieved June 5, 2014)

$$\sum_{t=1}^{T} R_t = \sum_{t=1}^{T_1} R_t + \sum_{t=T_1+1}^{T} R_t,$$

according to the above assumptions, in a full economic cycle with a 2-year recession and a 6-year boom, we list the revenue and expenditure for each period in Table 2.

[Table 2 about here]

Thereby we have

$$\sum_{t=1}^{T} G_t = \frac{G_1(1 - \frac{1}{(1+r)^T})}{1 - \frac{1}{1+r}} = \frac{G_1}{r}(1 + r - \frac{1}{(1+r)^{T-1}})$$

$$\sum_{t=1}^{T} R_t = \sum_{t=1}^{T_1} R_t + \sum_{t=T_1+1}^{T} R_t$$

$$= (1+\mu)R_0 \frac{1-(1+\mu)^{T_1}}{-\mu} + \frac{R_0(1+\mu)^{T_1}(1+\rho)[(1+\rho)^{T-T_1} - 1]}{\rho}.$$

Thus,

$$G_1 = \frac{r}{1 + \mathrm{r} - \frac{1}{(1+r)^{T-1}}}\left[(1+\mu)R_0 \frac{1-(1+\mu)^{T_1}}{-\mu} + \frac{R_0(1+\mu)^{T_1}(1+\rho)[(1+\rho)^{T-T_1} - 1]}{\rho} - r^* \sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i\right]$$

where

$$\sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i = T d_1 + (T-1) d_2 + \cdots + d_T.$$

Based on the above results, with $T_1 = 2$ and $T = 8$, we can formulate equations on the countercyclical debt level for each period as:

$$d_1 = G_1 - R_1 = G_1 - (1+\mu)R_0$$

$$= \frac{r}{1 + \mathrm{r} - \frac{1}{(1+r)^{T-1}}}\left[(1+\mu)R_0 \frac{1-(1+\mu)^{T_1}}{-\mu}\right.$$

$$\left. + \frac{R_0(1+\mu)^{T_1}(1+\rho)[(1+\rho)^{T-T_1}-1]}{\rho} - r^* \sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i\right] - (1+\mu)R_0$$

$$d_2 = G_2 - (1+\mu)^2 R_0 = \frac{G_1}{1+r} - (1+\mu)^2 R_0$$

$$d_3 = G_3 - (1+\mu)^2(1+\rho)R_0 = \frac{G_1}{(1+r)^2} - (1+\mu)^2(1+\rho)R_0$$

…

$$d_T = G_t - (1+\mu)^2(1+\rho)^{T-T_1}R_0 = \frac{G_1}{(1+r)^{T-1}} - (1+\mu)^2(1+\rho)^{T-T_1}R_0$$

The T-equation set illustrates the relationship between debt usages in each year. If we assume

$$r^* \sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i \cong 0;$$ [11]

then

[11] The interest of new accumulated countercyclical debt is:

$$r^* \sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i = r[Td_1 + (T-1)d_2 + \cdots + d_T]$$

Substitute $\sum_i^T d_i = 0$ into the equation,

$$r^* \sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i = r[(T-1)d_1 + (T-2)d_2 + \cdots + d_{T-1}]$$

After considering the interest rate for these countercyclical debts, the amount of debt should be a little less than what we calculate here. The exact amount can be calculated by a loop computation which will be done in Subsection 3.3.

$$d_1 = G_1 - (1+\mu)R_0$$

$$= \frac{r}{1 + \mathrm{r} - \frac{1}{(1+r)^{T-1}}}\left[(1+\mu)R_0 \frac{1-(1+\mu)^{T_1}}{-\mu} + \frac{R_0(1+\mu)^{T_1}(1+\rho)[(1+\rho)^{T-T_1} - 1]}{\rho}\right] - (1+\mu)R_0.$$

An example of this scenario is provided in Table 3. We assign values to each parameter as follows: $T_1 = 2$, $T = 8$, $r = 0$ or 1%, $\mu = -5\%$, and $\rho = 3\%$. The countercyclical use of debt for each period is listed in Table 3. [12]

[Table 3 about here]

We then assume different values to the important parameters (revenue shortfall μ, revenue and expenditure growth rate ρ, discount factor r) and run a simulation. The results are shown in Table 4.

[Table 4 about here]

The results are interesting and illuminating in several aspects. First, while the economy begins to recover in the third year (2-year recession and 6-year boom), following the optimal path, the net countercyclical debts are positive (issue more debt instead of retirement) in the first four years and become negative in the latter four years. This path keeps consistent regardless of the values of the parameters. Second, with a fixed revenue growth rate ρ and discount factor r, the change of revenue due to economic shock will only change the countercyclical debt burden

[12] Two specific conditions are $r = 0$ (no discount factor) and $\rho = 0$ (no revenue and expenditure growth).
If $r = 0$,

$$d_1 = \frac{1}{T}\left[(1+\mu)R_0 \frac{1-(1+\mu)^{T_1}}{-\mu} + \frac{R_0(1+\mu)^{T_1}(1+\rho)[(1+\rho)^{T-T_1} - 1]}{\rho}\right] - (1+\mu)R_0$$

If $\rho = 0$,

$$d_1 = \frac{r}{1 + \mathrm{r} - \frac{1}{(1+r)^{T-1}}}\left[(1+\mu)R_0 \frac{1-(1+\mu)^{T_1}}{-\mu} + (T - T_1)R_0 - r^* \sum_{t=1}^{T}\sum_{i=1}^{t-1} d_i\right] - (1+\mu)R_0$$

for the first year; the net countercyclical debt as a ratio of revenue keeps constant in the following years. Third, in the optimal path of countercyclical debt policy, a heavier recession does not imply a greater amount of debt issuance. On the contrary, the deeper recession incurs a lesser amount of countercyclical debt. The reason is that in the optimal path of debt use, governments need to retire all these debts over the subsequent boom years. This result requires a comprehensive consideration by governments of their fiscal conditions in order to use countercyclical debt. Fourth, and finally, based on the first four rows of simulation results, we can also find that the discount factor has a large impact on the countercyclical debt policy. In the first year of recession, a one percentage point change in the discount factor (r) will induce a 3-percent difference in the new net debt burden (measured by debt as a ratio of lagged revenue). This raises a concern on the scale of the discount factor. The often used discount factor is around 3%; but based on the simulation results, 1% is probably a better approximation, especially when we use real terms instead of nominal terms. If r is close to the benchmark at 1% or 0, the high debt burden in certain years (along the optimal path of debt) is around 10%, which is feasible and stays within statutory local debt limits.[13]

3.3 Toward a Comprehensive Scenario

Recall that until now we have not considered the interest of accumulated net debt. In this subsection, we will investigate a comprehensive framework that contains all related parameters including the interest rate of countercyclical debt (r^*), revenue growth (ρ), and discount factor (r). The interest of increased debt is $r^* \sum_{t=1}^{T} \sum_{i=1}^{t-1} d_i$. If we denote D_t as the stock of accumulated

[13] According to the data and statutory requirement in the state of Georgia, the capital budget as a percentage of total budget is around 30% ($\frac{C}{O+C} \cong 30\%$); the ratio of debt service (principal expenditure and interest) to local revenue was lowered from 15% to 10% after 1983 with the ratification of the 1983 Constitution ($\frac{d_t}{R_{t-1}} < 10\%$). Therefore, the extra capital investment induced by countercyclical debt use should be less than $\frac{10\%}{30\%} = 33.3\%$.

new debt (deviations from the trend), it is $D_t = \sum_{i=1}^{t-1} d_i$. The values of D_t for each year in Scenario (1) without considering the interest are listed in Table 5. Based on the listed values of D_t, with the interest rate r^*, the total interest for the countercyclical debts will be $\sum_{t=1}^{T} r^* D_t \cong 4r^* D$.

[Table 5 about here]

However, it is clear that with the interest considered, the countercyclical debt changes in each period. According to the balanced budget equation, we have a *T*-equation set.

$$G_1 = (1+\mu)R_0 + d_1,$$

$$G_2 = (1+\mu)R_0 + d_2 - r^* d_1,$$

$$G_3 = R_0 + d_3 - r^*(d_1 + d_2),$$

$$\dots$$

$$G_T = R_0 + d_T - r^*(d_1 + d_2 + \dots + d_{T-1})$$

Since $G_t = G_{t+1}$, we get

$$d_2 = (1+r^*)d_1$$

$$d_3 = (1+r^*)d_2 + \mu R_0$$

$$d_4 = (1+r^*)d_3$$

$$\dots$$

$$d_T = (1+r^*)d_{T-1}$$

Summarizing these equations, with $\sum_{t=1}^{T} d_t = 0$, we have

$$\sum_{t=1}^{T} d_t = d_1 - (1+r^*)d_T + \mu R_0 = 0.$$

Substituting $d_T = (1 + r^*)^{T-3} d_3 = (1 + r^*)^{T-3}[(1 + r^*)^2 d_1 + \mu R_1]$ into this equation, we get

$$d_1 = \frac{\mu R_0[(1 + r^*)^{T-2} - 1]}{1 - (1 + r^*)^T}.$$

If the duration of recession years is t_1, then

$$d_T = (1 + r^*)^{T-(t_1+1)} d_{t_1+1} = (1 + r^*)^{T-(t_1+1)}[(1 + r^*)^{t_1} d_1 + \mu R_0]$$

Thus,

$$d_1 = \frac{\mu R_0[(1 + r^*)^{T-t_1} - 1]}{1 - (1 + r^*)^T}$$

Following the results in Table 1 with $r_i^* = 5\%$, $\mu = -10\%$ and $t_1 = 2$, we provide in Table 6 a comparison between the two paths of optimal debt uses, with and without interest on these countercyclical debts. It shows that changes do exist even though they are relatively small.

[Table 6 about here]

In *Scenario (2)*, the situation is much more complicated. According to the balanced budget equation, we have a different *T*-equation set:

$$G_1 = (1 + \mu)R_0 + d_1,$$

$$G_2 = (1 + \mu)^2 R_0 + d_2 - r^* d_1,$$

$$\dots$$

$$G_{t_1} = (1 + \mu)^2 R_0 + d_{t_1} - r^*(d_1 + \dots + d_{t_1-1}),$$

$$G_{t_1+1} = (1 + \mu)^2 (1 + \rho) R_0 + d_{t_1+1} - r^*(d_1 + \dots + d_{t_1}),$$

$$\dots$$

$$G_T = (1 + \mu)^2 (1 + \rho)^{T-t_1} R_0 + d_T - r^*(d_1 + d_2 + \dots + d_{T-1}).$$

With discount factor r, the first order conditions are as follows: $G_{t+1} = \frac{1}{1+r} G_t$. Substituting this equation into the balanced budget equations, we get: [14]

$$d_2 = \left[\frac{1}{1+r} - (1+\mu)\right](1+\mu)R_0 + \left(\frac{1}{1+r} + r^*\right)d_1$$

…

$$d_{t_1} = \left[\frac{1}{1+r} - (1+\mu)\right](1+\mu)^{t_1-1}R_0 + \frac{1}{1+r}\left[d_{t_1-1} - r^*\left(d_1 + \cdots + d_{t_1-2}\right)\right]$$

$$+ r^*\left(d_1 + \cdots + d_{t_1-1}\right)$$

$$= \left[\frac{1}{1+r} - (1+\mu)\right](1+\mu)^{t_1-1}R_0 + \left(\frac{1}{1+r} + r^*\right)d_{t_1-1} + \frac{r}{1+r}r^*(d_1 + \cdots$$

$$+ d_{t_1-2})$$

$$d_{t_1+1} = \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1}R_0 + \left(\frac{1}{1+r} + r^*\right)d_{t_1} + \frac{r}{1+r}r^*(d_1 + \cdots + d_{t_1-1})$$

…

$$d_T = \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1}(1+\rho)^{T-(t_1+1)}R_0 + \left(\frac{1}{1+r} + r^*\right)d_{T-1}$$

$$+ \frac{r}{1+r}r^*(d_1 + \cdots + d_{T-2}).$$

Summarizing these equations, with $\sum_{t=1}^{T} d_t = 0$, we have

[14] If $t_1 = 1$,

$$d_2 = \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)R_0 + \left(\frac{1}{1+r} + r^*\right)d_1$$

and for $t > 2$,

$$d_t = \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)(1+\rho)^{t-(t_1+1)}R_0 + \left(\frac{1}{1+r} + r^*\right)d_{t-1} + \frac{r}{1+r}r^*(d_1 + \cdots + d_{t-2})$$

$$\sum_{t=1}^{T} d_t = d_1 + \left(\frac{1}{1+r} - 1 - \mu\right)(1+\mu)R_0 \frac{1-(1+\mu)^{t_1-1}}{-\mu}$$

$$+ \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1} R_0 \frac{1-(1+\rho)^{T-t_1}}{-\rho} - \left(\frac{1}{1+r} + r^*\right) d_T$$

$$+ \frac{r}{1+r} r^* [(T-2)d_1 + (T-3)d_2 + \cdots + d_{T-2}] = 0 \quad (2)$$

Since $\frac{r}{1+r} r^* \cong 0$, if the duration of recession years $t_1 \geq 2$,

$d_t = \left(\frac{1}{1+r} + r^*\right) d_{t-1} + (1+\mu)^{t_1-2} A$ for $t \leq t_1$

$d_t = \left(\frac{1}{1+r} + r^*\right) d_{t-1} + (1+\rho)^{t-(t_1+1)} B$ for $t > t_1$

where $A = \left[\frac{1}{1+r} - (1+\mu)\right](1+\mu)R_0$ and $B = \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1} R_0$.

$$d_T = \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1}(1+\rho)^{T-(t_1+1)} R_0$$

$$+ \left(\frac{1}{1+r} + r^*\right)\left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1}(1+\rho)^{T-1-(t_1+1)} R_0 + \cdots$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-(t_1+1)} \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1} R_0$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-t_1} \left(\frac{1}{1+r} - 1 - \mu\right)(1+\mu)^{t_1-1} R_0 + \cdots$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-2} \left[\left(\frac{1}{1+r} - 1 - \mu\right)(1+\mu)R_0 + \left(\frac{1}{1+r} + r^*\right) d_1\right]$$

$$= \left(\frac{1}{1+r} - 1 - \rho\right)(1+\mu)^{t_1} R_0 (1+\rho)^{T-(t_1+1)} \frac{1 - \left(\frac{\frac{1}{1+r} + r^*}{1+\rho}\right)^{T-t_1}}{1 - \left(\frac{\frac{1}{1+r} + r^*}{1+\rho}\right)}$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-t_1} \left(\frac{1}{1+r} - 1 - \mu\right)(1+\mu)^{t_1-1} R_0 \frac{1 - \left(\frac{\frac{1}{1+r} + r^*}{1+\mu}\right)^{t_1-1}}{1 - \left(\frac{\frac{1}{1+r} + r^*}{1+\mu}\right)}$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-1} d_1$$

$$= \left(\frac{1}{1+r} - 1 - \rho\right)(1+\rho)^{T-(t_1+1)}(1+\mu)^{t_1} R_0 \frac{1 - \left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]^{T-t_1}}{1 - \left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]}$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-t_1} \left(\frac{1}{1+r} - 1 - \mu\right)(1+\mu)^{t_1-1} R_0 \frac{1 - \left[\frac{1+(1+r)r^*}{(1+r)(1+\mu)}\right]^{t_1-1}}{1 - \left[\frac{1+(1+r)r^*}{(1+r)(1+\mu)}\right]}$$

$$+ \left(\frac{1}{1+r} + r^*\right)^{T-1} d_1$$

Substituting this into equation (2), we get

$$d_1 = \frac{1}{1-\left(\frac{1}{1+r}+r^*\right)^T}\left\{\left(\frac{1}{1+r}-1\right.\right.$$

$$\left.-\rho\right)(1+\rho)^{T-(t_1+1)}(1+\mu)^{t_1}R_0\frac{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]^{T-t_1}}{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]}\left(\frac{1}{1+r}+r^*\right)$$

$$+\left(\frac{1}{1+r}+r^*\right)^{T-t_1+1}\left(\frac{1}{1+r}-1-\mu\right)(1+\mu)^{t_1-1}R_0\frac{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\mu)}\right]^{t_1-1}}{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\mu)}\right]}$$

$$-\left(\frac{1}{1+r}-1-\mu\right)(1+\mu)R_0\frac{1-(1+\mu)^{t_1-1}}{-\mu}$$

$$\left.-\left(\frac{1}{1+r}-1-\rho\right)(1+\mu)^{t_1}R_0\frac{1-(1+\rho)^{T-t_1}}{-\rho}\right\}$$

Based on the above formula of d_1, the countercyclical debt of the following periods (d_2, d_3, … , d_t) can be calculated by using the debt equations. This is a comprehensive equation covering different parameter values on optimal countercyclical debt use.[15]

If $t_1 = 1$,

$$d_T = \left(\frac{1}{1+r}-1-\rho\right)(1+\rho)^{T-(t_1+1)}(1+\mu)R_0\frac{1-\left(\frac{\frac{1}{1+r}+r^*}{1+\rho}\right)^{T-t_1}}{1-\left(\frac{\frac{1}{1+r}+r^*}{1+\rho}\right)}+\left(\frac{1}{1+r}+r^*\right)^{T-1}d_1$$

Similarly,

[15] This equation also covers all specific conditions including $r = 0$ (without considering discounter factor), $r^* = 0$ (without considering interest of countercyclical debt) and $\rho = 0$ (without considering growth rate of government revenue). Note that for the term $\frac{1-(1+\rho)^{T-t_1}}{-\rho}$ in the equation, when $\rho = 0$, $\frac{1-(1+\rho)^{T-t_1}}{-\rho} \cong T - t_1$.

$$d_1 = \frac{1}{1-\left(\frac{1}{1+r}+r^*\right)^T}\left\{\left(\frac{1}{1+r}-1-\rho\right)(1+\rho)^{T-(t_1+1)}(1\right.$$

$$+\mu)R_0\frac{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]^{T-t_1}}{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]}\left(\frac{1}{1+r}+r^*\right)$$

$$\left.-\left(\frac{1}{1+r}-1-\rho\right)(1+\mu)R_0\frac{1-(1+\rho)^{T-t_1}}{-\rho}\right\}$$

$$= \frac{1}{1-\left(\frac{1}{1+r}+r^*\right)^T}\left\{\left(\frac{1}{1+r}-1-\rho\right)(1+\rho)^{T-2}(1+\mu)R_0\frac{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]^{T-1}}{1-\left[\frac{1+(1+r)r^*}{(1+r)(1+\rho)}\right]}\left(\frac{1}{1+r}\right.\right.$$

$$\left.\left.+r^*\right)-\left(\frac{1}{1+r}-1-\rho\right)(1+\mu)R_0\frac{1-(1+\rho)^{T-1}}{-\rho}\right\}$$

By substituting the values of related parameters into these equations, in Table 7, we calculate the optimal paths of countercyclical debt use in different scenarios. The results shed light on how governments can or should use countercyclical debt optimally after considering the relevant parameters.

[Table 7 about here]

Thus, we have obtained some preliminary conclusions about the optimal countercyclical debt. First, an optimal path of countercyclical debt exists under certain relaxed assumptions that are close to reality, and we can simulate these paths. Second, by the optimal path, a deeper recession does not necessarily indicate a higher level of countercyclical debt. For instance, to smooth government expenditure following a two-year recession that causes a revenue shortfalls of 5%, the countercyclical debts as percentages of previous year's revenue are 2.7%, 8.0%, 5.9%, and 3.0%. In comparison, the debt percentages are -1.6%, 8.2%, 6.5%, and 3.7% to tackle a revenue shortfall of 10% of the same recession duration. The reason is that governments need to *optimally* consider how to retire these debts in the subsequent boom years in order to prepare for the next economic downturn. If the recession is very deep, rational governments will use countercyclical debt strategically taking their fiscal condition into account rather than just borrow more to smooth outlays.

Moreover, the optimal debt path does not indicate that debt issuance in recession years are retire debt in *each and all* boom years. According to our results, localities may continue to borrow for a couple more years after a recession and start to retire only after the economy has recovered to the pre-recession level.[16] For example, if recession lasts two years followed by an expansion of six years, localities may use countercyclical debt for four consecutive years (two recession years plus two following years) and start to retire these debt in the following four years. Finally, the amount of optimal debt, as high as around 10% of total revenue, is reasonable by local fiscal conditions and feasible under most statutory debt limits.

Up to this point, we have provided a comprehensive picture of optimal debt use for local governments that aspire to combat outlay fluctuations due from recessionary revenue shortfalls. Based on the literature and our theoretical framework, we have three concerns or extensions with regard to the optimal path of countercyclical debt use. First, since the duration of a bust or boom in an economic cycle is not as predicted in the model, local governments need to adjust their parameter values accordingly. Second, our framework assumes post-recession revenue growth on the pre-downturn basis, but in reality the timeline and base of revenue recovery varies a lot, which casts an impact on debt policy. Third, Barro (1979) assumed $\gamma \leq \rho < r$ (to facilitate the

[16] In the case of 2-year recession and 6-year boom, localities borrow for four years (2 recession years and 2 recovery years) and then retire during the following four years. Hou (2013) also points out that debt issue may continue, because reversing the direction takes time.

calculation of indefinite periods), but the real terms of government revenue and expenditure increase over time ($r < \gamma \leq \rho$), i.e., the nominal terms increase faster than inflation rates.

4. Data

We use two data sets for empirical analyses. We first conduct an aggregate analysis of different types of U.S. local governments. This exercise uses aggregated data from 1960 to 2008, from the U.S. Census from. The set includes variable aggregates of the U.S. total and state totals of all local governments including counties, cities and townships, school districts, and special districts. Figures 1 and 2 describe the trend of nominal and real total (and per capita) debt outstanding for the U.S. local total and different types of localities.

[Figures 1 and 2 about here]

Then we conduct a detailed investigation of localities in the State of Georgia, using a balanced panel data set that includes all counties and municipalities in the state from 1985 to 2011. After removing missing observations, the set covers 156 counties and 548 municipalities. The data sources are Georgia Department of Community Affairs, Georgia Department of Revenue, and the U.S. census. The variables include debt amount, property value, revenue and expenditure structures, and other local fiscal characteristics. Local data boast advantages over state aggregates (Farnham, 1985; McEachern, 1978): when examined at the state level, local behavior can lead to an aggregation bias in econometric analyses (Feige & Watts, 1972; Kmenta, 1971). This problem is even more severe when the variation is substantial in characteristics among local governments. Using local data can more precisely estimate the large variations in variables at the local level, such as population and income. Using the Georgia data set, we examine the operational mechanisms of debt issuance over the booms and busts of the three full economic cycles in our sample period.

5. Methodology and Results

In this section, we elaborate on methodologies and report our results on the optimal use of local debt. For the aggregated analysis, we follow the framework by Barro (1979) and Hou (2013); for individual localities, we conduct a panel data analysis and also a time series analysis for each locality.

5.1 Aggregated Analysis

Following Barro (1979) and Hou (2013), the estimation equation is as follows:

$$\ln\left(\frac{B_t}{B_{t-1}}\right) = \alpha_0 + \alpha_1 \pi_t + \alpha_2 \left[\frac{P_t(G_t - \bar{G}_t)}{\bar{B}_t}\right] - \alpha_3 [\ln(\frac{Y_t}{\bar{Y}_t})(P_t \bar{G}_t + r\bar{B}_t)/\bar{B}_t] + \varepsilon$$

where B_t is the stock of nominal debt at the end of a calendar year *t*, $\bar{B}_t$ is the average amount of debt outstanding during year *t*, π_t is the average anticipated rate of inflation during year *t*, P_t is the average price level for year t, G_t is real federal government expenditure (excluding interest payments on debt stock) during the year, Y_t is the aggregate real income for year t, and $\bar{Y}_t$ is the level of nominal income during year *t*.

Note that these are Barro's (1979) definitions, about some which clarifications are need. First, B_t is the value of a fiscal year – a point Hou (2013) clarifies. Second, since G_t is *real* expenditure, for the terms in the equations $(P_t(G_t - \bar{G}_t)/\bar{B}_t$ and $(P_t\bar{G}_t + r\bar{B}_t)/\bar{B}_t)$ we caution that P_t, price should be at the accumulated level instead of on a yearly basis, (i.e., $P_t = \prod_{i=1}^{t} \frac{P_i}{P_{i-1}} = \frac{P_t}{P_0}$). Barro (1979) also points out that if government expenditures are expected to grow faster than income for some periods, the term $P_t\bar{G}_t/\bar{B}_t$ could be added to pick up the downward effect on the constant.

[Figure 3 about here]

As shown in Figure 3, from year 1950 to 1973, expenditure growth was larger than that of income; thus, during this period the estimation equation is:[17]

$$\ln\left(\frac{B_t}{B_{t-1}}\right) = \alpha_0 + \alpha_1\pi_t + \alpha_2\left[\frac{P_t(G_t - \bar{G}_t)}{\bar{B}_t}\right] - \alpha_3\left[\frac{\ln\left(\frac{Y_t}{\bar{Y}_t}\right)(P_t\bar{G}_t + r\bar{B}_t)}{\bar{B}_t}\right] + \alpha_4\frac{P_t\bar{G}_t}{\bar{B}_t} + \varepsilon$$

Due to data availability, we use direct general expenditure instead of total expenditure for G_t, to cover a multi-year window.[18] For anticipated inflation π_t, we use either the 20-Bond Buyer Index (20BBI) as in Hou (2013) or the implicit price deflator (IPD) as in Barro (1979) in alternate regressions. As indicated in the literature, the average amount of debt outstanding $\bar{B}_t$ is measured as $\bar{B}_t = \sqrt{B_t \cdot B_{t-1}}$. $\bar{G}_t$, and $\bar{Y}_t$ for each locality i is calculated as a time trend using the following formulas:

$$\bar{G}_{it} = \alpha_i + \beta_i t + \gamma_i t^2 + \omega_i t^3 \quad \text{and} \quad \bar{Y}_{it} = \alpha_i + \beta_i t + \gamma_i t^2 + \omega_i t^3$$

where the higher order terms are used to capture the more rapid increase during recent years. Summary statistics for the independent variables are shown in Table 8.

[Table 8 about here]

For expected values of the coefficients, as illustrated in the literature, α_0 would equal the growth rate ρ if the local expenditure and real income grow at the same rate; if government expenditure grows faster than real income, a downward effect $(\rho - \gamma)$ is expected on the constant α_0. The coefficient α_1 of anticipated inflation rate (20BBI or IPD) should equate to

[17] Hou (2013) creates two dummies (years 1950-1975 and 1973-1982) to control for the periods of extraordinary growth in government expenditure and personal income.
[18] There is no big difference when we use total expenditure.

unity with a one-to-one effect on the growth rate of *nominal* debt. Countercyclical debt use indicates that the coefficient of $P_t(G_t - \bar{G}_t)/\bar{B}_t$ should be positive and that the coefficient of $\ln(Y_t/\bar{Y}_t)(P_t\bar{G}_t + r\bar{B}_t)/\bar{B}_t$ should be negative – both below unity, implying that when personal income is below the normal trend, a government tends to issue more debt to increase public expenditure. For $P_t\bar{G}_t/\bar{B}_t$, debt burden – normal government spending relative to debt outstanding, the coefficient should be negative if localities take the debt burden into consideration when they issue new debts.

As to the aggregated analysis of different locality types, the results in Table 9 show that the constant α_0 ranges from 5% to 20%; different locality types have different debt growth rates. Consistent with the results in Figure 1, school districts and special districts have much higher debt growth than other localities, especially from the 1980s to the late 2000s. As expected, the coefficient α_1 is more or less around unity in most regressions. The coefficients α_2 and α_3 are mostly not significant, indicating that changes in debt stock were following the trend, showing little influence from economic fluctuations. For the U.S. local total, the coefficient of $P_t(G_t - \bar{G}_t)/\bar{B}_t$ is even negative, which implies that the local use of debt in aggregate is procyclical – issuing more debt in boom years and less in boom years. Special districts are an exception in that they take debt burden into consideration in making debt issue decisions (negative α_4), while their debt use is not necessarily countercyclical either.

[Table 9 about here]

We then run the same regressions at the aggregated region and state levels. We use the eight census regions. [19] In Table 10, the results indicate that at the aggregated regional level, local governments did not systematically implement any countercyclical debt use policy. Most

[19] The eight census regions are: Far West, Great Lakes, Mid Atlantic, New England, Plains, Rocky Mountains, Southeast, and Southwest.

coefficients α_2 and α_3 are not significant; additionally, some α_2 are significantly negative, and some α_3 are significantly positive (positive α_2 and negative α_3 indicate a countercyclical debt use policy). These results are consistent with the above conclusion that debt use by local governments in the aggregated national level was not countercyclical; some of them were even procyclical. At the aggregated state level, similarly, α_2 and α_3 are not significant for most states. Local governments in some states used debt counter-cyclically (positive α_2 and/or negative α_3); these include Iowa, Michigan, Nebraska, North Carolina, and Wisconsin. Some states, such as Alaska, Florida, Maine, Maryland, West Virginia, and Wyoming, decreased new debt issuance or increased debt retirement when their debt burdens were high (negative α_4).

[Table 10 about here]

5.2 Individual Locality Analysis

For analyses of individual localities, besides the empirical framework in the literature (Barro, 1979; Hou, 2013), we design a different, simplified way to capture countercyclical debt use and investigate the determinants of countercyclical debt policy for counties and cities.

We assume that local revenue and long-term debt outstanding both follow a time trend.[20] Amidst a recession, revenue falls below the trend; whether debt outstanding sits below or above the trend depends on the debt policy of the local government. With a countercyclical debt policy in place, the debt outstanding should be above the trend during recession years; if the debt is

[20] In this subsection, *debt* refers to only long-term debt to fit the proposition of countercyclical debt financing. The data of long-term debt outstanding are not available in our data set; instead, we use the interest payment of long-term debt each year as a proxy for debt outstanding. In calculation, we assume the interest rate is 5%; thus calculated long-term debt outstanding is equal to the next year's interest payment of long-term debt multiplied by 20. For instance, the debt outstanding in fiscal year 2000 would be as follows: $Debt_{2000} = interest_{2001}/5\%$. Note that the assumed interest rate will not influence the results since it is only a fixed multiplier, and as described below, the sign of deviation matters rather than the absolute debt magnitude, especially in the logit regression.

procyclical or the government has no policy whatsoever, the debt outstanding will be below the trend during recession years.

The trend of local revenue and debt outstanding are described as follows:[21]

$$\bar{R}_{it} = \alpha_i + \beta_i t + \gamma_i t^2$$

$$\bar{D}_{it} = \alpha_i + \beta_i t + \gamma_i t^2$$

where $\bar{R}_t$ is the trend of local revenue and $\bar{D}_t$ is the trend of long-term debt outstanding. i is a county or city indicator, and t is year. Figure 4 provides an illustration of the trend of long-term debt outstanding and total local revenue for Fulton County and Augusta City in Georgia. After obtaining the trends based on regressions, deviations are calculated, and the relationships between the deviations are gained from the following regressions:

$$D_{it} - \bar{D}_{it} = \alpha_i^* + \beta_i^* (R_{it} - \bar{R}_{it}).$$

[Figure 4 about here]

As illustrated above, for each locality i, a negative β_i^* implies a countercyclical debt use policy, while a positive β_i^* implies a procyclical policy. We then generate a new dummy variable *counter* that is equal to 1 if β_i^* is positive and 0 if β_i^* is negative. With this *counter*, we are able to identify localities that used a countercyclical debt policy and those that did not. With the dummy as dependent variable, we use logistic regressions to examine the determinants of using a countercyclical debt policy, that is, why some localities use debt countercyclically and others do not. As described in the following equation,

$$counter = \beta_0 + \beta_1 propratio + \beta_2 grantratio + \beta_3 capitalratio + \boldsymbol{\beta_5 X} + \epsilon,$$

[21] The cube term is not included, since the time periods are not very long and the cube terms are often dropped in the regression. This time trend regression is somewhat simple here. We may try Kernel density estimation in the next step. Also, another effective way is to regress the equation $\ln(\bar{R}_{it}) = \alpha_i + \beta_i t$ which we use as a robustness check, and the results are similar.

the independent variables include the percentage of property tax in own source revenue ($propratio$); the percentage of intergovernmental grants in total revenue ($grantratio$); the percentage of public works expenditure in total expenditures (capital expenditure ratio, $capitalratio$); and the vector $\boldsymbol{X}$ are control variables**,** including per capita property tax (in 1000), per capita infrastructure expenditure (in 1000), per capita expenditure (in 1000), per capita personal income (in 1000), per capita income square, unemployment rate, infrastructure expenditure as a ratio of total local expenditure, fund balance ratio measured as surplus as a ratio of revenue, debt burden measured as debt outstanding as a ratio of total revenue, population, population growth, and form of government.[22] Table 11 provides summary statistics of relevant variables. For three different variables of *counter*, all averages are larger than 0.5; the data indicate that more counties used debt procyclically rather than countercyclically. Especially when local revenue is below the trend in bust years, about two thirds of Georgia counties decreased their long-term debt to below the trend.

[Table 11 around here]

For counties in Georgia, the regression results regarding the determinants of countercyclical debt use are shown in Table 12. The results illustrate the fiscal behaviors of local governments. Note that for the binary dependent variable, 0 indicates a negative relationship between revenue and debt (countercyclical) and 1 indicates a positive relationship (procyclical). Thus, in these regressions, negative coefficients indicate a higher probability of debt use for countercyclical purposes. The coefficients provided in the table are marginal effects based on logistic regressions.

[22] In terms of control variables, for both counties and municipalities, per capita property tax, per capita infrastructure expenditure, and per capita expenditure are highly correlated; thus, only per capita infrastructure expenditure is included in the regression. The forms of government (for counties) includes: (1) commission, (2) council-administrator, (3) council-manager, and (4) council-elected executive.

[Table 12 around here]

According to Table 12, during the entire period, counties with higher property tax ratios are less likely to use debt countercyclically. The reason may be that these counties have a more stable revenue that is less influenced by economic fluctuation; thus, their debt stock increases in boom years and decreases in recession years. When revenue is above the trend (i.e., in boom years), the coefficients for the property tax ratios are not significant. When revenue is below the trend (i.e., in recession years), the effects of non-metro counties dominate the coefficients. Non-metro counties with a higher property tax ratio suffer less from recessions; they may not need to increase their debt as much as other counties to compensate for the decreased expenditures. The property tax ratio indicates the stability of local revenue, while other own revenue sources are much more volatile. Since other revenue sources are more procyclical than property tax, counties with a lower property tax ratio are under more stress and need to borrow more to offset revenue loss. This further confirms that counties with volatile revenue sources need to resort more to countercyclical debt.

Most debt, especially long-term debt, is used for capital expenditure. Thus, the coefficient of capital expenditure as a ratio of local expenditure is statistically significant. While infrastructure investments are often initiated in boom years, which render the positive coefficients (procyclical) in the third column, in bust years, the coefficient shows that the larger the share of infrastructure investment the more likely it is that a county uses countercyclical debt. The results imply that if counties use countercyclical debt, infrastructure expenditure is an appropriate path to use for these debts to combat fiscal stress. In this sense, non-metro counties with a potential for more infrastructure investment are in a better position to use debt for countercyclical purposes. The combination of the procyclical use of infrastructure investment in boom years and countercyclical use in bust years explains the rapid accumulation of debt outstanding. The coefficients of infrastructure investment indicate that counties were borrowing more in recession years but did not retire these debts even after their fiscal condition recovers, causing an overburden of debt. The consistently positive coefficients of debt burden remind us that a high debt burden does impede the use of countercyclical debt. The reason may be that counties with high debt burdens do not have the fiscal/debt capacity to borrow more in recession years.

Municipalities are telling a different story. As shown in Table 13, infrastructure investment and fund balance are two important factors of countercyclical debt policy. Property tax is not statistically significant for the use of countercyclical debt since property tax is only around 10% of own source revenue in municipalities as shown in Table 11. Consistently, the ratio of infrastructure expenditure plays a very important role in using countercyclical debt. Counties with higher ratios of infrastructure investment in total expenditures are more likely to use debt countercyclically. The effects of fund balance are interesting in that the coefficient is negative in boom years, i.e., municipalities are retiring more debt when they have a high fund balance; while the coefficient is positive in bust years, indicating that municipalities are still using high fund balances to pay off debt even during a recession. The latter phenomenon highlights the much stronger fiscal capacity of municipalities than counties.

[Table 13 around here]

The difference in the determinants of countercyclical debt use for counties versus cities further reminds us of the possible invalidity and bias in aggregated analysis. Though the signs of coefficients have switched back and forth between positive and negative, we can explain the internal reasons for countercyclical debt uses on the basis of two contradictory factors. One is fiscal capacity, and the other is borrowing cost. In recession years, if localities use countercyclical debt, they first need to have enough fiscal capacity for more debt, and second, to take borrowing cost into consideration. The more fiscal capacity localities have the lower the borrowing cost (interest payment) and, also, the lower the need for debt. Therefore, during recession years, localities with higher property tax ratios may have enough fiscal capacity or credentials to borrow more debt for countercyclical use, but meanwhile, their demand for expenditure smoothing is smaller, since their revenue structure is more stable. The sign of the coefficient will depend on the dominance of the two factors. According to the regression results, counties with a higher property tax ratio have a smaller probability of using countercyclical debt, mostly because they do not need to. The fact that non-metro counties use more countercyclical debt confirms this conclusion since they have higher demands. In contrast, cities with higher infrastructure investment ratios are more likely to use countercyclical debt because they stand in a better position to use countercyclical debt to achieve two goals simultaneously: to combat recession and invest in infrastructure. With better fiscal capacities, municipalities can do a better job maintaining appropriate fund balances; thus, they can do more about expenditure smoothing to meet city residents' demands to keep the qualities and quantities of public services consistent. Infrastructure expenditure is still the first alternative to using debt. When the share of infrastructure expenditure is high, localities have more potential room for countercyclical debt use; however, when infrastructure expenditure is low, there is less room to use countercyclical debt and, thus, smooth expenditure.

Regarding the effects of counter-cyclical debt on expenditure stability. The brief statistics in Table 14 provide strong evidence that counter-cyclical debt effectively decreased expenditure volatility. The results indicates that localities tend to use countercyclical debt when revenue is in larger deviations, i.e., during big recessions or unexpected boom years. The effects of counter-cyclical debt are evident. For localities using countercyclical debt, they have much smaller expenditure gaps even when they keep larger revenue gaps, indicating that counter-cyclical debt effectively lowered expenditure volatility. *Difference* in the table measures the difference of expenditure gap and revenue gap for each type of local governments. Obviously, localities using counter-cyclical debt possess much bigger differences, especially during bust years. Difference-in-difference (DID) is calculated by the difference between expenditure gap and revenue gap for localities using countercyclical debt minus the difference for other localities. In this context, countercyclical debt is mainly used to mitigate expenditure volatility given revenue fluctuations. We conduct left side t-tests to investigate whether the effects of countercyclical debt on decreased expenditure volatility are statistically significant. The results indicate that it is very effective in reducing expenditure volatility, especially in bust years when counties and cities using counter-cyclical debt reduced their expenditure gaps by 8.8% and 4.4%, respectively.[23]

With regard to the robustness of methodology issues, one concern is that, sometimes, the deviations of local revenue from the trend are not necessarily due to economic fluctuations, but may be due to random reasons, such as incorrect revenue forecasting or temporary grant changes. As a robustness check, we calculate the relationship between deviations of local revenue and deviations of debt outstanding only for those localities in which the local revenue was below or above the trend for at least three consecutive years. We can be confident that a deviation for at least three consecutive years is not due to a random change but to systematic fluctuations. The results are very similar to the regressions covering all observations.

[23] For the statistical validity, there is a possible selection bias that in many other fiscal perspectives, localities that are using counter-cyclical debt may be different from those that are not. More details can be found in the next chapter by running several complete regressions including comprehensive control variables. The results are similar as described here.

6. Conclusion and Discussion

In this paper, we have constructed a theoretical framework of an optimal debt path over the economic cycle and illustrate how countercyclical debt use may smooth local expenditure. We described different scenarios of countercyclical debt use to derive a comprehensive scenario of the optimal debt policy. We assigned different values for parameters in the framework, and ran simulations. Our results are intuitive and show that a countercyclical debt policy is feasible for localities and will not induce unacceptable debt burden even in a deep recession, partly because the consideration of fiscal stress in the retirement of countercyclical debt will force localities to choose appropriate level of borrowing to fight the recession. This framework provides an important guidance for localities and can be used by government official as long as they have a good estimate of the parameters.

The empirical results indicate that different localities assume distinct debt policies due to two key elements – fiscal capacity and borrowing cost. Localities with higher fiscal capacity have decent debt-paying ability but lower need to borrow, whereas localities with lower fiscal capacity face higher need to borrow but have to incur higher interest cost because of lower credit rating. These two sides accompanied with the regression results tell us that counties design debt policies mainly conditional on their debt-paying ability, while municipalities make debt policies based on their need for borrowing since they usually have strong fiscal capacity. Thus, municipalities use countercyclical debt policies more often than counties, and municipalities and townships have a much higher per capita debt outstanding than counties. The effect of countercyclical debt policy is also evident: *T* tests show that localities with countercyclical debt policy have much lower coefficient of variation in per capita expenditure than those that do not, and the difference is significant at 1% level.

This paper contributes to the literature by theoretically calculating the optimal path of countercyclical debt use in the context of different values of parameters and empirically extend the analyses from national and subnational levels to the local level. The property tax, as the main own-source revenue for localities, has a determining impact on countercyclical debt policy. The effect of property tax highlights the importance of the stability of revenue sources for local governments. A policy implication from this study is that local governments should compose and implement appropriate debt policies on the basis of their distinct revenue structure. To effectively implement countercyclical debt policy, localities should improve their revenue structure to achieve higher fiscal flexibility.

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Table 1: Different amounts of countercyclical debt use

Year	Revenue	Option 1		Option 2 (optimal)		Option 3	
		Expenditure	**Debt ***	**Expenditure**	**Debt**	**Expenditure**	**Debt**
0	100	100	0	100	0	100	0
1	90	95	6	**97.5**	**7.5**	100	10
2	90	95	6	**97.5**	**7.5**	100	10
3	100	98	-2	**97.5**	**-2.5**	96.67	-3.33
4	100	98	-2	**97.5**	**-2.5**	96.67	-3.33
5	100	98	-2	**97.5**	**-2.5**	96.67	-3.33
6	100	98	-2	**97.5**	**-2.5**	96.67	-3.33
7	100	98	-2	**97.5**	**-2.5**	96.67	-3.33
8	100	98	-2	**97.5**	**-2.5**	96.67	-3.33

Notes: * Debt here refers only to new debt used as a countercyclical tool, which excludes normal debt use in local finance. The same definition applies throughout the paper.

Table 2: Government revenue and expenditure for each year with countercyclical debt use

Year in a full cycle	Revenue	Expenditure	Debt	Debt outstanding (at the beginning of year)
1	$R_0(1+\mu)$	$R_0(1+\mu)+d_1$	d_1	0
2	$R_0(1+\mu)^2$	$R_0(1+\mu)^2+d_2-r^*d_1$	d_2	d_1
3	$R_0(1+\mu)^2(1+\rho)$	$R_0(1+\mu)^2(1+\rho)+d_3-r^*(d_1+d_2)$	d_3	d_1+d_2
4	$R_0(1+\mu)^2(1+\rho)^2$	$\dots$	d_4	$\dots$
5	$R_0(1+\mu)^2(1+\rho)^3$	$\dots$	d_5	$\dots$
6	$R_0(1+\mu)^2(1+\rho)^4$	$\dots$	d_6	$\dots$
7	$R_0(1+\mu)^2(1+\rho)^5$	$\dots$	d_7	$\dots$
8	$R_0(1+\mu)^2(1+\rho)^6$	$R_0(1+\mu)^2(1+\rho)^6+d_8-r^*\sum_{i=1}^{7}d_i$	d_8	$\sum_{i=1}^{7}d_i$

Table 3: Optimal debt use with assumptions relaxed

Panel A: discounter factor r = 0					
Year	**Revenue**	**Growth rate**	**Expenditure**	**Debt**	**Debt/Rev$_{t-1}$**
0	100		100	0	0
1	95.00	-5%	98.32	**3.32**	**3.3%**
2	90.25	-5%	98.32	**8.07**	**8.5%**
3	92.96	3%	98.32	**5.36**	**5.9%**
4	95.75	3%	98.32	**2.57**	**2.8%**
5	98.62	3%	98.32	**-0.30**	**-0.3%**
6	101.58	3%	98.32	**-3.26**	**-3.3%**
7	104.62	3%	98.32	**-6.31**	**-6.2%**
8	107.76	3%	98.32	**-9.45**	**-9.0%**
Panel A: discounter factor r = 1%					
Year	**Revenue**	**Growth rate**	**Expenditure**	**Debt**	**Debt/Rev$_{t-1}$**
0	100		100	0	0
1	95.00	-5%	101.78	**6.78**	**6.8%**
2	90.25	-5%	100.77	**10.52**	**11.1%**
3	92.96	3%	99.77	**6.81**	**7.5%**
4	95.75	3%	98.78	**3.04**	**3.3%**
5	98.62	3%	97.80	**-0.81**	**-0.9%**
6	101.58	3%	96.84	**-4.74**	**-4.8%**
7	104.62	3%	95.88	**-8.75**	**-8.6%**
8	107.76	3%	94.93	**-12.84**	**-12.3%**

Table 4: Simulation of countercyclical debt uses

Scenarios			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Parameters	**Revenue fall**	μ	-5%	-5%	-5%	-5%	-5%	-10%	-10%	-10%	-10%	-10%
	Growth rate	ρ	1%	2%	3%	4%	3%	1%	2%	3%	4%	3%
	Discount factor	r	0%	0%	0%	0%	1%	0%	0%	0%	0%	1%
Years	**1**	d_1	-1.7%	0.7%	3.3%	6.0%	6.8%	-5.7%	-3.5%	-1.2%	1.2%	2.0%
	2	d_2	3.2%	5.8%	8.5%	11.3%	11.1%	3.7%	6.1%	8.7%	11.4%	11.2%
	3	d_3	2.3%	4.1%	5.9%	7.9%	7.5%	3.1%	4.8%	6.7%	8.6%	8.3%
	4	d_4	1.3%	2.0%	2.8%	3.6%	3.3%	2.0%	2.7%	3.5%	4.3%	4.0%
	5	d_5	0.3%	0.0%	-0.3%	-0.6%	-0.9%	1.0%	0.7%	0.4%	0.1%	-0.2%
	6	d_6	-0.7%	-2.0%	-3.3%	-4.5%	-4.8%	0.0%	-1.3%	-2.6%	-3.9%	-4.1%
	7	d_7	-1.7%	-4.0%	-6.2%	-8.4%	-8.6%	-1.0%	-3.3%	-5.6%	-7.7%	-8.0%
	8	d_8	-2.7%	-5.9%	-9.0%	-12.0%	-12.3%	-2.0%	-5.3%	-8.4%	-11.4%	-11.7%

Notes: * The values here refer to newly issued countercyclical debt as a percentage of previous year revenue, d_t/rev_{t-1}.

Table 5: Net debt increase (above the trend) with countercyclical debt use

t (year)	1	2	3	4	5	6	7	8
d_t (net debt increase)	$\frac{1}{2}D$	$\frac{1}{2}D$	$-\frac{1}{6}D$	$-\frac{1}{6}D$	$-\frac{1}{6}D$	$-\frac{1}{6}D$	$-\frac{1}{6}D$	$-\frac{1}{6}D$
D_t (outstanding)	$\frac{1}{2}D$	D	$\frac{5}{6}D$	$\frac{4}{6}D$	$\frac{3}{6}D$	$\frac{2}{6}D$	$\frac{1}{6}D$	0

Table 6: Comparison between optimal debt uses with and without interest

Year	Rev	Optimal debt use			Optimal debt use (with interest)			
		Exp	Debt	Debt/Rev$_{t-1}$	Exp	Debt	Debt/Rev$_{t-1}$	Debt outstanding
0	100	100	0	0	100	0	0	0
1	90	97.5	**7.5**	**7.5%**	97.12	**7.12**	**7.1%**	0.00
2	90	97.5	**7.5**	**7.7%**	97.12	**7.48**	**8.3%**	7.12
3	100	97.5	**-2.5**	**-2.6%**	97.12	**-2.15**	**-2.4%**	14.60
4	100	97.5	**-2.5**	**-2.6%**	97.12	**-2.25**	**-2.3%**	12.46
5	100	97.5	**-2.5**	**-2.6%**	97.12	**-2.37**	**-2.4%**	10.20
6	100	97.5	**-2.5**	**-2.6%**	97.12	**-2.49**	**-2.5%**	7.83
7	100	97.5	**-2.5**	**-2.6%**	97.12	**-2.61**	**-2.6%**	5.35
8	100	97.5	**-2.5**	**-2.6%**	97.12	**-2.74**	**-2.7%**	2.74

Table 7: Scenarios of revenue and expenditure with optimal countercyclical debt use[24]

Scenarios			Recession=1 year				Recession=2 years			
Parameters		μ ρ r r^*	**-5%** **3%** **0%** **5%**				**-5%** **3%** **0%** **5%**			
			Revenue	Expenditure	d_t	d_t/Rev_{t-1}	Revenue	Expenditure	d_t	d_t/Rev_{t-1}
Years	**0**		100	100	0	0	100	100	**0**	**0**
	1	d_1	**95.00**	104.80	**9.80**	**9.8%**	**95.00**	97.75	**2.75**	**2.7%**
	2	d_2	97.85	104.80	**7.44**	**7.8%**	**90.25**	97.75	**7.64**	**8.0%**
	3	d_3	100.79	104.80	**4.88**	**5.0%**	92.96	97.75	**5.31**	**5.9%**
	4	d_4	103.81	104.80	**2.10**	**2.1%**	95.75	97.75	**2.79**	**3.0%**
	5	d_5	106.92	104.80	**-0.91**	**-0.9%**	98.62	97.75	**0.05**	**0.1%**
	6	d_6	110.13	104.80	**-4.16**	**-3.9%**	101.58	97.75	**-2.90**	**-2.9%**
	7	d_7	113.43	104.80	**-7.68**	**-7.0%**	104.62	97.75	**-6.09**	**-6.0%**
	8	d_8	116.84	104.80	**-11.46**	**-10.1%**	107.76	97.75	**-9.54**	**-9.1%**
$D = Sum\ (d_t)$					**0**				**0**	
Scenarios			**Recession=1 year**				**Recession=2 years**			
Parameters		μ ρ r r^*	**-10%** **3%** **0%** **5%**				**-10%** **3%** **0%** **5%**			
			Revenue	Expenditure	d_t	d_t/Rev_{t-1}	Revenue	Expenditure	d_t	d_t/Rev_{t-1}
Years	**0**		100	100	0	0	100	100	**0**	**0**
	1	d_1	**90.00**	99.29	**9.29**	**9.3%**	**90.00**	88.43	**-1.57**	**-1.6%**
	2	d_2	92.70	99.29	**7.05**	**7.8%**	**81.00**	88.43	**7.35**	**8.2%**
	3	d_3	95.48	99.29	**4.62**	**5.0%**	83.43	88.43	**5.29**	**6.5%**
	4	d_4	98.35	99.29	**1.99**	**2.1%**	85.93	88.43	**3.05**	**3.7%**
	5	d_5	101.30	99.29	**-0.86**	**-0.9%**	88.51	88.43	**0.62**	**0.7%**
	6	d_6	104.33	99.29	**-3.95**	**-3.9%**	91.17	88.43	**-2.00**	**-2.3%**
	7	d_7	107.46	99.29	**-7.27**	**-7.0%**	93.90	88.43	**-4.84**	**-5.3%**
	8	d_8	110.69	99.29	**-10.86**	**-10.1%**	96.72	88.43	**-7.90**	**-8.4%**
$D = Sum\ (d_t)$					**0**				**0**	

Notes: The parameters are: μ, revenue fall; ρ, growth rate; r, discount factor; r^*, interest rate.

[24] See more comparisons in Appendix Table A1.

Appendix Table A1: Optimal countercyclical debt uses in different scenarios

Panel (1): recession=2 years

Scenarios			**(1)**		**(2)**		**(3)**		**(4)**		**(5)**		**(6)**	
Parameters		μ	-5%		-5%		-5%		-10%		-10%		-10%	
		ρ	2%		2%		3%		2%		3%		3%	
		r	0%		0%		0%		0%		0%		1%	
		r^*	4%		5%		5%		4%		5%		5%	
			d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}
Years	**1**	d_1	0.47	0.5%	0.41	0.4%	2.75	2.7%	**-3.64**	**-3.6%**	**-1.57**	**-1.6%**	1.32	1.3%
	2	d_2	5.24	5.5%	5.18	5.5%	7.64	8.0%	5.22	5.8%	7.35	8.2%	9.48	10.5%
	3	d_3	3.65	4.0%	3.63	4.0%	5.31	5.9%	3.81	4.7%	5.29	6.5%	6.63	8.2%
	4	d_4	1.95	2.1%	1.97	2.1%	2.79	3.0%	2.31	2.8%	3.05	3.7%	3.57	4.3%
	5	d_5	0.15	0.2%	0.19	0.2%	0.05	0.1%	0.71	0.8%	0.62	0.7%	0.28	0.3%
	6	d_6	-1.76	-1.8%	-1.71	-1.8%	-2.90	-2.9%	-0.98	-1.1%	-2.00	-2.3%	-3.24	-3.7%
	7	d_7	-3.78	-3.9%	-3.75	-3.8%	-6.09	-6.0%	-2.77	-3.2%	-4.84	-5.3%	-7.01	-7.7%
	8	d_8	-5.93	-5.9%	-5.93	-6.0%	-9.54	-9.1%	-4.67	-5.2%	-7.90	-8.4%	-11.03	-11.8%
Sum (d_t)			**0**		**0**		**0**		**0**		**0**		**0**	

Notes: The parameters are: μ, revenue fall; ρ, growth rate; r, discount factor; r^*, interest rate.

Panel (2): recession=1 year

Scenarios			(1)		(2)		(3)		(4)		(5)		(6)	
Parameters		μ	-5%		-5%		-5%		-10%		-10%		-10%	
		ρ	2%		2%		3%		2%		3%		3%	
		r	0%		0%		0%		0%		0%		1%	
		r^*	4%		5%		5%		4%		5%		5%	
			d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}	d_t	d_t/Rev_{t-1}
Years	**1**	d_1	6.51	6.5%	6.41	6.4%	9.80	9.8%	6.17	6.2%	9.29	9.3%	12.54	12.5%
	2	d_2	4.87	5.1%	4.83	5.1%	7.44	7.8%	4.61	5.1%	7.05	7.8%	9.46	10.5%
	3	d_3	3.13	3.2%	3.13	3.2%	4.88	5.0%	2.96	3.2%	4.62	5.0%	6.14	6.6%
	4	d_4	1.27	1.3%	1.31	1.3%	2.10	2.1%	1.21	1.3%	1.99	2.1%	2.57	2.7%
	5	d_5	-0.69	-0.7%	-0.64	-0.6%	-0.91	-0.9%	-0.66	-0.7%	-0.86	-0.9%	-1.25	-1.3%
	6	d_6	-2.78	-2.7%	-2.73	-2.7%	-4.16	-3.9%	-2.63	-2.7%	-3.95	-3.9%	-5.34	-5.3%
	7	d_7	-4.98	-4.8%	-4.96	-4.7%	-7.68	-7.0%	-4.72	-4.8%	-7.27	-7.0%	-9.72	-9.3%
	8	d_8	-7.32	-6.8%	-7.35	-6.9%	-11.46	-10.1%	-6.94	-6.8%	-10.86	-10.1%	-14.40	-13.4%
Sum (d_t)			**0**		**0**		**0**		**0**		**0**		**0**	

Table 8: Summary statistics of values of independent variables

Year	π (20BBI)	π (IPD)	$\ln\left(\frac{Y}{\bar{\bar{Y}}}\right)$	$P\bar{G}/\bar{B}$	$P(G-\bar{G})/\bar{B}$	$\ln\left(\frac{Y}{\bar{\bar{Y}}}\right)P(\bar{G}+rB)/\bar{B}$	$G/\bar{Y}$
1955	0.025	0.021	-0.046	0.850	-0.127	-0.039	0.068
1956	0.028	0.063	-0.028	0.819	-0.112	-0.023	0.070
1957	0.033	0.043	-0.044	0.816	-0.105	-0.036	0.072
1961	0.035	0.027	-0.079				0.081
1962	0.032	0.026	-0.062	0.731	-0.033	-0.046	0.083
1963	0.032	0.023	-0.061	0.716	-0.030	-0.044	0.084
1964	0.032	0.019	-0.038	0.712	-0.027	-0.027	0.085
1965	0.033	0.030	-0.011	0.711	-0.017	-0.008	0.087
1966	0.038	0.048	0.009	0.711	0.006	0.006	0.091
1967	0.039	0.051	0.015	0.722	0.023	0.011	0.093
1968	0.044	0.058	0.035	0.746	0.022	0.026	0.094
1969	0.057	0.068	0.041	0.761	0.058	0.031	0.099
1970	0.063	0.081	0.028	0.768	0.077	0.022	0.102
1971	0.055	0.077	0.029	0.767	0.120	0.023	0.108
1972	0.053	0.064	0.061	0.755	0.165	0.046	0.115
1973	0.052	0.066	0.083	0.773	0.140	0.064	0.112
1974	0.062	0.110	0.044	0.823	0.102	0.037	0.107
1975	0.070	0.096	0.015	0.867	0.121	0.013	0.110
1976	0.066	0.048	0.029	0.906	0.142	0.026	0.112
1977	0.057	0.063	0.038	0.938	0.114	0.036	0.109
1978	0.060	0.065	0.054	0.978	0.076	0.053	0.105
1979	0.065	0.088	0.032	1.059	0.031	0.033	0.101
1980	0.086	0.108	-0.013	1.135	-0.032	-0.015	0.096
1981	0.114	0.100	-0.025	1.187	-0.079	-0.030	0.093
1982	0.116	0.061	-0.043	1.187	-0.098	-0.051	0.092
1983	0.095	0.044	-0.038	1.131	-0.097	-0.043	0.092
1984	0.101	0.043	-0.007	1.092	-0.096	-0.007	0.092
1985	0.091	0.041	-0.002	1.048	-0.076	-0.002	0.094
1986	0.073	0.029	0.008	0.972	-0.035	0.008	0.098
1987	0.077	0.047	0.005	0.914	-0.014	0.005	0.101
1988	0.077	0.033	0.013	0.910	-0.012	0.012	0.101
1989	0.072	0.048	0.014	0.941	-0.013	0.013	0.102
1990	0.073	0.050	-0.003	0.962	-0.006	-0.003	0.103
1991	0.069	0.035	-0.033	0.971	-0.004	-0.032	0.103
1992	0.064	0.036	-0.027	0.974	-0.006	-0.026	0.104
1993	0.056	0.026	-0.041	0.984	-0.019	-0.041	0.103
1994	0.062	0.027	-0.038	0.990	-0.036	-0.037	0.101
1995	0.060	0.027	-0.037	1.000	-0.033	-0.037	0.102
1996	0.058	0.023	-0.030	1.019	-0.042	-0.031	0.101

Year	π (20BBI)	π (IPD)	$\ln\left(\frac{Y}{\bar{\bar{Y}}}\right)$	$P\bar{G}/\bar{B}$	$P(G-\bar{G})/\bar{B}$	$\ln\left(\frac{Y}{\bar{\bar{Y}}}\right)P(\bar{G}+rB)/\bar{B}$	$G/\bar{Y}$
1997	0.055	0.022	-0.017	1.014	-0.041	-0.018	0.101
1998	0.051	0.020	0.015	1.002	-0.025	0.015	0.103
1999	0.054	0.040	0.018	0.995	-0.013	0.018	0.105
2000	0.057	0.048	0.040	0.994	-0.012	0.040	0.105
2001	0.052	0.035	0.025	0.984	0.002	0.024	0.107
2002	0.050	0.021	0.005	0.954	0.022	0.004	0.109
2003	0.047	0.035	-0.008	0.938	0.022	-0.007	0.110
2004	0.047	0.051	0.000	0.922	0.016	0.000	0.109
2005	0.044	0.057	-0.003	0.915	0.004	-0.003	0.108
2006	0.044	0.050	0.014	0.917	0.003	0.013	0.108
2007	0.044	0.052	0.017	0.899	0.024	0.015	0.111
2008	0.049	0.051	-0.016	0.888	0.023	-0.014	0.111

Table 9: Results of aggregated analysis of U.S. totals for different locality types

π (20BBI) α_1		**π (IPD)** α_1		$P(G-\overline{G})/\overline{B}$ α_2		$\ln\left(\frac{Y}{\overline{Y}}\right) P(\overline{G}+rB)/\overline{B}$ α_3		$P\overline{G}/\overline{B}$ α_4		**Constant** α_0		**Obs.**	R^2
US local total													
0.509***	(0.188)			-0.0513	(0.0794)	0.150	(0.173)			0.0404***	(0.0121)	47	0.191
0.810***	(0.258)			-0.101	(0.0834)	0.172	(0.170)	-0.0769	(0.0463)	0.0893***	(0.0318)	47	0.241
		0.432**	(0.168)	-0.182**	(0.0783)	0.150	(0.174)			0.0511***	(0.00881)	47	0.179
		0.457**	(0.186)	-0.201**	(0.0966)	0.156	(0.177)	-0.0126	(0.0375)	0.0610*	(0.0306)	47	0.182
US counties													
0.975*	(0.490)			-0.122	(0.212)	0.106	(0.489)			0.0364	(0.0364)	21	0.250
1.043**	(0.444)			-0.219	(0.197)	0.338	(0.454)	0.0918**	(0.0419)	-0.0863	(0.0649)	21	0.423
		0.964*	(0.522)	-0.283	(0.211)	0.216	(0.493)			0.0551*	(0.0295)	21	0.230
		0.616	(0.663)	-0.321	(0.217)	0.348	(0.519)	0.0513	(0.0595)	0.00706	(0.0631)	21	0.264
US cities and townships													
-0.0901	(0.344)			-0.151	(0.236)	-0.331	(0.563)			0.0863***	(0.0250)	21	0.114
-0.432	(1.042)			-0.191	(0.267)	-0.244	(0.630)	0.0898	(0.257)	0.0487	(0.111)	21	0.121
		-0.421	(0.349)	-0.0601	(0.231)	-0.385	(0.543)			0.102***	(0.0198)	21	0.180
		-0.531	(0.412)	-0.0338	(0.241)	-0.331	(0.564)	0.0514	(0.0964)	0.0728	(0.0583)	21	0.195
US school districts													
-2.220***	(0.446)			-0.0725	(0.0716)	-0.198	(0.160)			0.214***	(0.0301)	22	0.610
-3.040***	(0.444)			0.0312	(0.0666)	-0.356**	(0.139)	0.0357***	(0.0112)	0.175***	(0.0273)	22	0.756
		-1.427**	(0.503)	0.122	(0.0795)	-0.234	(0.205)			0.146***	(0.0289)	22	0.359
		-1.439**	(0.511)	0.0758	(0.106)	-0.185	(0.220)	-0.00984	(0.0146)	0.172***	(0.0482)	22	0.376
US special districts													
0.591	(0.825)			0.720	(1.007)	0.552	(3.414)			0.0440	(0.0622)	21	0.142
-0.540	(0.884)			**1.790***	**(1.011)**	-3.703	(3.559)	**-1.214****	**(0.525)**	0.343**	(0.141)	21	0.357
		1.174	(0.760)	0.522	(0.865)	-0.445	(2.924)			0.0227	(0.0446)	21	0.225
		0.625	(0.764)	0.988	(0.841)	-2.110	(2.857)	**-0.892***	**(0.468)**	0.212*	(0.108)	21	0.368

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 10: Results of aggregated analysis of local totals, by Census region and state

Region / State	**π (IPD)** α_1		**$P(G-\bar{G})/\bar{B}$** α_2		**$\ln\left(\frac{Y}{\bar{Y}}\right)P(\bar{G}+rB)/\bar{B}$** α_3		**$P\bar{G}/\bar{B}$** α_4		**Constant** α_0		**Obs.**	**R^2**
Region												
Far West	0.222	(0.555)	-0.142	(0.0975)	0.0650	(0.248)	0.0531	(0.0837)	0.0134	(0.0576)	43	0.160
Great Lakes	-0.394	(0.332)	0.0743	(0.0947)	-0.116	(0.145)	0.0350	(0.0274)	0.0465**	(0.0212)	43	0.053
Mid Atlantic	0.366	(0.562)	-0.132	(0.0937)	0.447**	(0.196)	-0.00879	(0.0691)	0.0464	(0.0360)	43	0.125
New England	0.341	(0.290)	-0.0530	(0.0598)	0.288***	(0.0892)	-0.0235	(0.0219)	0.0626**	(0.0243)	43	0.233
Plains	0.205	(0.349)	0.200	(0.202)	-0.0859	(0.286)	-0.134	(0.0943)	0.172*	(0.0872)	43	0.132
Rocky Mountains	2.681***	(0.635)	0.0356	(0.248)	-0.170	(0.366)	0.224**	(0.0961)	-0.263**	(0.106)	43	0.439
Southeast	0.920***	(0.305)	-0.306*	(0.156)	0.577*	(0.322)	-0.0233	(0.0667)	0.0421	(0.0466)	43	0.263
Southwest	0.240	(0.358)	0.0134	(0.294)	1.094***	(0.303)	-0.0720	(0.0892)	0.111*	(0.0624)	43	0.539
State												
Alabama	-0.394	(0.731)	0.299	(0.398)	-0.302	(1.264)	-0.00953	(0.0326)	0.0992**	(0.0453)	43	0.026
Alaska	-1.164	(1.294)	-0.624**	(0.269)	1.578**	(0.670)	**-0.216****	**(0.0874)**	0.230**	(0.0953)	43	0.209
Arizona	1.337*	(0.743)	-0.0775	(0.283)	1.223	(0.783)	-0.0756	(0.0624)	0.0425	(0.0550)	43	0.232
Arkansas	-1.250	(1.081)	1.008	(0.679)	-1.518	(1.228)	-0.0411	(0.0489)	0.171**	(0.0720)	43	0.076
California	1.383**	(0.571)	-0.375**	(0.176)	0.295	(0.341)	-0.00882	(0.0322)	0.00275	(0.0322)	43	0.198
Colorado	1.170*	(0.685)	0.193	(0.301)	0.236	(0.455)	-0.0229	(0.0628)	0.0268	(0.0467)	43	0.160
Connecticut	0.0516	(0.368)	0.0810	(0.101)	0.281	(0.174)	-0.00947	(0.0108)	0.0526**	(0.0242)	43	0.103
Delaware	-1.117	(0.995)	0.106	(0.371)	-1.475	(0.961)	-0.0435	(0.0414)	0.148**	(0.0679)	43	0.094
D. C.	-1.487	(0.989)	0.0170	(0.326)	0.437	(0.619)	**-0.103****	**(0.0490)**	0.236***	(0.0731)	43	0.226
Florida	2.310***	(0.551)	-0.174	(0.149)	1.235*	(0.633)	**-0.0723***	**(0.0416)**	-0.0170	(0.0366)	43	0.365
Georgia	0.741	(0.533)	0.0299	(0.154)	0.425	(0.404)	-0.0256	(0.0287)	0.0540	(0.0357)	43	0.091
Hawaii	-1.355	(1.449)	0.489	(0.330)	0.0715	(0.814)	0.168*	(0.0885)	0.0835	(0.0697)	43	0.152
Idaho	1.079	(0.797)	0.0538	(0.216)	0.254	(0.299)	0.0180	(0.0168)	-0.0270	(0.0538)	43	0.115
Illinois	-0.136	(0.487)	0.348	(0.289)	-0.452	(0.368)	0.0438*	(0.0225)	0.0394	(0.0271)	43	0.159
Indiana	-0.225	(0.760)	0.438	(0.370)	-0.131	(0.439)	0.0155	(0.0302)	0.0757	(0.0486)	43	0.037
Iowa	-0.564	(0.820)	**1.045****	**(0.449)**	-0.425	(0.324)	-0.0220	(0.0256)	0.111*	(0.0551)	43	0.154
Kansas	1.313*	(0.702)	1.251	(0.939)	1.161	(0.772)	-0.0395	(0.0544)	0.00190	(0.0513)	43	0.222

Region / State	π (IPD) α_1		$P(G-\overline{G})/\overline{B}$ α_2		$\ln\left(\frac{Y}{\overline{Y}}\right) P(\overline{G}+rB)/\overline{B}$ α_3		$P\overline{G}/\overline{B}$ α_4		Constant α_0		Obs.	R^2
Kentucky	-0.220	(0.634)	0.758	(1.220)	0.701	(1.086)	-0.144	(0.0951)	0.137***	(0.0471)	43	0.100
Louisiana	0.202	(0.564)	0.283	(0.495)	0.770	(0.485)	-0.0374	(0.0248)	0.0653*	(0.0379)	43	0.268
Maine	0.464	(0.665)	0.0692	(0.279)	0.605	(0.394)	**-0.0466****	**(0.0226)**	0.0833*	(0.0455)	43	0.181
Maryland	0.934***	(0.329)	0.0276	(0.126)	-0.0647	(0.274)	**-0.0489*****	**(0.0142)**	0.0263	(0.0228)	43	0.381
Massachusetts	0.430	(0.406)	-0.250	(0.182)	0.512**	(0.224)	-0.0244	(0.0205)	0.0460*	(0.0266)	43	0.156
Michigan	-1.509***	(0.531)	**1.077*****	**(0.386)**	**-0.590***	**(0.316)**	0.00723	(0.0220)	0.160***	(0.0330)	43	0.257
Minnesota	0.143	(0.846)	-1.225	(0.977)	-0.977	(0.903)	-0.0872	(0.0625)	0.103*	(0.0586)	43	0.093
Mississippi	0.588	(1.005)	0.0442	(0.490)	0.0993	(0.668)	-0.0185	(0.0348)	0.0426	(0.0666)	43	0.020
Missouri	-0.961*	(0.542)	0.313	(0.275)	-0.0794	(0.429)	0.0247	(0.0200)	0.104***	(0.0325)	43	0.099
Montana	1.059	(2.121)	-0.278	(0.731)	0.659	(0.734)	-0.00735	(0.0620)	-0.00133	(0.155)	43	0.027
Nebraska	-1.353	(0.811)	**1.406***	**(0.698)**	1.792*	(0.977)	-0.0282	(0.0540)	0.164***	(0.0572)	43	0.242
Nevada	0.0798	(1.148)	0.0516	(0.265)	-0.326	(0.603)	0.0400	(0.0921)	0.0847	(0.0587)	43	0.038
New Hampshire	0.154	(0.508)	0.0306	(0.111)	0.477***	(0.161)	-0.0211	(0.0149)	0.0832**	(0.0342)	43	0.220
New Jersey	0.546	(0.360)	-0.249	(0.231)	0.349	(0.264)	-0.0152	(0.0151)	0.0401	(0.0242)	43	0.147
New Mexico	1.773**	(0.782)	-0.617	(0.560)	3.127***	(0.916)	-0.0377	(0.0356)	-0.0117	(0.0573)	43	0.327
New York	0.145	(0.419)	0.166	(0.230)	0.626**	(0.282)	0.0217	(0.0221)	0.0381	(0.0248)	43	0.187
North Carolina	1.321**	(0.591)	0.0486	(0.145)	**-0.960***	**(0.476)**	-0.0139	(0.0314)	0.00821	(0.0399)	43	0.318
North Dakota	-0.147	(0.852)	0.303	(0.377)	0.0758	(0.259)	-0.0285	(0.0414)	0.0835	(0.0614)	43	0.033
Ohio	0.571	(0.433)	-0.401	(0.259)	0.244	(0.238)	-0.00364	(0.0155)	0.0233	(0.0248)	43	0.089
Oklahoma	1.365	(0.911)	0.544	(0.560)	0.276	(0.355)	-0.0295	(0.0374)	-0.00414	(0.0645)	43	0.171
Oregon	0.146	(0.629)	0.286	(0.462)	0.206	(0.440)	-0.00968	(0.0313)	0.0812*	(0.0422)	43	0.070
Pennsylvania	0.438	(0.876)	-0.407	(0.848)	0.392	(1.420)	-0.0619	(0.0686)	0.0654	(0.0551)	43	0.025
Rhode Island	-0.471	(0.529)	0.0795	(0.152)	-0.234	(0.258)	0.00918	(0.0144)	0.0657*	(0.0357)	43	0.048
South Carolina	1.161	(1.633)	-0.0380	(0.408)	0.269	(1.647)	-0.0419	(0.103)	0.0510	(0.0995)	43	0.020
South Dakota	0.249	(1.126)	-0.144	(0.472)	0.426	(0.380)	0.00659	(0.0430)	0.0549	(0.0864)	43	0.033
Tennessee	0.453	(1.282)	-0.880	(1.156)	1.382	(2.166)	-0.0243	(0.0826)	0.0538	(0.0811)	43	0.016
Texas	0.182	(0.376)	0.332	(0.291)	1.244***	(0.371)	-0.00414	(0.0351)	0.0724***	(0.0239)	43	0.477
Utah	4.666***	(1.069)	-0.486	(0.349)	2.851*	(1.579)	-0.0309	(0.108)	-0.186**	(0.0811)	43	0.404

Region / State	**π (IPD)** α_1		**$P(G-\overline{G})/\overline{B}$** α_2		**$\ln\left(\frac{Y}{\overline{Y}}\right)P(\overline{G}+rB)/\overline{B}$** α_3		**$P\overline{G}/\overline{B}$** α_4		**Constant** α_0		**Obs.**	**R^2**
Vermont	1.371	(1.212)	0.229	(0.376)	-0.358	(0.654)	-0.0201	(0.0330)	0.000949	(0.0841)	43	0.064
Virginia	-0.0198	(0.456)	0.258	(0.220)	0.0191	(0.483)	0.0103	(0.0257)	0.0700**	(0.0289)	43	0.056
Washington	0.156	(0.692)	0.0560	(0.529)	0.758	(1.022)	-0.0898	(0.0604)	0.0841*	(0.0495)	43	0.070
West Virginia	1.242	(0.958)	0.0819	(0.488)	1.863***	(0.619)	**-0.156****	**(0.0608)**	0.0714	(0.0749)	43	0.266
Wisconsin	-0.00871	(0.410)	**0.552****	**(0.261)**	-0.265	(0.208)	0.000907	(0.0177)	0.0660**	(0.0262)	43	0.133
Wyoming	0.686	(1.050)	-0.306	(0.183)	0.654**	(0.248)	**-0.0551****	**(0.0232)**	0.0687	(0.0762)	43	0.225

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 11: Summary statistics of dependent and independent variables, Georgia localities

Variables	N	Mean	SD	Min	Max
Counties					
Counter	134	0.58	0.50	0	1
Counter (boom)	125	0.51	0.50	0	1
Counter (bust)	131	0.66	0.48	0	1
property tax / own revenue	147	0.43	0.10	0.12	0.64
taxes / total revenue	147	0.70	0.07	0.49	0.84
grants / total revenue	147	0.08	0.04	0.02	0.20
infrastructure / total expenditure	147	0.21	0.07	0	0.39
per capita property tax (,000)	147	0.34	0.54	0.01	5.26
per capita expenditure (,000)	147	0.84	1.61	0.02	16.13
per capita infrastructure expenditure (,000)	147	0.18	0.45	0	4.49
debt outstanding / revenue	147	0.19	0.18	0	1.39
fund balance / revenue	147	0.11	0.10	-0.34	0.29
per capita income (,000)	147	20.22	4.50	8.79	45.95
population (,000)	147	56.97	114.29	1.69	811.99
population growth (1/1000)	147	0.01	0.01	-0.05	0.06
government form	147	2.66	0.90	1	4
Municipalities					
Counter	369	0.52	0.50	0	1
Counter (boom)	337	0.49	0.50	0	1
Counter (bust)	359	0.51	0.50	0	1
property tax / own revenue	418	0.11	0.10	0	0.66
grants / total revenue	418	0.08	0.10	0	0.84
infrastructure / total expenditure	418	0.48	0.17	0	0.89
per capita property tax (,000)	416	0.12	0.21	0	3.57
per capita expenditure (,000)	416	1.10	0.99	0.01	9.45
per capita infrastructure expenditure (,000)	416	0.59	0.74	0	8.48
debt outstanding / revenue	418	0.43	0.51	0	4.49
fund balance / revenue	418	0.12	0.10	-0.32	0.56
population (,000)	416	7.49	27.82	0.09	440.23
population growth (1/1000)	414	0.20	1.85	-0.40	31.33

Table 12: Determinants of countercyclical debt use for Georgia counties

Variables	**(1)** All	**(2)** Boom	**(3)** Boom (metro)	**(4)** Boom (non-metro)	**(5)** Bust	**(6)** Bust (metro)	**(7)** Bust (non-metro)
property tax / own revenue	**4.19***	4.58	8.92	5.45	**5.92***	5.43	**11.44***
	(2.45)	(3.01)	(5.66)	(4.81)	**(3.35)**	(5.64)	**(6.58)**
grants / total revenue	**10.33***	9.06	7.58	8.56	0.37	-26.47	**33.71***
	(6.23)	(7.10)	(12.80)	(10.71)	(7.98)	(21.70)	**(17.33)**
infrastructure / total expenditure	-4.23	2.85	**12.85***	-5.70	**-7.52***	-3.97	-7.33
	(3.36)	(3.54)	**(6.95)**	(6.19)	**(4.12)**	(7.54)	(7.45)
per capita expenditure (,000)	0.01	-0.01	1.02	-0.13	0.26	0.39	0.30
	(0.12)	(0.13)	(0.75)	(0.46)	(0.23)	(0.45)	(0.30)
debt outstanding / revenue	**3.46****	**3.21***	**9.56****	2.23	0.21	-2.40	**9.89****
	(1.61)	**(1.79)**	**(4.00)**	(2.84)	(1.84)	(2.76)	**(4.87)**
fund balance / revenue	2.15	**4.80***	2.81	**7.70***	0.47	-5.62	-1.16
	(2.62)	**(2.84)**	(5.02)	**(4.53)**	(3.49)	(7.04)	(5.45)
Constant	-7.70	-1.47	-2.84	-13.79	2.56	-13.32	-20.13
	(5.35)	(6.13)	(5.80)	(21.11)	(5.14)	(8.70)	(25.85)
Observations	127	97	49	48	91	47	44
Pseudo R^2	0.170	0.115	0.142	0.154	0.152	0.329	0.316

Notes: Logit regressions are used, and the coefficients are marginal effects. Only counties with at least 5 year's data were included in the regression, which is why the number of observations is smaller than the number of counties (159). Controls variables are included but not shown here.
Standard errors in parentheses. Significance levels are: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13: Determinants of countercyclical debt use for cities

	(1) All	(2) Boom	(3) Bust
property tax / own revenue	-0.80	-1.26	-1.54
	(1.25)	(1.82)	(1.42)
grants / total revenue	-1.57	1.32	-2.64
	(1.79)	(2.54)	(1.90)
infrastructure / total expenditure	**-1.88****	**-2.31***	-1.38
	(0.82)	**(1.20)**	(1.02)
per capita expenditure (,000)	0.13	-0.12	0.15
	(0.13)	(0.15)	(0.14)
debt outstanding / revenue	-0.26	0.04	-0.12
	(0.21)	(0.36)	(0.25)
fund balance / revenue	1.44	**-3.75****	**2.72***
	(1.26)	**(1.87)**	**(1.44)**
population (,000)	-0.01	-0.00	-0.01
	(0.01)	(0.00)	(0.00)
population growth	-0.51	-0.79	-0.10
	(0.46)	(1.00)	(0.13)
Constant	1.10^{*}	1.83^{**}	0.79
	(0.59)	(0.86)	(0.68)
Observations	349	227	295
Pseudo R^2	0.031	0.037	0.029

Notes: Logit regressions are used, and the coefficients are marginal effects. Only municipalities with at least 5 year's data were included in the regression.
Robust standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 14: The effects of countercyclical debt use on expenditure stability

Counter-cyclical debt	All		Boom		Bust	
	Yes	**No**	**Yes**	**No**	**Yes**	**No**
County						
No. of counties	52	101	75	77	45	108
Revenue Gap	14.7%	10.5%	9.8%	9.0%	-18.2%	-12.7%
Expenditure Gap	12.6%	10.4%	10.0%	12.2%	-3.2%	-6.3%
Difference	-2.1%	-0.1%	0.2%	3.3%	-15.0%	-6.5%
DID (*t-test, left*)	-2.0%		-3.1%		**-8.8%****	
City						
No. of cities	177	192	173	164	175	184
Revenue Gap	21.9%	20.3%	15.1%	15.4%	-26.5%	-22.4%
Expenditure Gap	15.7%	16.8%	2.7%	2.8%	-11.1%	-11.4%
Difference	-6.3%	-3.4%	-12.4%	-12.6%	-15.4%	-11.0%
DID (*t-test, left*)	**-2.8%***		0.2%		**-4.4%***	

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes:

1. The gaps of revenue and expenditure (and debt) are calculated as the percentage in deviations of real values from predicted values based on the time trend.
2. Whether a localities is using debt counter-cyclically or pro-cyclically are based on the relationships between debt gap and revenue gap. If revenue gap and debt gap both are positive or negative, the debt is pro-cyclical; if one is positive and the other is negative, the debt is counter-cyclical. The results are based on coefficients of regressions.
3. For the first two columns, gaps are in absolute values, i.e., all deviations are recalculated as positive values including those of recession years. The other columns are in original gaps (positive or negative).
4. The number of counties and cities are different due to the data availability. Some counties/cities do not have debt at all and some of them only have debt in occasional years which is not enough to calculate whether it is pro-cyclical or counter-cyclical.

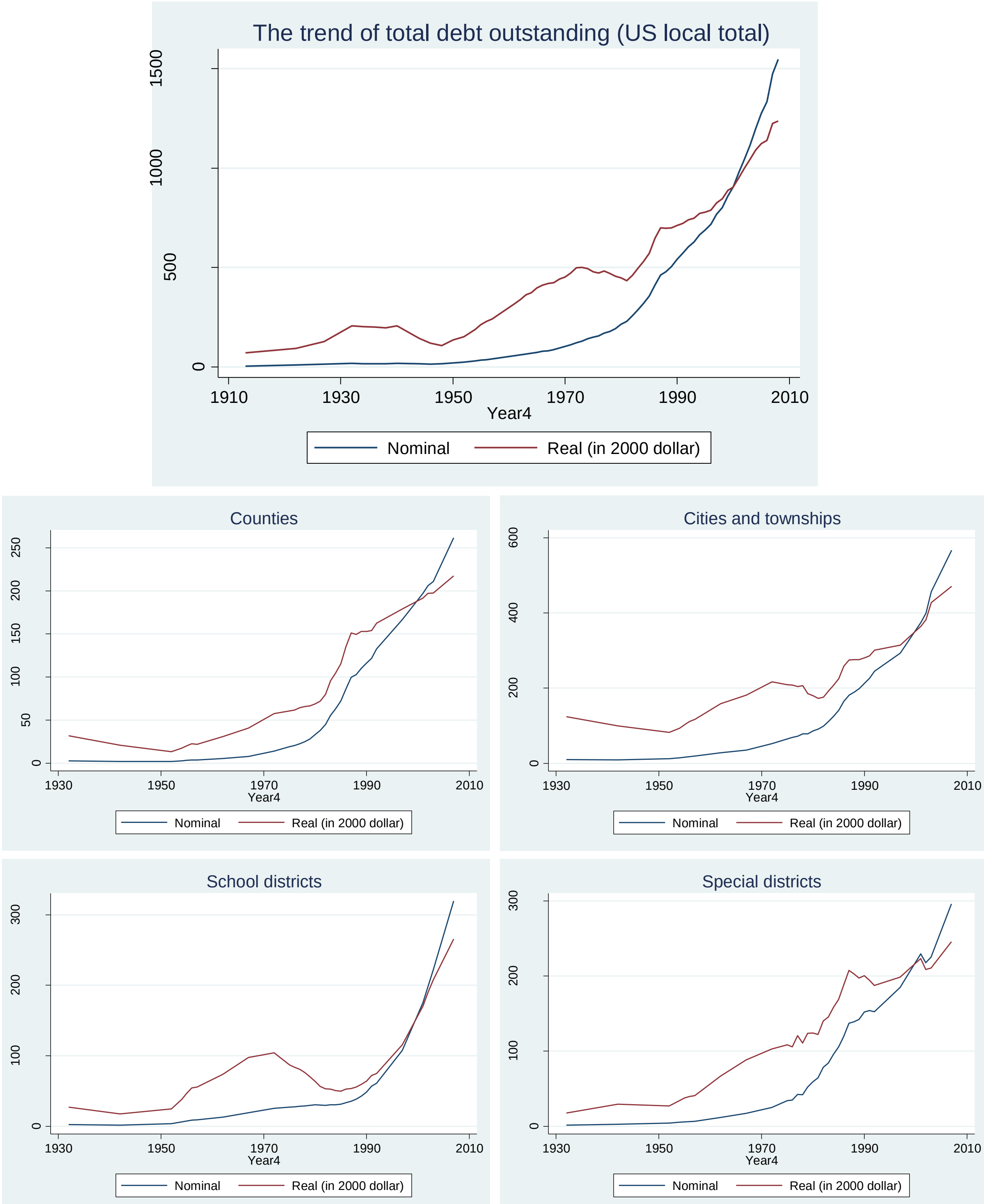

Figure 1: The trend of total debt outstanding for the U.S. local totals for different localities

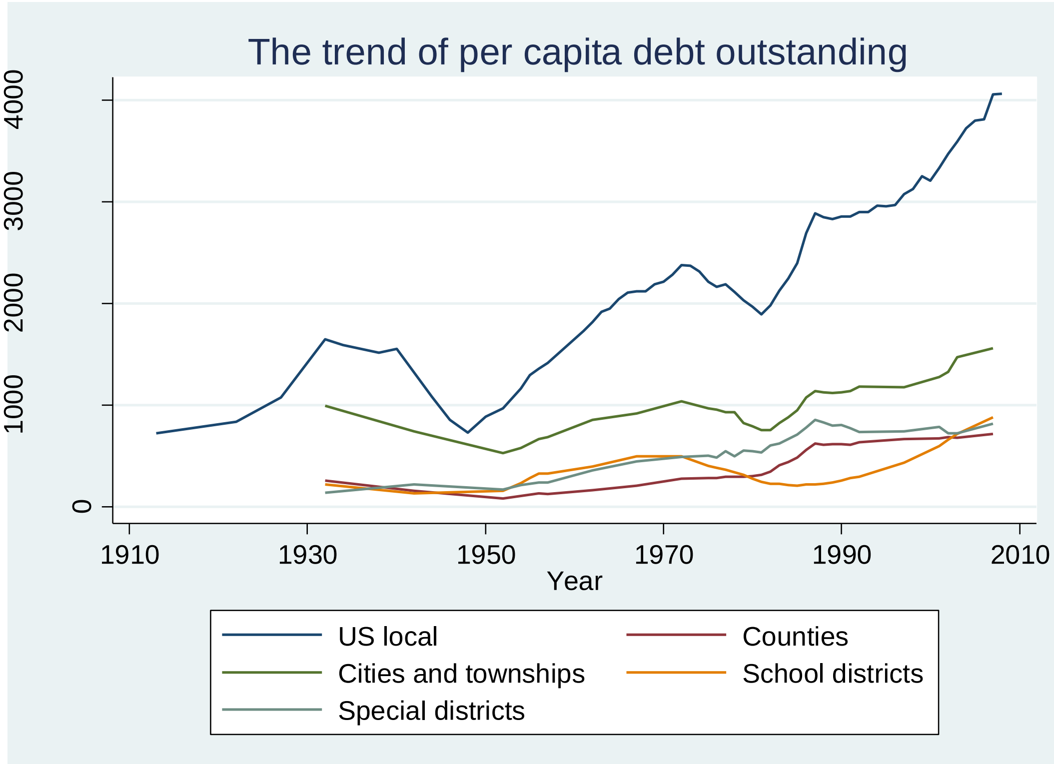

Figure 2: The trend of per capita debt outstanding (in year-2000 dollars) for localities

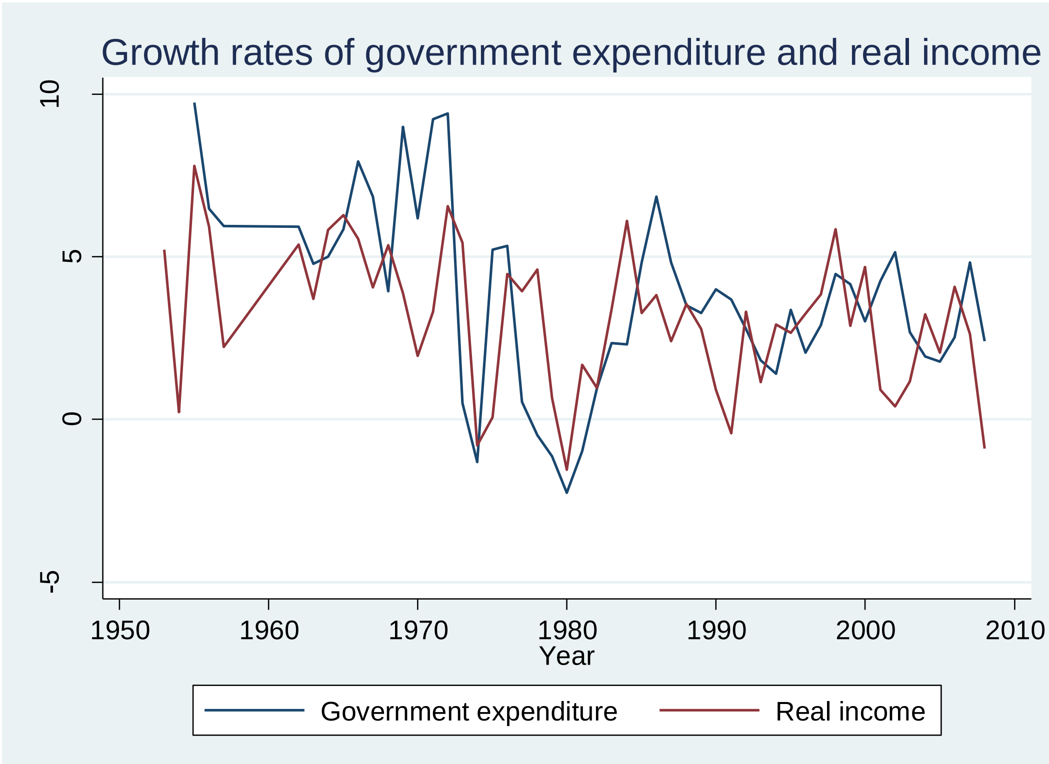

Figure 3: Trend growth rate of real government expenditure and real income, U.S. local total

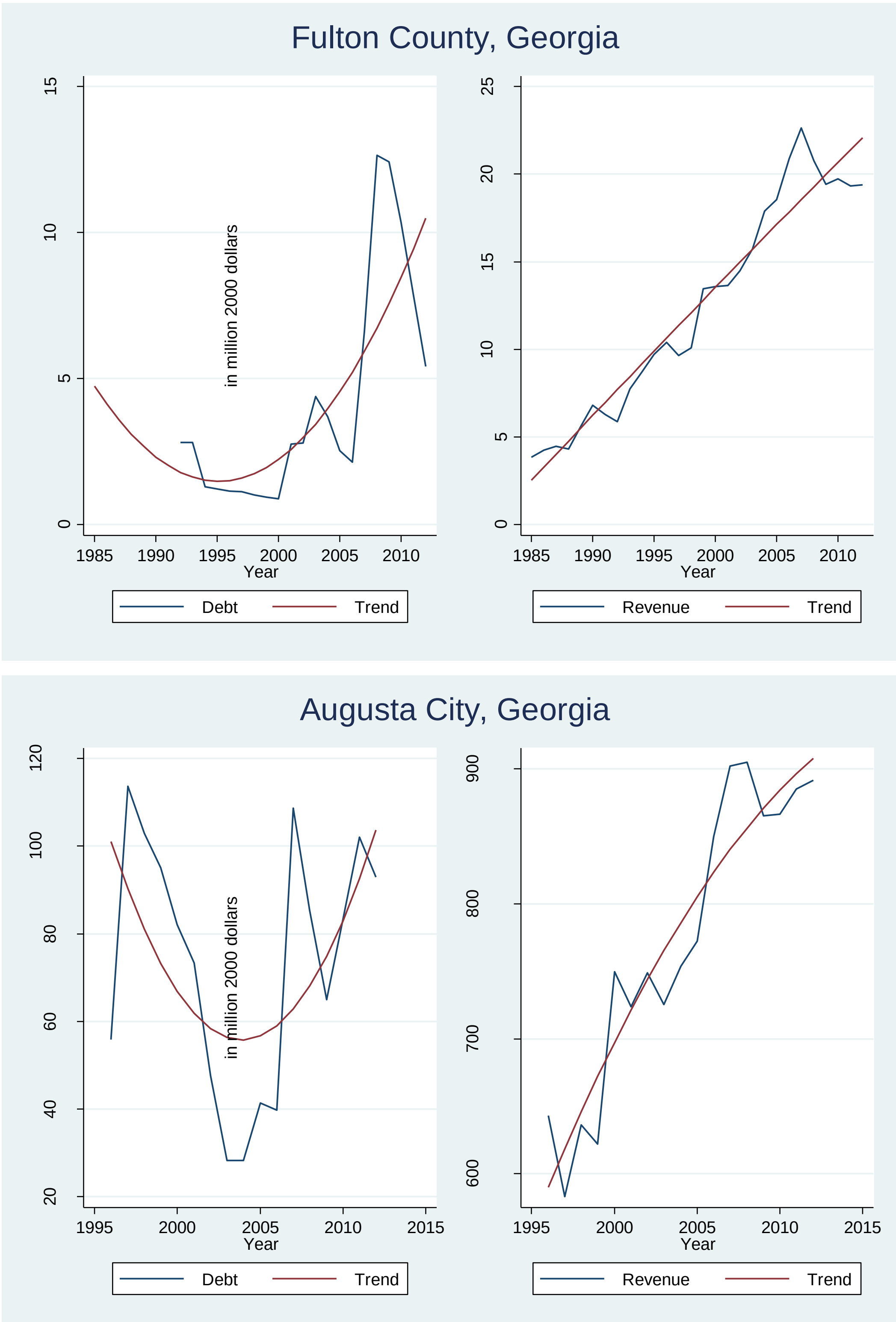

Figure 4: Trends of long-term debt outstanding and total local revenue for two sample localities