

The Optimal Deterrence of Tax Evasion: The Trade-off Between Information Reporting and Audits

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Abstract

Despite the widespread recognition of the effectiveness of information reporting to increase tax compliance, existing tax theory considers tax audits to be the main tool to prevent evasion. The paper extends tax theory by modeling information reporting as an additional enforcement instrument that allows acquiring information about taxpayers' income. The paper solves the problem of the IRS commissioner who, having limited resources to deter tax evasion, has to maximize tax revenue by allocating resources between audits and information reporting. I derive the optimal resource allocation which is governed by the value of information reporting - how much the acquired information helps to facilitate tax audits. I find that the value of information reporting, because of the strategic behavior of taxpayers, depends on the number of audits as an inverse U-shaped function. As a result, the optimal level of information depends on the budget such that it first increases, but then decreases with the budget, while the optimal number of audits always increases with the budget. This implies that there is a budget at which it is no longer optimal to expand the information reporting system. Applying the terminology of utility theory, we would say that information reporting is not a normal good.

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1 Introduction

Information reporting is considered a dependable tool for improving tax compliance.¹ When matched with tax records, it provides information that enables the tax authority to verify the amount of income reported by taxpayers in their returns. Consequently, many countries use information reporting. Most OECD countries have information reporting for dividend and interest incomes – some for incomes from independent personal services, royalties, and patents.² The United States has a substantial information reporting system and has expanded it recently. It now requires banks to report the gross amount of merchant payment card transactions and brokers to report the adjusted cost basis for certain securities.

But, information reporting has its costs that are usually overlooked. Those costs include expenses needed to publicize rules, to educate taxpayers, and to perform and improve matching of information returns with tax returns. For example, during the period 2009 through 2012 the IRS spent about \$110 million just for developing the matching program needed to implement the new information reporting requirements.³ Unless the tax agency is able to employ new resources, it has to divert resources from tax audits.

These observations show that information reporting is highly valuable for improving compliance. But, at the same time, information reporting imposes some tangible costs involving trading off resources with tax audits. This raises some serious questions. How many resources should the tax authority devote to information reporting expansion? More generally, what is the optimal tax enforcement policy?

The existing tax enforcement literature does not answer these questions, because it has not modeled information reporting – it considers tax audits as the main tool to prevent tax evasion. To address these limitations, this paper extends optimal tax enforcement theory by considering information reporting as an additional tax enforcement instrument, and introducing it into the existing tax evasion framework. The paper focuses on the problem of the tax authority that has to maximize tax revenue collection by optimally deterring tax evasion.⁴ The tax authority has a limited budget, and therefore it must trade-off its resources between auditing people and improving information reporting.

This paper models information reporting as an instrument that allows acquiring informa-

¹According to the existing empirical literature, income sources that are subject to some information reporting are characterized by a higher tax compliance than income sources that are not (Kleven et al. (2011), Gordon and Li (2009), Alm and McKee (2009)).

²In some of these countries some of income sources are also subject to withholding. See OECD (2004).

³For more details see GAO (2011).

⁴This paper leaves aside equity issues of taxation and enforcement.

tion about taxpayers' income. Specifically, in the model, information reporting provides the tax authority with signals. Based on a signal, the tax authority can construct predictions about the taxpayer income distribution. Through improving information reporting and its expansion, a more accurate signal can be obtained. This allows constructing a more accurate prediction about the taxpayer income distribution and facilitates conducting tax audits, because it helps to divide taxpayers into more distinct audit classes.

Based on this model, I find that optimal tax enforcement policy involves a combination of auditing and information reporting. Moreover, the optimal level of information reporting depends on the tax agency's budget such that it first increases, but then decreases with the budget, while the optimal number of audits always increases with the budget. This implies that there is a level of budget at which it is no longer optimal to improve information reporting. It is because of a strategic behavior of taxpayers that leads to a particular substitution pattern between signal accuracy and the number of audits. Specifically, the number of audits that can be substituted by an increase in signal accuracy, provided that the tax revenue stays the same, first rises but then declines with the number of people audited.

In the core of the model lies the difference between audits and information reporting, which I conceptualize in the amount of information each of these tools reveals. Both audits and information reporting help the tax authority to access information about taxpayers' income, but they do this differently. An audit helps to reveal full information about one taxpayer, while information reporting helps to reveal partial information about many, or all, taxpayers. For example, information reporting on payments to independent contractors helps the IRS to discover cases when the recipients of these payments underreport their gross profits.⁵ However, it does not help to discover information about independent contractors' expenses, and thus their taxable income is still uncertain.⁶ In general, information reporting helps to reduce only some uncertainty about true taxable income. This justifies modeling information reporting as signals that help to make a more accurate prediction of the income distribution.

As a first step in understanding the relationship between tax audits and information reporting, this paper analyzes how the audit policy depends on the accuracy of information about taxpayers. This paper extends the model of Sanchez and Sobel (1993) and considers audit probability as a function of reported income, as well as the signal. Based on the signals, taxpayers can be divided into audit classes, which allows audit function to be chosen

⁵In the US, information about payments to independent contractors is reported on 1099-MISC form.

⁶Of course, information reporting on different income sources is helpful to discover taxable income to different extents.

separately for each audit class. Within an audit class, the optimal audit strategy is to examine those taxpayers whose reported income is below the cutoff income specific to this audit class.

I find that optimal audit policy requires the marginal revenue of an audit to be the same for all signals. This result has valuable practical implications. Specifically, it implies that the tax authority should focus audits on those taxpayers about whom it has less accurate signals; in other words, on those who have more opportunity to evade taxes. For example, if we consider two groups of taxpayers – wage earners and self-employed individuals – then audits should be focused on the self-employed, i.e., those who characterized by less accurate information.

The second step of the analysis focuses directly on the trade-off between how much of the resources to spend on improving signal accuracy and how much to spend on conducting tax audits. The trade-off arises because the tax authority has limited resources. The decision to allocate resources depends on how effectively an increase in signal accuracy, as compared to an increase in the number of audits, helps to reduce tax evasion and raises tax revenue. When a certain amount of tax revenue has to be collected, an increase in signal accuracy allows saving on tax audits. Based on this observation, the value of signal accuracy can be defined in terms of the number of “saved” audits through the increase in signal accuracy. An analysis of the marginal rate of substitution between signal accuracy and the number of audits shows that the value of signal accuracy first increases but then decreases as the number of audits increases. This substitution pattern arises because optimal auditing has to account for taxpayers’ strategic behavior.

The intuition for the declining value of signal accuracy is the following. Recall that without a signal only low income taxpayers are audited. Signals help to divide taxpayers into audit classes and therefore help to redirect tax audits toward some high-income taxpayers. As signal accuracy increases, taxpayers can be divided into more distinct audit classes and more high-income taxpayers can be examined. However, the value of dividing taxpayers into audit classes declines as more audits can be conducted, because more taxpayers with higher income have been audited. Moreover, when the number of audits increases, the marginal tax revenue of increasing number of classes declines faster than the additional tax revenue of increasing number of audits.

To examine the property of the optimal solution, this paper studies comparative statics with respect to the budget and costs. When a change in the tax authority’s budget is considered, I find that the optimal signal accuracy depends on the tax agency’s budget such that it first rises with the budget, but declines later on. This implies that there is a level of budget at which it is no longer optimal to expand information reporting. At the same time,

the optimal number of audits always increases with the budget.

This paper discusses several important implications of the results. The first implication is that investments in information reporting are especially critical for a tax authority with scarce resources. This suggests that developing countries could benefit by using information reporting. The second implication concerns tax authorities that have substantial resources for tax enforcement. These tax authorities might be close to the point at which it is no longer optimal to expand information reporting. Therefore, for them it is especially important to conduct a careful estimation of potential benefits and costs of further expanding information reporting.

The rest of the paper is organized as follows. Section 2 reviews the related literature to show how this paper builds on existing theory and extends it. Section 3 presents the model that incorporates information reporting into the tax evasion framework by introducing the concept of signal accuracy. Section 4 first characterizes the optimal tax audit rule and how it depends on signal accuracy. Then, taking into account the dependence of the optimal audit rule on signal accuracy, the paper determines the optimal level of signal accuracy and the optimal number of audits. Finally, the analysis of the comparative statics with respect to the tax authority's budget and costs reveals policy relevant findings. Section 5 discusses policy implications of the results. Section 6 concludes.

2 Related Literature

This section overviews the existing optimal tax enforcement literature in order to lay the groundwork for the model in this paper. Moreover, this section helps to ascertain a concept of the accuracy of information, which is a new notion to the literature. While existing literature apprehends that tax authorities can condition audit strategy on information beyond reported income, it does not reckon that the accuracy of information can be changed. It is this endogenizing of the accuracy of information that is implemented in the rest of this paper.

Sanchez and Sobel (1993) solve for the optimal audit rule in a model where the tax authority observes only reported income and chooses the probability of audit based on that reported income. The optimal audit strategy in this case is a cutoff rule: to audit with high enough probability the taxpayers with reported income below a certain cutoff and to not audit those with a reported income above that cutoff.

Scotchmer (1987) notes that, in practice, the tax authority can observe both reported income and correlates like profession, age, and gross income as reported independently by

the employer. Given that such information allows taxpayers to be segregated into audit classes, “the enforcement agency will find it lucrative to condition the probability of an audit on audit class, as well as on reported income, particularly if the audit class is a good signal of income” (p. 229). Her model examines audit rules within an audit class. The other paper that allows conditioning of tax audits on information about taxpayers is Macho-Stadler and Perez-Castrillo (2002). In contrast to Scotchmer (1987), they assume only three discrete income levels and that signals are unobserved by the taxpayers. Nevertheless, the main findings in those papers are similar. However, both papers treat signal as free and exogenous, which precludes the possibility of addressing the optimal improvement in the accuracy of observed information.

Similar to the model in this paper, Menichini and Simmons (2014) consider a costly state verification model where a principal entering a contract with an ex-post informed agent, as well as auditing, has the option of acquiring costly imperfect information about future revenues. In contrast to the current paper, their model has only two income realizations and two signal values.⁷ The model in this paper has a continuous distribution of incomes and signals. Also, their optimization problem of maximizing of an agent expected income subject to the principal’s participation constraint differs from the current paper optimization problem that solves for the maximum tax revenue collection given that agents maximize their expected income. Lastly, they do not allow the principal to choose the size of the correlation between the signal and state realization, while this paper allows the principal (tax authority) to choose signal accuracy.

To describe the accuracy with which the tax authority can observe true information (relevant for tax administration purposes), a new term, “observability” of the tax base, has been recently introduced by Slemrod and Traxler (2010). They define observability as the accuracy of the tax base measurement that the tax authority can obtain. In their model, the tax authority can influence observability by changing the amount of investment spent on tax enforcement. They determine the optimal level of observability that is defined by the trade-off between the social costs from allowing more inaccuracy and the net revenue gains from having a cheaper-to-administer, but more capricious, tax system. Their model is similar to the model in this paper in its idea to endogenize information observed by the tax authority; however, their model does not allow for tax evasion and, thus, tax enforcement is beyond its scope.

Kleven and Kopczuk (2011) note that the accuracy of information concerning applicants

⁷The extension of the model has three income realizations.

to social programs is an important factor affecting program design. They show that the government can influence accuracy of this information by setting a complexity of screening tests. Their model characterizes the optimal social program in which policy makers can choose the rigor of screening, along with a benefit level and eligibility criterion. While their settings are sufficiently different from those considered in this paper, the idea that government can influence and, hence, optimally choose the accuracy of information is parallel.

Besley and Persson (2011) argue that tax bases like income taxes and value added taxes can only become effective through extensive government investments in tax compliance. This supports the importance to improve information reporting from a high level of generalization. According to them, in the process of administrative infrastructure development the countries are able to move from collecting around 10 percent of national income towards collecting around 40 percent, and to shift from trade taxes and excise taxes towards labor income and other broad bases. In their framework a forward-looking government will decide to invest in fiscal capacity in order to build a more effective tax system. However, the ways to improve administrative infrastructure are beyond their scope.

Boserup and Pinje (2013) introduce information reporting into a model of tax evasion in a way that is different than this paper. They assume that one part of taxpayer income – income reported by third parties – is completely observed and that other part of income is completely unobserved by the tax authority. This is a reasonable assumption, but it precludes the general case when information reporting provides only partial information about taxable income. Also, in their model information reporting is exogenous and the tax authority cannot affect its extent.

As has been discussed so far, the importance of accuracy of observed information for the state policy and, in particular, for tax policy has been acknowledged in many studies. The next natural move is to consider information accuracy as one tax enforcement instrument and to determine its optimal level. I present this move in what follows.

3 Model

This section presents a model that describes taxpayers who evade their taxes and the tax authority that fights tax evasion. The key innovative part of the model is signals; they represent information collected by the tax authority through information reporting. This model presents a way to incorporate information reporting into the existing tax evasion framework.

Consider an economy consisting of risk-neutral individuals characterized by their income, i . Income is exogenous and distributed on \mathbb{R} according to c.d.f. $F_0(\cdot)$, which is assumed to be continuously differentiable with density $f_0(i) = F_0'(i)$. Each individual is subject to an income tax at rate t and is required to file a tax report.

The tax authority collects taxes and performs audits to enforce tax compliance. The tax authority does not observe taxpayers' true income, and a priori believes that taxpayers' true income is distributed according to $F_0(\cdot)$. However, the tax authority receives a signal about each taxpayer's income, $s \in \mathbb{R}$.⁸ Given a signal, s , about a taxpayer's income, the tax authority forms an updated belief about the taxpayer's income distribution, which is the conditional distribution function $F(i | s)$. The associated conditional density function is denoted by $f(i | s)$. The tax authority uses a signal when it chooses the probability of an audit, $p(r, s)$. So, the probability of an audit depends not only on taxpayer's reported income, r , but also on the signal, s .

In practice, the tax authority has different types of information that might be helpful to predict a taxpayer's true income, including a taxpayer's age, occupation, marital status, information from previous periods, as well as from third-party reporting of different income types. Third-party information reporting is especially valuable in predicting a taxpayer's true income. I assume that the tax authority is able to aggregate this information to construct a univariate prediction of true income. This prediction is implied by the signal, s .

To characterize the dispersion of the conditional distribution of income, I introduce the signal accuracy, a . Correspondingly, I denote the conditional distribution of income by $F^a(i | s)$, indicating that it depends on the signal accuracy, a . I define signal accuracy, a , as the inverse of the standard deviation of the conditional distribution of income, which requires imposing additional assumptions. Specifically, assume that for any signal, s , the conditional distribution of income is

$$F^a(i|s) = G(a(i - s)), \tag{1}$$

where G is a symmetric distribution function with zero expectation and unit variance, and the expectation of the conditional distribution of income is equal to s . The conditional p.d.f., $f^a(i|s)$, is correspondingly equal to $ag(a(i - s))$. As can be seen from the definition of $G(a(i - s))$, signal accuracy, a , is equal to the inverse of the standard deviation of the conditional income.

Importantly, condition (1) assumes that all signals are characterized by the same accuracy,

⁸Signal s is one-dimensional.

a. This implicitly assumes that all taxpayers have identical income sources. For example, half may be wage and salary income and half rental income. However, this assumption is needed only for the analysis in Section 4.2. As we go along, I will point out when this assumption is imposed and when not.

The distribution of signals also depends on accuracy, a . Let us denote the c.d.f. of signals by $H^a(s)$ and p.d.f. of signals by $h^a(s)$. An integration of the conditional distribution of income over all signals should give the p.d.f. of income, that is,

$$f_0(i) = \int ag(a(i-s))h^a(s)ds. \quad (2)$$

I assume that, when signal accuracy changes, the distribution of income, $f_0(i)$, does not change. For this to be true, the distribution of signals, $h^a(s)$, has to change with signal accuracy. Indeed, when signal accuracy changes, the conditional distribution of income - beliefs based on signals - changes. To satisfy condition (2), the distribution of signals, $h^a(s)$, should correspondingly adjust. Moreover, in order for the assumption on the conditional distribution of income (1) and the assumption that the distribution of income, $f_0(i)$, does not depend on signal accuracy to be satisfied, the support of the distribution of income should be unbounded.⁹ This permits a taxpayer to have a negative income, in which case she is entitled to receive a tax refund of $t|i|$.¹⁰

An example that satisfies all the assumptions imposed above obtains when the conditional distribution of income is normal with expectation equaled to s and variance equaled to $\frac{1}{a^2}$. Letting $\Phi(\cdot)$ and $\varphi(\cdot)$ to denote the c.d.f. and the p.d.f. of the standard normal distribution, the conditional c.d.f. of income is $F^a(i|s) = \Phi(a(i-s))$ and the conditional p.d.f. is $f^a(i|s) = a\varphi(a(i-s))$. Additionally, the distribution of true income is also normal with expectation μ and variance Ω^2 . The distribution of signals that conforms to these assumptions is a normal distribution with expectation μ and variance $\Sigma^2 = \Omega^2 - \frac{1}{a^2}$ (i.e., the c.d.f. is $H(s) = \Phi(\frac{s-\mu}{\Sigma})$, the p.d.f. is $h(s) = \frac{1}{\Sigma}\varphi(\frac{s-\mu}{\Sigma})$).¹¹

The tax authority may increase signal accuracy at some resource costs, for example, by

⁹You can request a proof of this assertion from the author.

¹⁰Having negative income does not change the incentives of a taxpayer. A taxpayer with negative income has incentives to understate the true income because understatement of true income allows to claim a higher tax refund.

¹¹In the case of the standard normal distribution it can be shown that the notion of signal accuracy agrees with the notion of informativeness introduced by Blackwell: more accurate signals are also more informative. According to one of the Blackwell's definitions of the more informative experiment, if the posteriors constructed after observing the outcome of experiment P are mean-preserving spread of the posteriors constructed after observing the outcome of experiment Q then experiment P is called to be more informative than experiment Q . More information about Blackwell's informativeness can be found in Borgers (2009).

improving information reporting. Specifically, the tax authority can achieve signal accuracy, a , by investing $K(a)$. The cost function, $K(\cdot) > 0$, is assumed to be increasing (i.e., $K'(\cdot) > 0$). Later, I have an in-depth discuss of what these costs include.

A taxpayer is assumed to know the signal and its accuracy. The taxpayer knows that the probability of audit is based on her reported income, r , and on the known signal, s . If a taxpayer's report is not audited, then the taxpayer owes the tax based on the reported income, tr . If her report is audited, then the taxpayer owes taxes based on her true income and also a penalty proportional to the concealed taxes at rate π , $ti + (1 + \pi)t(i - r)$. Each taxpayer minimizes her expected tax payment:

$$\min_r \{tr + p(r, s)(1 + \pi)t(i - r)\} \quad (3)$$

Define this minimum as $T(i, s, p(\cdot))$ and the optimal reported income that minimizes the above taxpayer's problem as $r(i, s)$. Note that the signal affects the optimal report only though the audit probability.

The tax authority chooses the probability of an audit, $p(r, s)$, and signal accuracy, a , to maximize the tax revenue given limited resources to conduct tax audits and invest in signal accuracy.¹² While the tax authority does not observe the true income of each individual, it knows the conditional distribution of income based on signals, $F^a(i | s)$, and the distribution of signals over the entire population, $H^a(s)$. Thus, the tax authority problem is

$$\begin{aligned} & \max_{p(r,s),a} \left\{ \int \int T(i, s, p(\cdot)) dF^a(i | s) dH^a(s) \right\} \\ & s.t. \int \int c \cdot p(r(i, s), s) dF^a(i | s) dH^a(s) + K(a) \leq B, \end{aligned} \quad (4)$$

where c is the cost of an audit, and B is a budget of the tax authority. In what follows the tax authority is assumed to have a limited budget (i.e., $0 \leq B < \frac{c}{1+\pi}$) meaning that the tax authority is unable to audit every taxpayer with probability at least $\frac{1}{1+\pi}$, in which case all evasion would be detected.

¹²First, formulating the tax authority problem in this way rather than maximizing total welfare allows my analysis to focus on tax enforcement problem. Second, I consider constraint tax revenue maximization rather than *net* tax revenue maximization because, as the model of hierarchical policy described by Sanchez and Sobel (1993) shows, society benefits when the government controls the budget of the tax authority rather than allows the tax authority to maximize net tax revenue. In their model, the government first selects a tax function, a level of public good expenditure, and the tax authority's budget and then delegates the responsibility to collect taxes to the tax authority. The model shows that the budget provided by the government is always less than the net revenue-maximizing budget. An extra dollar in the tax authority's budget not just raises extra tax revenue, but also imposes heavier tax burden on society and hence decreases social welfare. Since I consider only tax authority's enforcement problem, I assume that the tax authority is subject to the budget constraint.

The costs of improving signal accuracy, $K(a)$, includes several components. First, if improving signal accuracy is achieved through expanding information reporting, then $K(a)$ includes administrative costs of information reporting expansion. Those costs arise because the tax authority has to publicize new rules in social media, create and publish new tax forms, etc. Second, the cost of improving signal accuracy includes the cost of processing the information returns collected from third parties. This is an important component of information reporting cost that is usually left in the shadow, but actually deserves a careful examination. Each year the IRS receives an enormous number of information returns. For example, in 2013 the number of information returns received was 2 billion.¹³ To match all these information returns with tax returns, a sophisticated matching program had to be developed and used. In 2009 the IRS initiated the so-called Information Reporting and Document Matching (IRDM) program in order to implement the two new information reporting requirements - 1099-K and 1099-B information returns. To develop and support this program, IRS spent approximately \$110 million during the period 2009 through 2012.¹⁴

Even without expanding information reporting, there are costs associated with processing and improving information reporting. An improvement in information reporting that increases signal accuracy can be achieved through a better examination of information reports, but this requires investments in computer, software, and programs. Additionally, the IRS has been modernizing its system in order to facilitate the information returns filing and submitting process.¹⁵ Moreover, during the last decade the IRS has been constantly working to improve the matching process of existing programs and refining the methodology of selecting cases for taxpayer contact. However, the GAO (2009) report pointed out that the Automated Underreporter (AUR) program – a program responsible for matching 1099-MISC information returns – “currently has a narrow reach and pursues less than half of 1099-MISC-related cases in the AUR inventory” (p. 5). Thus, a scope for further improvement of information reporting remains.

The third important component of information reporting costs is the compliance costs imposed on third parties (firms/taxpayers). Compliance costs include the costs of getting taxpayer identification numbers, buying software, tracking reportable payments, filing returns, and mailing copies to taxpayers or paying to tax preparer. Contrary to common belief, the GAO (2007) report found that the size of existing compliance costs was relatively

¹³See SOI Tax Stats - Information Reporting Program - IRS Data Book Table 14, available at <http://www.irs.gov/uac/SOI-Tax-Stats-Information-Reporting-Program-IRS-Data-Book-Table-14>.

¹⁴For more details see GAO (2011).

¹⁵For further details see IRS IT Modernization Vision & Strategy (October 2007).

small. Specifically, “[o]ne small business employing under five people told GAO of possibly spending 3 to 5 hours per year filing Form 1099 information returns manually, using an accounting package to gather the information. An organization with more than 10,000 employees estimated spending less than .005 percent of its yearly staff time on preparing and filing Forms 1099, including record-keeping. Two external parties reported prices for preparing and filing Forms 1099 with IRS of about \$10 per form for 5 forms to about \$2 per form for 100 forms, with one of them charging about \$.80 per form for 100,000 forms” (p. 3). Even though the size of the compliance costs seems to be moderate, it should not be excluded from consideration. The cost of improving signal accuracy, $K(a)$, may include a compensation for imposed compliance costs to third parties.

4 Analysis

This section describes the solution to problem (4). To facilitate the analysis, I subdivide it into two steps.¹⁶ First, by holding signal accuracy fixed, I determine how the optimal audit policy can depend on signal accuracy. Second, taking into account the dependence of the optimal audit policy on signal accuracy, I derive the optimal level of signal accuracy and the optimal number of tax audits. The main focus in that subsection is on the trade-off between the number of tax audits and level of signal accuracy. The analysis is concluded by comparative statics which examines how change in the budget and costs affect the solution. Findings from this subsection bring policy relevant implications.

4.1 The Optimal Audit Policy: How Does It Depend on Signal Accuracy?

In this subsection signal accuracy is taken as fixed and the optimal audit policy is determined. For notational simplicity, the superscript a is omitted. So, the tax authority problem is

$$\begin{aligned} & \max_{p(r,s)} \left\{ \int \int T(i, s, p(\cdot)) dF(i|s) dH(s) \right\} \\ & \text{s.t. } \int \int c \cdot p(r(i, s), s) dF(i|s) dH(s) + K(a) \leq B, \end{aligned} \tag{5}$$

A simple solution strategy to tackle problem (5) can be discovered by examining the nature of the problem. Recall that signals are observed by the tax authority and by taxpayers as well. Consequently, signals cannot be manipulated by taxpayers. Moreover, signals provide

¹⁶This two-step approach allows relying on the previous results for the optimal audit rule.

information about income distribution, which allows the tax authority to segregate taxpayers into audit classes. Thus, the tax authority can treat all taxpayers with signal s as belonging to one audit class. For each audit class, i.e., for each signal, the tax authority can determine the optimal audit function given the amount of resources assigned to conduct audits within this audit class. Let's denote this amount of resources by $B(s) = \int c \cdot p(r(i, s), s) dF(i|s)$. The sub-problem of choosing optimal audit function within an audit class given resources $B(s)$ can be analyzed using arguments employed by Sanchez and Sobel (1993). Then, the allocation of resources among audit classes, i.e., the size of $B(s)$ for any s , can be chosen to maximize the total tax revenue subject to the constraint $\int B(s) dH(s) \leq B - K(a)$.

In pursuance of this strategy, a solution of problem (5) which is described in Proposition 1 can be found. Note that while Proposition 1 relies on arguments employed by Sanchez and Sobel (1993), it extends them to the case when the support of the income distribution is unbounded.¹⁷ Additionally, Proposition 1 does not require any assumptions about conditional distribution of income $F(i|s)$ except that for each signal s the conditional inverse hazard rate $\gamma(i | s) = \frac{1-F(i|s)}{f(i|s)}$ is strictly decreasing in i . So, the conditional distribution function could have bounded or unbounded support, which w.l.o.g. is denoted by $[l(s), h(s)]$. Also, signals need not to have the same accuracy, hence the variance of the conditional distribution of income could be different for different signals.

Proposition 1. *Assume that for each signal s the conditional inverse hazard rate $\gamma(i | s) = \frac{1-F(i|s)}{f(i|s)}$ is strictly decreasing in i . The optimal audit function which solves (5) satisfies:*

$$p^*(r, s) = \begin{cases} \frac{1}{1+\pi}, & \text{if } r < \beta(s) \\ 0, & \text{if } r \geq \beta(s) \end{cases}, \quad (6)$$

where the optimal audit trigger $\beta(s)$ satisfies

$$\gamma(\beta(s) | s) = \begin{cases} = \frac{\lambda c}{t(1+\pi)} & \text{if } l(s) \leq \beta(s) \leq h(s) \\ < \frac{\lambda c}{t(1+\pi)} & \text{if } \beta(s) = l(s) \end{cases}, \quad (7)$$

and λ - Lagrange multiplier - is determined so that the budget constraint is satisfied:

¹⁷Sanchez and Sobel (1993) consider only the case when the support of the income distribution is bounded.

$$\int F(\beta(s) | s)dH(s) = \frac{(1 + \pi)[B - K(a)]}{c}. \quad (8)$$

Proof. See proof in Appendix.

Proposition 1 shows that the optimal audit rule depends both on reported income and signal. For a given signal s , the optimal audit policy is a cutoff rule: audit with probability $\frac{1}{1+\pi}$ if reported income is below the cutoff $\beta(s)$, and do not audit if reported income is above the cutoff $\beta(s)$.¹⁸ Note that with such an audit policy in place, a taxpayer optimal strategy is to choose reported income equal to true income, $r(i, s) = i$, if her true income i is less than the cutoff $\beta(s)$ and to choose reported income equal to the cutoff $\beta(s)$, $r(i, s) = \beta(s)$, if her true income i is greater or equal to the cutoff $\beta(s)$.¹⁹

The dependence of the audit trigger $\beta(s)$ on the signal is crucial. Because of this dependence, it is not low reports, but low reports *conditional on signals* that are audited by the tax authority. This conditioning on signals makes the derived audit rule more realistic than the simple cutoff rule like one in Sanchez and Sobel (1993) which was criticized for mismatching the reality.²⁰ Indeed, the derived audit rule which conditions audit probability on signal/audit class easily agrees with the experience that audit probability seems to increase with reported income. Even when the tax authority audits taxpayers with low income reports within an audit class with higher probability than it audits high-report taxpayers, the observed probability of audit which pools audit classes could rise with reported income. Taxpayers with high reported income might have high probabilities of audit because they are in audit classes that signal high income.

Conditions (7) and (8) which determine the audit trigger $\beta(s)$ have intuitive interpretations. Condition (8) guarantees that resources spent on audits are equal to the available resources for audits. Condition (7) basically requires the marginal revenue of an audit to be the same among all signals, i.e., among all audit classes. This can be seen by considering a small hypothetical increase of the cutoff from $\beta(s)$ to $\beta(s) + \epsilon$, for a given signal s . In the result of such an increase, those who have income greater than $\beta(s)$, whose mass is

¹⁸I call $\beta(s)$ the “audit trigger” or “cutoff income”, and use these terms interchangeably. The term audit trigger is more self-explanatory, the term cutoff income is used in the literature.

¹⁹The cutoff rule results in the solution of the model because this paper following mechanism design literature adopts a commitment principle assuming that the principal announces an audit strategy sufficient to induce truthful reports by the agent and then sticks to it despite it being costly.

²⁰Melumad and Mookherjee (1989) point out that this type of solution has credibility problem. This rule is not credible because the tax authority has to audit those who do not evade. However, if we interpret this static model from a dynamic prospective - the tax authority has to audit those who not evade today in order that they do not evade tomorrow - this rule looks more reasonable.

equal to $(1 - F(\beta(s)|s))$, will increase the paid taxes on $t\epsilon$. But, this will require the tax authority to conduct $\frac{f(\beta(s)|s)\epsilon}{1+\pi}$ additional audits at cost c . Condition (7) requires that the ratio of marginal revenue to marginal cost of the hypothetical increase of the cutoff income, $\frac{t\epsilon(1-F(\beta(s)|s))}{cf(\beta(s)|s)\epsilon(1+\pi)^{-1}} = \frac{t(1+\pi)}{c}\gamma(\beta(s)|s)$, equals λ (the Lagrange multiplier) for signals that require auditing at the optimum. Because marginal cost of an audit, c , is constant, condition (7) implies that the marginal revenue of an audit is the same for all signals.

Condition (7) also covers the case when taxpayers with a certain signal are not audited. It is the case when $\beta(s) = l(s)$. If for some signals the function $\gamma(i|s)$ is lower than $\frac{\lambda c}{t(1+\pi)}$, even for the lowest income $l(s)$, then the audit trigger $\beta(s)$ is equal to $l(s)$.²¹ This situation can arise if $l(s)$ is finite and some signals are relatively more accurate than others, so that audits of the taxpayers with such signals cannot generate high enough tax revenue. In turn, this can happen when some taxpayers have high degree of voluntary compliance because of higher information reporting, and thus less opportunity to evade. For instance, those taxpayers might be wage earners, and it might be optimal to concentrate audits only on self-employed people, leaving wage earners unaudited. A rigorous example of a situation when some taxpayers are not audited is considered in the Appendix A.

The result that optimal audit policy requires the marginal revenue of an audit to be the same for all signals has valuable practical implications. Specifically, it implies that the tax authority should focus audits on those taxpayers about whom it has less accurate signals which means on those who have more opportunity to evade taxes. To see this, consider two groups of taxpayers such that the first group is characterized by signal accuracy a_1 and second group is characterized by signal accuracy a_2 which is greater than the first signal accuracy (i.e., $a_1 < a_2$). Assume that the conditional distributions of income are $F^{a_1}(i|s_1) = G(a_1(i - s_1))$ and $F^{a_2}(i|s) = G(a_2(i - s_1))$ for the first and second groups correspondingly. Then, the following corollary shows that the number of people audited in the group with signal accuracy a_1 is greater than the number of people audited in the group with signal accuracy a_2 when $a_1 < a_2$.

Corollary. *If $a_1 < a_2$ then $G(a_1(\beta(s_1) - s_1)) > G(a_2(\beta(s_2) - s_2))$.*

Proof. See proof in Appendix.

This corollary shows that optimal audit policy prescribes the tax authority to conduct more audits among taxpayers with less accurate signals than among taxpayers with more accurate signals. Intuitively, prior to audits the expected audit revenue from taxpayers with

²¹ $\gamma(i|s)$ shows the proportion of people with income greater than i to people with income exactly equal to i for a given signal s .

a low accurate signal is higher than from taxpayers with a high accurate signal. Therefore, the tax authority should conduct more audits among the former ones in order for the marginal revenue of an audit to be the same for both groups. This result that the tax authority should focus audits on those taxpayers about whom it has less accurate signals closely matches reality. However, prior to this paper there has been no theory which would provide a sound ground for it.

4.2 The Optimal Signal Accuracy and the Optimal Number of Audits

In this subsection I determine the optimal signal accuracy as well as the optimal number of audits. To do this, I return to the case when signal accuracy is not fixed and can be changed by the tax authority by investing resources. Also I rely on the result from the previous subsection of how the optimal audit rule depends on signal accuracy.

The optimal policy involves a trade-off of how much to invest in signal accuracy and how much of resources to spend on audits. To be able to focus on this trade-off, we need to use assumption (1) imposed on the conditional distribution of income, which assumes that all signals are characterized by the same accuracy. This assumption is that $F(i|s) = G(a(i-s))$.

Given this assumption, the cutoff function that characterizes the audit policy can be derived in explicit form. To make inference about the cutoff function that is defined by (7) and (8), it is helpful to analyze the conditional inverse hazard rate function, $\gamma(i | s) = \frac{1-G(a(i-s))}{ag(a(i-s))}$. As the distribution function on its own, the conditional inverse hazard rate function, $\gamma(i | s) = \frac{1-G(a(i-s))}{ag(a(i-s))}$, depends on i and s additively. Therefore, the FOC (7), which is $\frac{1-G(a(\beta(s)-s))}{ag(a(\beta(s)-s))} = \frac{\lambda c}{t(1+\pi)}$, implies that $\beta(s) - s$ is a constant that does not depend on s and depends only on a . By defining $\delta \equiv \beta(s) - s$, the other FOC (8) – the budget constraint – can be simplified to $G(a\delta) = \frac{1+\pi}{c} (B - K(a))$. Defining the right-hand side by P (i.e., $P \equiv \frac{1+\pi}{c} (B - K(a))$) and noticing that P is the number of taxpayers audited (with probability $\frac{1}{1+\pi}$), helps to derive the cutoff income which is

$$\beta(s) = s + \delta = s + \frac{1}{a}G^{-1}(P). \quad (9)$$

For sake of brevity, I will refer to P as the number of audits. Note that $0 \leq P < 1$, because $0 \leq B < \frac{c}{1+\pi}$.

As (9) shows, the cutoff income increases in the magnitude of signal, s , and in the number of audits, P . Intuitively, the higher is the magnitude of signal, s , the higher is the expected

true income, and therefore the higher is the cutoff income. Also, the greater is the number of audits, P , that are conducted, the higher is the cutoff income. Simply, because to audit more people, the tax authority needs to have a high cutoff income.

The cutoff income (9) is also affected by signal accuracy, a . When signal accuracy increases, the conditional density of income given signal, $G(a(i-s))$, shrinks toward its expected value, s , and the cutoff income does the same. Specifically, if $P \leq \frac{1}{2}$ (i.e., $G^{-1}(P) \leq 0$), then an increase in a leads to an increase in the cutoff income. If $P > \frac{1}{2}$ (i.e., $G^{-1}(P) > 0$), then an increase in a leads to a decrease in the cutoff income.²² Intuitively, as signal accuracy increases, a signal provides more accurate prediction of income which is reflected by the conditional density of income becoming more concentrated (lower variance) around that signal. This implies that conditional on the signal the probability of income being in close proximity of the signal is higher, therefore the cutoff income moves toward the signal.

Importantly, when signal accuracy increases, shrinking of the conditional density of income occurs along with widening of the density of signals. The density of signals become more dispersed around the mean. For instance, when all distributions are normal, the distribution of signals is $H(s) = \Phi\left(\frac{s-\mu}{\sqrt{\Omega^2 - \frac{1}{a^2}}}\right)$, with variance $\Omega^2 - \frac{1}{a^2}$ increasing in signal accuracy, a . Thus, as signal accuracy increases, signals become more distinct and better predict income.

Using expression (9) for the cutoff income, we can calculate the tax revenue. We start by calculating the tax revenue collected from a taxpayer whose signal is s and then we calculate the total tax revenue by integrating over all signals. The tax revenue which is collected from a taxpayer with signal s is $T(s) = t \left[\int_{-\infty}^{\beta(s)} i a g(a(i-s)) di + \beta(s) \int_{\beta(s)}^{\infty} a g(a(i-s)) di \right] = t \left[s + \frac{1}{a} P \mathbb{E}_G[z|z \leq G^{-1}(P)] + \frac{1}{a} (1-P) G^{-1}(P) \right]$, where $\mathbb{E}_G[z|z \leq G^{-1}(P)]$ is an expectation of a random variable with the c.d.f. $G(\cdot)$, denoted by z , conditional on $z \leq G^{-1}(P)$, i.e., $\mathbb{E}_G[z|z \leq G^{-1}(P)] = \int_{-\infty}^{G^{-1}(P)} z \frac{g(z)}{P} dz$. The total tax revenue is obtained by integrating $T(s)$ over all signals and is equal to

$$\begin{aligned} TR &= \int_{-\infty}^{\infty} T(s) h(s) ds = \\ &= t \left[\mu + \frac{1}{a} (P \mathbb{E}_G[z|z \leq G^{-1}(P)] + (1-P) G^{-1}(P)) \right]. \end{aligned} \tag{10}$$

Let us define $R(P)$ as $R(P) \equiv P \mathbb{E}_G[z|z \leq G^{-1}(P)] + (1-P) G^{-1}(P)$. The expression $t \cdot R(P)$ has a simple interpretation and is equal to the expected tax collection from a taxpayer with signal $s = 0$ if $a = 1$ and correspondingly the cutoff income is equal to $G^{-1}(P)$. Specifically, with probability P such a taxpayer has a true income lower than $G^{-1}(P)$, in which case she honestly reports her true income equal in expectation to $\mathbb{E}_G[z|z \leq G^{-1}(P)]$. With

²²Here, I consider an exogenous change in a meaning that the number of audits, P , fixed.

probability $(1-P)$ her true income is greater than $G^{-1}(P)$, in which case she reports the cutoff income, $G^{-1}(P)$. Because $R(P) \leq \mathbb{E}_G[z] = 0$, the tax revenue, $TR = t(\mu + \frac{1}{a}R(P))$, increases with a . Because $R'(P) = \frac{1-P}{g(G^{-1}(P))} > 0$, the tax revenue increases with P . Interestingly, $R'(P)$ is equal to inverse hazard rate function evaluated at $i = G^{-1}(P)$ given $s = 0$ and $a = 1$, i.e., $R'(P) = \gamma(G^{-1}(P)|s = 0, a = 1)$.

Finally, the tax revenue is

$$TR(a, P) = t(\mu + \frac{1}{a}R(P)) \quad (11)$$

As (11) shows, to raise higher tax revenue, the tax authority can use two tools: to increase signal accuracy and to increase the number of audits. First, an increase in signal accuracy, a , raises the tax revenue by proportionally shrinking $R(P)$ function. It is because an increase in signal accuracy increases the average income of those who are audited and, in case when $P \leq \frac{1}{2}$, increases reported income of those who are not audited, which is equal to the cutoff income. When $P > \frac{1}{2}$, an increase in signal accuracy, on the opposite, decreases the cutoff income, but it is still increases the average income of those who are audited. Second, an increase in the number of audits raises the tax revenue by increasing $R(P)$ function. It is because an increase in the number of audits increases the cutoff income.

Recall that $P = \frac{1+\pi}{c}(B - K(a))$ and notice that this expression is exactly equivalent to the budget constraint $\frac{c}{1+\pi}P + K(a) = B$. Given this, we can state the tax authority problem of finding the optimal signal accuracy, a , and the optimal number of audits, P , which maximize the tax revenue, as

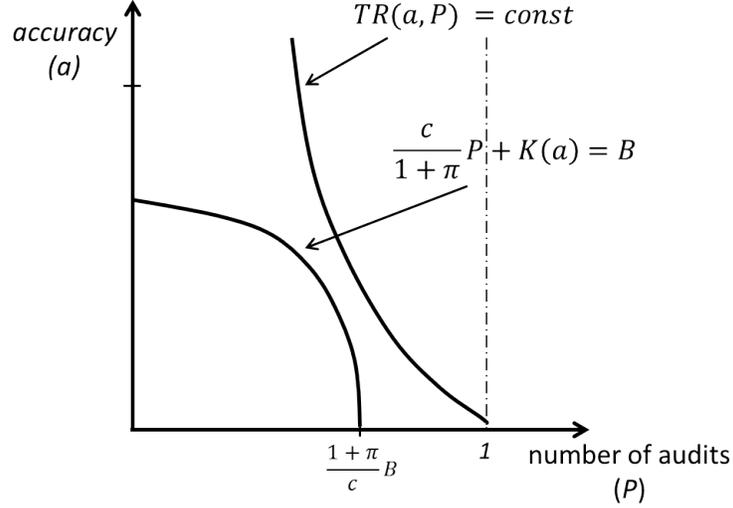
$$\begin{aligned} & \max_{a_L \leq a, 0 \leq P} t(\mu + \frac{1}{a}R(P)) \\ & s.t. \frac{c}{1+\pi}P + K(a) = B, \end{aligned} \quad (12)$$

where $a_L = \frac{1}{\Omega}$. To insure that accuracy a is not smaller than $\frac{1}{\Omega}$ – the inverse of the standard deviation of unconditional income distribution, I impose $a \geq a_L$ and assume $K(a) = 0$ for $a \leq a_L$.

Problem (12) considers maximization of tax revenue, $TR(a, P) = t(\mu + \frac{1}{a}R(P))$ subject to the budget constraint $\frac{c}{1+\pi}P + K(a) = B$. As Figure 1 shows, this problem can be presented graphically in (P, a) space in terms of finding the highest iso-revenue curve $TR(a, P) = const$ given the budget constraint $\frac{c}{1+\pi}P + K(a) = B$. An iso-revenue curve can be expressed as $a|_{TR=const} = \frac{-R(P)}{\mu - TR/t}$. From this expression it is clear that iso-revenue curves are strictly convex if $R(P)$ is strictly concave.²³ If $K(a)$ is convex, then the budget constraint curve

²³Note that, when $G(\cdot)$ is equal to the c.d.f. of the standard normal distribution, $\Phi(\cdot)$, as in the example

Figure 1: Iso-revenue curve and the budget constraint



is concave. The strict convexity of the iso-revenue curves and the concavity of the budget constraint curve ensure that there exists a unique optimal accuracy, a^* . If there is a point where an iso-revenue curve is tangent to the budget constraint curve, then it is the interior solution; otherwise, the solution is a corner solution. Formally, the ratio the FOCs for the maximization problem (12) can be written as follows:²⁴

$$\frac{\frac{\partial TR}{\partial a}}{\frac{\partial TR}{\partial P}} - \frac{1 + \pi}{c} K'(a^*) = -\frac{R(P^*)}{a^* R'(P^*)} - \frac{1 + \pi}{c} K'(a^*) = \begin{cases} = 0 & \text{if } a^* > a_L, P^* > 0 \\ > 0 & \text{if } a^* > a_L, P^* = 0 \\ < 0 & \text{if } a^* = a_L, P^* > 0. \end{cases} \quad (13)$$

The solution of (12) is determined by equation (13) and the budget constraint:

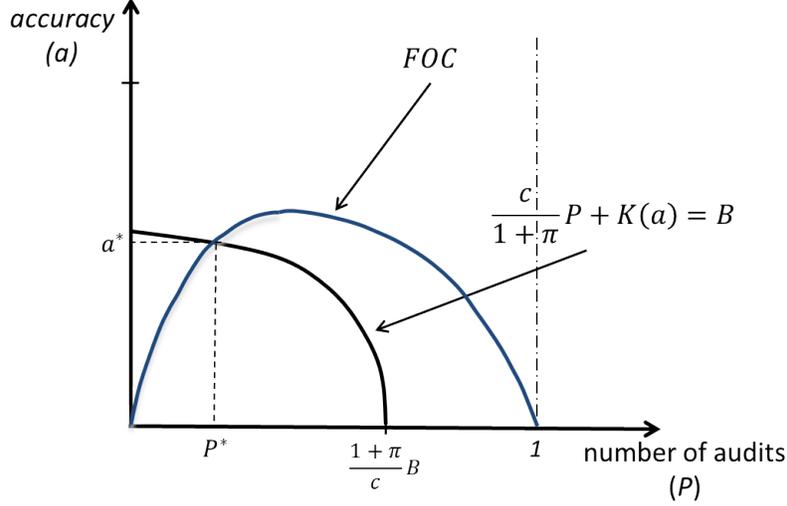
$$\frac{c}{1 + \pi} P^* + K(a^*) = B. \quad (14)$$

Equation (13) has a simple interpretation. At an interior optimum, the ratio of the marginal tax revenue from increasing signal accuracy to the marginal tax revenue from increasing the number of audits is equal to the ratio of the marginal cost of increasing signal accuracy to the marginal cost of increasing the number of audits.

above, $R(P)$ is indeed concave. It can be shown that $R''(P) = \frac{R(P)}{\varphi^2(\Phi^{-1}(P))} < 0$

²⁴There are two FOCs for problem (12) w.r.t. a and w.r.t. P which include a Lagrange multiplier. We can divide one by the other to eliminate the Lagrange multiplier.

Figure 2: The optimal solution (P^*, a^*) as intersection of the FOC condition and the budget constraint



The following proposition formalizes the conditions defining the optimal solution.

Proposition 2. *i) If $\gamma(z) = \frac{1-G(z)}{g(z)}$ is strictly decreasing and $K(a)$ is convex then the point – the accuracy a^* and the number of audits P^* – which satisfies to the condition (13) and (14) is the unique solution of (12);*

ii) If additionally $K'(a_L) = 0$ then the optimal solution is an interior solution characterized by

$$-\frac{R(P^*)}{a^* R'(P^*)} = \frac{1 + \pi}{c} K'(a^*). \quad (15)$$

Proof. In the appendix.

Proposition 2 provides conditions which ensure the existence of a unique interior optimum. Relying on this result and imposing these assumptions hereafter we can examine the properties of the optimum.

Figure 2 illustrates how the interior solution can be determined as the intersection of FOC (15) and budget constraint (14). As Figure (2) shows, FOC (15) has an inverse U-shape. This indicates a specific relationship between signal accuracy and the number of audits which can be described as the following. By the analogy to utility theory, let us define the marginal rate of substitution of signal accuracy, a , for the number of audits, P , as the ratio of the marginal tax revenue from increasing signal accuracy to the marginal tax revenue from increasing the

number of audits, that is,

$$MRS_{a,P} = \frac{\frac{\partial TR}{\partial a}}{\frac{\partial TR}{\partial P}} = -\frac{R(P)}{aR'(P)}. \quad (16)$$

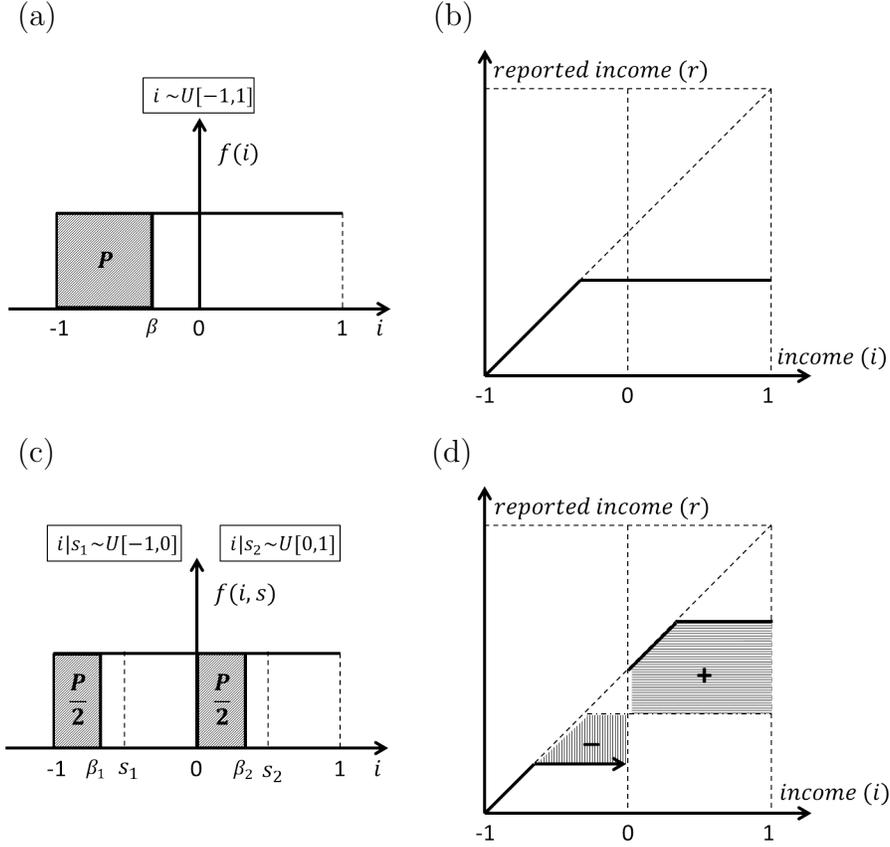
The marginal rate of substitution of signal accuracy, a , for the number of audits, P , shows how many audits could be saved through an increase in signal accuracy keeping tax revenue the same. Based on analogy with utility theory for two normal goods, one might expect that the marginal rate of substitution would decrease with signal accuracy and increase with the number of audits. $MRS_{a,P}$ is indeed decreases with signal accuracy, a . However, $MRS_{a,P}$ first increases with the number of audits, P , but then it decreases the number of audits. This causes a particular substitution pattern between signal accuracy and the number of audits. This substitution pattern can be described by considering how many audits the tax authority can save through an increase in signal accuracy provided that the tax revenue is the same. This number of “saved” audits initially increases with the number of audits, but then decreases with the number of audits. Since, the left hand side of FOC (15) is exactly equal to $MRS_{a,P}$, the shape of the FOC curve in Figure 2 reflects this substitution pattern.

Intuition To build intuition behind this substitution part, recall that in this model signals facilitates tax audits, because they allows the tax authority to divide taxpayers into audit classes and to conduct audits within each audit class separately. To illustrate how the tax revenue is increased by dividing people into audits classes, I consider a simple example. It also illustrates how this additional revenue depends on the number of audits conducted.

The example considers two cases when the tax authority conducts audits without obtaining any signals about taxpayers income, and when the tax authority does obtain signals allowing to divide taxpayers into audit classes and conduct audits within each audit class. To compare these two cases, I describe how the audits are conducted and what the reported income is. For simplicity, the example assumes that taxpayer’s true income is distributed uniformly on $[-1, 1]$. The rest of the assumptions about taxpayer are the same.

First, consider a case when the tax authority does not have any signals about taxpayers’ income and therefore all taxpayers belong to one audit class. According to Proposition 1, the audit rule in this case is to audit those taxpayers whose reported income is less than the audit trigger β , with probability $\frac{1}{1+\pi}$, and not audit those whose reported income is above the audit trigger β . If we denote the total number of audits, which the tax authority can conduct by P , then the audit trigger β is equal to $2P - 1$. This is indicated in Figure 3 (a). The next panel of the figure – Figure 3 (b) – shows how reported income depends on true income. When true income is less than the audit trigger β , taxpayers report income equaled

Figure 3: The example illustrating the value of dividing taxpayer into audit classes

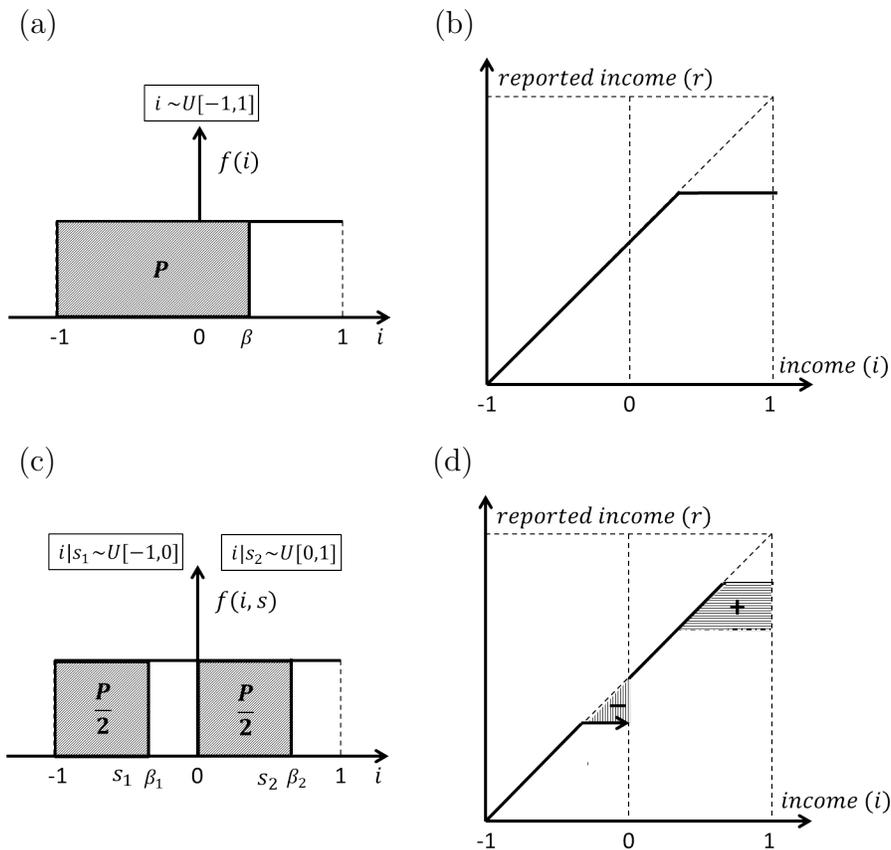


to true income. When true income is above the audit trigger β , reported income is equal to the audit trigger β . Given reported income, we can calculate the tax revenue in this case. It is equal to $TR_1 = -t(1 - P)^2$. This case provides a baseline to which the following case is compared.

Consider the second case when the tax authority is able to obtain two signals such that conditional on the first signal, s_1 , income is uniformly distributed on $[-1, 0]$ and conditional on the second signal, s_2 , income is uniformly distributed on $[0, 1]$. The probability of each signal is $\frac{1}{2}$. These signals allow the tax authority to divide taxpayers into two audit classes and conduct audits within each class separately. Figure 3 (c) illustrates how the audits are allocated within each audit class. For an easy comparison with Figure 3 (a), Figure 3 (c) shows the joint distribution of income and signal (i.e., $f(i, s) = f(i|s)\frac{1}{2}$).

The optimal audit strategy in each audit class is calculated according to Proposition 1. If a taxpayer's signal is s_1 then she is audited if her reported income is less than the audit trigger β_1 . If taxpayer's signals is s_2 then she is audited if her reported income is less than

Figure 4: The example illustrating the value of dividing taxpayer into audit classes: the case of large number of audits



the audit trigger β_2 . If the total number of audits is P , then the audit trigger β_1 is equal to $P - 1$ and the audit trigger β_2 is equal to P . This implies that $\frac{P}{2}$ audits are conducted in the first audit class and $\frac{P}{2}$ audits are conducted in the second audit class. The number of audits conducted in each class is the same because the marginal revenue of an additional audit should be the same in both audit classes.²⁵

This audit strategy induces reported income as shown in Figure 3 (d). Reported income is equal to true income when true income i belongs to either $[-1, \beta_1]$ or $[0, \beta_2]$, reported income is constant and equal to β_1 when $i \in [\beta_1, 0]$, and reported income is constant and equal to β_2 when $i \in [\beta_2, 1]$.²⁶ As can be seen, reported income in this case differs from reported income in the baseline case shown in Figure 3 (b). When audits are based on signals, reported income is substantially higher for $i \in [0, 1]$ and lower for $i \in [\beta_1, 0]$. So, dividing taxpayer into audits

²⁵Recall that the cost of an audit, c , is constant for all taxpayers.

²⁶When true income is equal to 0, reported income could be equal to β_1 or 0 depending on whether the signal is s_1 or s_2 .

classes helps to deter evasion by some high-income taxpayers.

The tax revenue in this case is higher than tax revenue in baseline case and is equal to $TR_2 = -\frac{t}{2}(1 - P)^2$. Interestingly, in the formula $TR_2 = -\frac{t}{2}(1 - P)^2$ the division of $-t(1 - P)^2$ by 2 resembles the division of $R(P)$ by accuracy a in the formula for tax revenue $TR(P) = t(\mu + \frac{1}{a}R(P))$ in general model. Overall, the comparison of the case when audits are based on signals with the baseline case shows that ability to divide taxpayers into audit classes helps to increase tax revenue because audits can be redirected toward some high-income taxpayers.

The additional tax revenue obtained by dividing taxpayers into audit classes decreases with the number of audits P . To show this, Figure 4 presents exactly the same example as Figure 3 but when the number of audits P is bigger. As can be seen, there are more high-income taxpayers who are audited in the baseline case when the number of audits P is bigger. As a result, the additional revenue from dividing taxpayers into audit classes is smaller. This also can be seen direct from the formula $TR_2 - TR_1 = \frac{t}{2}(1 - P)^2$. Indeed, $TR_2 - TR_1$ decreases with P .

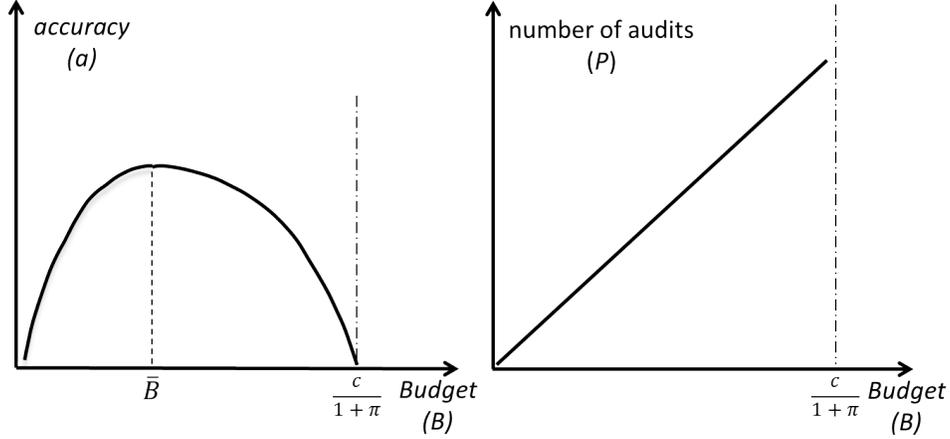
What this example cannot illustrate is how the revenue from increasing the number of audit classes changes relative to the revenue from increasing the number of audits. This relative change is important for understanding the substitution pattern, because the substitution between signal accuracy and the number of audits is determined by the relative change in revenue. By definition, the substitution pattern depends on how marginal revenue from increasing signal accuracy changes relative to the marginal tax revenue from increasing the number of audits. For understanding the behavior of the relative change in revenue, we have to rely on the general model.

The consequences of this substitution pattern are revealed in the next subsection where I analyze how the optimal solution depends on the budget of the tax authority.

4.3 Comparative statics

In this section I analyze how the budget and costs affect the optimal signal accuracy and the optimal number of audits. I examine first an effect of a change in the budget and second an effect of a change in the costs of audits and costs of improving signal accuracy. The finding of inverse U-shaped dependence of the optimal signal accuracy on the budget leads to important policy implications.

Figure 5: The effect of budget, B , change on the optimal solution (P^*, a^*)



4.3.1 Budget Change

The effect of the tax authority's budget change can be analyzed using Figure 2. An increase in the budget corresponds to a horizontal shift of the budget constraint curve to the right. As the budget constraint curve shifts to the right, the optimal solution moves along the FOC curve to the right. As a result, the optimal accuracy first increases and then decreases with the budget, B , while the number of audits always increases with the budget. The following proposition formally states this result.

Proposition 3. *Assume that assumptions i) and ii) of Proposition 2 are satisfied. There exist a threshold budget, \bar{B} , such that when the budget is small, $B < \bar{B}$, the optimal signal accuracy, a^* , increases with the budget, B . However, when the budget is large, $B > \bar{B}$, the optimal signal accuracy, a^* , decreases with the budget. The optimal number of audits, $P^* = P(a^*)$, always increases with the budget, B .²⁷*

Proof. In the appendix.

²⁷To address a potential curiosity regarding whether the solution to the net revenue maximization problem lie to the left or to the right of the threshold budget, \bar{B} , I provide the answer. It could lie either to the left or to the right of the threshold budget, \bar{B} . The reason for this is the following. The solution to the net revenue maximization problem the point on the FOC curve depicted in Figure 2 for which the Lagrange multiplier from Proposition 1 is equal to one. This point can be determined as an intersection the FOC curve depicted in Figure 5 with $a = \frac{t(1+\pi)}{c}R'(P)$ which is decreasing and convex curve. Note that the slope of $a = \frac{t(1+\pi)}{c}R'(P)$ depends on the value of t , π , and c . Therefore, the intersection can occur to the left or to right of the pick of the arc.

The results of Proposition 3 are illustrated in Figure 5. This relationship between the optimal solution and the size of the budget can be explained by applying to the substitution pattern between signal accuracy and the number of audits. Recall that the marginal rate of substitution decreases with signal accuracy, a , and first increases but then decreases the number of audits, P . We can measure the value of an increase in signal accuracy in terms of the number of audits which can be saved through the increase in signal accuracy provided that the tax revenue is the same. Given this measure, the value of signal accuracy initially increases, but then decreases with the number of audits; and it always decreases with signal accuracy. As a result, when the budget increases and the optimal number of audits also increases, the optimal signal accuracy after some raise starts to decline.

The intuition for this is the following. An improvement in signal accuracy allows dividing taxpayers into more distinct audit classes, which helps the tax authority to compensate for “non-increasing audit probability” rule and to reach by audits relatively high income taxpayers. However, the value of this additional subdivision into audit classes depends on the number of audited taxpayers, because the more high income taxpayers are audited the less is the value of having more income classes. Therefore, an improvement in signal accuracy is relatively more valuable when the number of audits is small than when it is large. The number of audited taxpayers, in turn, increases with the size of the budget.

This result has important policy implications. To describe implications, we step up from the model to real life by equating investments in signal accuracy with investments in information reporting. First implication concerns the tax authorities which have relatively scarce resources to enforce tax compliance. It is especially important for them to invest in information reporting. As follows from Proposition 3, tax authorities with small budget on average rely on improving information relative to increasing audits more than tax authorities with large budget. Second implication concerns the tax authorities which in terms of their resources approach the turning point (\bar{B}) where it becomes non-optimal to expand information reporting. Such tax authorities should be especially diligent in evaluating any proposition of expanding information reporting.

4.3.2 Cost Increases

In order to examine the effect of an increase in the cost of improving signal accuracy on the optimal signal accuracy and the optimal number of audits, I introduce an additional parameter δ . Precisely, I assume that the tax authority budget constraint is $\frac{c}{1+\pi}P + \delta K(a) = B$.

The only difference of this budget constraint formula from the preceding one is in parameter δ which augments the cost function $K(a)$. When $\delta = 1$, this formula is identical to the preceding one. This addition of δ into the budget constraint allows us model a raise in the cost of improving signal accuracy as an increase in parameter δ . The effect of an increase in the cost of improving signal accuracy is described in the following proposition. Also, the following proposition characterizes changes in the optimal signal accuracy and the optimal number of audits when the cost of an audit, c , increases or the penalty rate, π , decreases.

Proposition 4. *Assume that assumptions i) and ii) of Proposition 2 are satisfied.*

i) *When δ increases, i.e., the cost of improving signal accuracy increases, the optimal signal accuracy decreases, $\frac{da^*}{d\delta} < 0$, the optimal number of audits may increase, decrease, or not change, $\frac{dP^*}{d\delta} \gtrless 0$, depending on the shape of the cost function $K(a)$.*

ii) *When $\frac{c}{1+\pi}$ increases, i.e., the cost of an audit increases or the penalty rate decreases, the optimal number of audits decreases, $\frac{dP^*}{d\frac{c}{1+\pi}} < 0$, while the optimal signal accuracy may increase, decrease, or not change, $\frac{da^*}{d\frac{c}{1+\pi}} \gtrless 0$, when budget B is less than \bar{B} , and the optimal signal accuracy increases, $\frac{da^*}{d\frac{c}{1+\pi}} > 0$, when budget B is greater than \bar{B} .*

Proof. In the appendix.

The results of this proposition is easy to interpret by considering two types of effects, analogous to the substitution and income effects from utility theory. An increase in δ can be interpreted as an increase in the “price” of signal accuracy. The substitution effect is the result of an increase in the *relative* “price” of signal accuracy and causes a decreases in signal accuracy and an increase in the number of audits. The income effect of the “price” raise is due to the decrease in real budget (income) and causes a decreases in both signal accuracy and the number of audits. Therefore, when the “price” of signal accuracy increases, the optimal signal accuracy decreases, because both substitution and income effects are negative. However, the effect of an increase in the “price” of signal accuracy on the optimal number of audits is uncertain, because the substitution effect is positive, but income effect is negative. Which effect dominates depends on the shape of cost function $K(a)$. For example, when $K(a)$ is equal to qa^m where $q > 0$ and $m \geq 1$, the optimal number of audits does not change with δ (i.e., $\frac{dP^*}{d\delta} = 0$).

Similar, an increase in $\frac{c}{1+\pi}$ can be interpreted as an increase in the “price” of audits. Therefore, when the “price” of audits increases, the optimal number of audits decreases, because both substitution and income effects are negative. For the optimal signal accuracy, the substitution effect is positive, however, the sign of the income effect depends on the size of the budget. If $B < \bar{B}$ the income effect is negative, if $B > \bar{B}$ the income effect is positive. This is the consequence of Proposition 3. Hence, the change in the optimal signal accuracy is uncertain, $\frac{da^*}{d\frac{c}{1+\pi}} \begin{cases} \geq 0 \\ < 0 \end{cases}$, when budget B is less than \bar{B} , and the optimal signal accuracy increases, $\frac{da^*}{d\frac{c}{1+\pi}} > 0$, when budget B is greater than \bar{B} .

5 Discussion

In this section I discuss the results, their policy implications, and connections with the real world practice. I also critically examine assumptions of the model to assess its applicability.

The model of this paper gives us new insights regarding the optimal enforcement of tax evasion. First, we learned how audit policy should be conducted when information reporting is used to a different extent among people with different income sources such as wage earners and self-employed individuals. It is optimal for the tax authority to concentrate tax audits on those taxpayers about whom the tax authority has less accurate information. This result confirms our experience: audit probability seems to be higher for self-employed people rather than for wage earners.

Second result of the model explains what the optimal allocation of resources between tax audits and information should be when the tax authority has a limited budget. The optimal allocation of resources depends on the tax authority’s budget such that the optimal level of information reporting initially increases when the budget increases, but decreases later on, while the optimal number of audits always increases with the budget. Other words, the optimal level of information reporting depends on the budget as inverse U-shape function. This implies that there is a level of budget at which it is no longer optimal to expand information reporting.

This result has two important implications. One implication is that investment in information reporting are especially important when the tax authority has scarce resources to conduct tax enforcement. Another implication is for a tax authority which already has broad information reporting system. Such a tax authority could be close to a point at which its no longer optimal to expand information reporting and therefore has to carefully examine any decision to enlarge information reporting system.

Before we discuss these implications in the light of real world practice, it is helpful to determine applicability of the model by recalling its main assumptions. First, when the trade-off between tax audits and signal accuracy is considered, it is assumed that all signals are characterized by one accuracy, which implies that the accuracy of each signal improves in a similar manner. Therefore, this part of model applies to tax enforcement among a group of people with the same income source. The model can be easily extended to the case when there are several groups of people with different income sources such as wage earners, self-employed, etc. This problem with several income source groups can be solved in two stages. The first stage will have to determine how to allocate resources *within* a group. The solution to this stage can be obtained by using the model in this paper. The second stage will have to determine how to allocate resources *among* the groups.

Two more assumptions are worth to discuss. One assumption concerns costs of improving information reporting. In practice, one part of these costs – compliance cost – are born by third parties (firms/taxpayers). In the model, I assume that all costs of improving information reporting are imposed on the tax authority. However, I still account for compliance cost because I assume that the total costs of improving information reporting include a compensation to third parties for compliance cost.²⁸ Another assumption concerns the framework. I consider a one-period model, which does not allow accounting for “durable” property of information reporting. Once information reporting is put in place, it can be used in the next periods at a smaller costs. However, in order to correctly model information reporting in dynamic settings, we need a better understanding of size and shape of the cost of information reporting over the time. Unfortunately, little is known about the costs of improving information reporting because of the absence of empirical studies that estimate this.

Keeping in mind these assumptions, we can return to the result implications and discuss their connection to real world practice. The first implication that it is especially efficient to invest in information reporting when resources on enforcement are small most likely applies to developing countries. While it is hard to estimate the use of information reporting by developing countries due to the lack of data, according to general observations developing countries seems to underestimate the benefits of using information reporting. If so, the policy of adaptation of information reporting could be highly beneficial for the developing countries. The findings of Gordon and Li (2009) support this claim. Their study stresses that for developing countries the role of financial institutions as third parties in tax enforcement is significant. Additionally, Bird and Zolt (2008) point out that recent technological progress significantly

²⁸I do not model third parties explicitly, and therefore cannot account for the compliance cost directly.

simplified the adaptation of information reporting in developing countries. Moreover, some developing countries have been successfully using tax withholding which in many senses is similar to information reporting. In her study of the provision and proposals for withholding on business income, Soos (1990) finds that developing countries seem to make greater use of withholding systems than those of industrialized countries.”²⁹

Second implication, which concerns countries that have substantial information reporting, is probably more relevant for developed countries. They might be close to the point where it is no longer optimal to expand information reporting system. For them it becomes critical to perform a thorough cost-benefit analysis when an expansion of information reporting is considered. A decision whether this expansion is worthwhile should be made based on this analysis. Moreover, the cost calculation should definitely account for indirect cost of information reporting which is diverting resources from audits.

The US is an example of a developed country which recent has expanded its information reporting system. It introduced requirement on banks to report the gross amount of merchant payment card transactions (1099-K Form), and brokers to report the adjusted cost basis for certain securities (1099-B Form). It also extended requirements on businesses regarding payments to vendors, subcontractors, independent contractors (1099-MISC). To implement these new information reporting requirements, the IRS had to develop a new program that matches information returns with tax returns. The cost of this project constituted about \$110 million during the period 2009 through 2012.³⁰ The exact revenue benefits of this expansion are still to be estimated. Slemrod et al. (forthcoming) investigate some preliminary responses and find that introduction of 1099-K has led to a relatively small overall change in reported tax liability, while it has significantly increased tax reports within a small subgroup of sole proprietors.

6 Conclusion

The accuracy of information observed by the tax authority about taxpayers income directly affects the level of tax evasion – a phenomenon which is mainly caused by the asymmetry of information between taxpayers and the tax authority. Information reporting helps to reduce this information asymmetry and thus to deter tax evasion, because it requires third parties to

²⁹Indonesia, Pakistan, and Egypt withhold on payments for goods and supplies as well as on fees for services. The Philippines, Japan, and Ireland provide for withholding on fees for services. The United States has proposed for withholding on fees for services. The United Kingdom, Ireland, and Australia have narrow provisions which limit withholding to fees for work or services in certain industries.

³⁰For more details see GAO (2011).

send reports to the tax authority with information about income payments to payees which allows verifying payees' tax returns.

Nevertheless, the optimal tax enforcement theory has not considered information reporting as an instrument to improve tax compliance focusing primarily on tax audits. This paper extends tax theory by introducing a way to model information reporting and incorporating it into tax evasion framework as an additional enforcement instrument. Moreover, this paper determines how the optimal tax enforcement policy should trade off resources between tax audits and information reporting, when both of these instruments are costly and the tax authority has limited budget.

Specifically, the model assumes that through information reporting the tax authority can obtain signals, which enable constructing more accurate prediction of taxpayers' income distributions. More information reporting corresponds to signals characterized by higher accuracy which means that they produce more accurate prediction of income distribution. However, an increase in signal accuracy is costly and diverts resources from tax audits. When the tax authority has limited budget, it has to decide how to allocate resources between tax audits and signal accuracy in order to maximize tax revenue. The model solves this trade-off by determining the optimal level of signal accuracy and the optimal number of audits, and it also explains how the choice of one instrument depends on the choice of the other.

First, the model explains how the optimal tax audit policy depends on signal accuracy. Note that in the model the audit probability is a function of the taxpayer's reported income as well as of the signal about the taxpayer's income. In the optimum, the tax authority should concentrate tax audits on those taxpayers about whom it has less accurate signals, so that the marginal tax revenue is the same among all taxpayers. The practical flavor of this result reveals when it is applied to people subject to information reporting in different extent. When two groups of taxpayers differ in the extent of information reporting, like wage earners and self-employed individuals, the optimal policy instructs to focus tax audits on the group with a lower extent of information reporting, like self-employed.

Second, this paper derives the optimal allocation of resource between information reporting and audits. On the cost side, to raise signal accuracy the tax authority has to deprive resources from tax audits. On the benefit side, as signal accuracy rises, the taxpayers can be divided into more distinct audit classes, which facilitates tax audits. The model enable measuring the benefit of an increase in signal accuracy. The value of an increase in signal accuracy can be measured in the number of audits which can be saved thorough the increase the signal accuracy, provided that the amount of tax revenue collection is the same. I find that the value of an increase in signal accuracy depends on the number of audits so that it

initially increases, but then decreases as the number of audits decreases. This substitution pattern between signal accuracy and the number of audits leads to a particular dependence of the optimal solution on the tax authority's budget. The optimal signal accuracy depends on the budget as an inverse U-shaped function, while the optimal number of audits always increases with the budget. This indicates that at some budget it becomes no longer optimal to increase signal accuracy.

This finding has two important implications. First, the investment in information reporting appears to be especially critical for developing countries. Second, countries which have extensive information reporting system should base their decision of whether to expand it further on a careful examination of cost and benefits, since it might not be beneficial for them to expand information reporting system.

This paper clarifies that though both audits and information reporting help the tax authority to access information about taxpayers, their mechanisms of accessing information are different. As a result of this difference, the problem of optimal tax enforcement should be considered as a problem of *joint* choice of the number of audits and information reporting scope. This paper starts to address this question and highlights the necessity for further research in this area.

Appendix

A An example of an optimal audit strategy characterized by leaving some taxpayers unaudited

Assume that income, i , is uniformly distributed on $[0, I]$. Also suppose that a signal is generated according to a process such that $s = i + \epsilon$, where noise ϵ is uniformly distributed on $[-\frac{1}{a}, \frac{1}{a}]$. In what follows it is assumed that the lowest a is $a_L = \frac{2}{I}$. Based on this, the density of signal s can be calculated and expressed as

$$h(s) = \begin{cases} \frac{\min\{I, s + \frac{1}{a}\} - \max\{0, s - \frac{1}{a}\}}{2I\frac{1}{a}}, & \text{if } -\frac{1}{a} \leq s \leq I + \frac{1}{a} \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{s + \frac{1}{a}}{2I\frac{1}{a}}, & \text{if } -\frac{1}{a} \leq s \leq \frac{1}{a} \\ \frac{1}{I}, & \text{if } \frac{1}{a} \leq s \leq I - \frac{1}{a} \\ \frac{I - s + \frac{1}{a}}{2I\frac{1}{a}}, & \text{if } I - \frac{1}{a} \leq s \leq I + \frac{1}{a} \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

The conditional density of income given signal s is

$$f(i|s) = \begin{cases} \frac{1}{\min\{I, s + \frac{1}{a}\} - \max\{0, s - \frac{1}{a}\}}, & \text{if } \max\{0, s - \frac{1}{a}\} < i < \min\{I, s + \frac{1}{a}\} \\ 0, & \text{otherwise} \end{cases}, \quad (18)$$

the conditional c.d.f. is

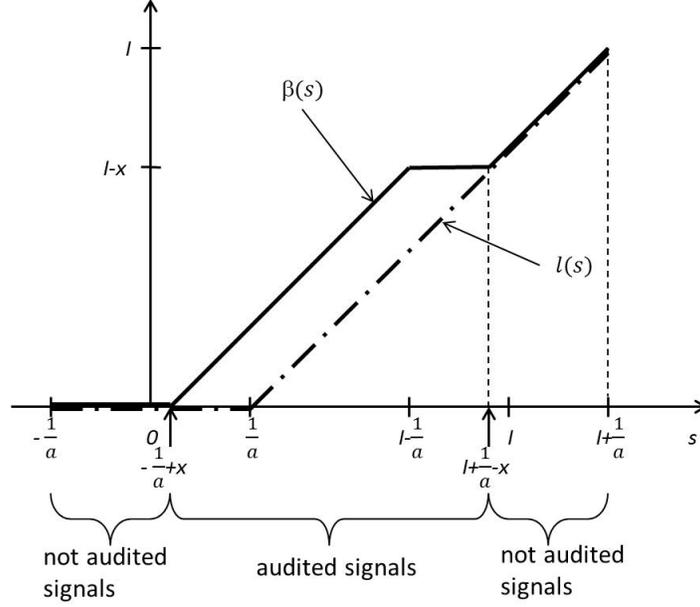
$$F(i|s) = \begin{cases} \frac{i - \max\{0, s - \frac{1}{a}\}}{\min\{I, s + \frac{1}{a}\} - \max\{0, s - \frac{1}{a}\}}, & \text{if } \max\{0, s - \frac{1}{a}\} < i < \min\{I, s + \frac{1}{a}\} \\ 0, & \text{otherwise} \end{cases}, \quad (19)$$

and $\gamma(i|s) = \min\{I, s + \frac{1}{a}\} - i$, for $\max\{0, s - \frac{1}{a}\} \leq i \leq \min\{I, s + \frac{1}{a}\}$.

In this example, signal $s = -\frac{1}{a}$ can be produced only by income $i = 0$. More generally, very low (high) signals can be produced only by low (high) enough incomes. This is because the support of the income distribution is bounded, $i \in [0, I]$. Given this relation between signals and incomes, when receiving low (high) signals the tax authority can infer that a taxpayers with such signals can have only low (high) enough income. As a result, as seen from (18), for low signals, $s \in [-\frac{1}{a}, \frac{1}{a}]$, and for high signals, $s \in [I - \frac{1}{a}, I + \frac{1}{a}]$ the conditional density of income is higher and is distributed on a shorter interval than for the rest of the signals. This indicates that for a given a low signals, $s \in [-\frac{1}{a}, \frac{1}{a}]$, and high signals, $s \in [I - \frac{1}{a}, I + \frac{1}{a}]$ are relatively more accurate, i.e., having a lower variance, than the rest of the signals. Moreover, the lower the signal within interval $[-\frac{1}{a}, \frac{1}{a}]$ (the higher the signal within interval $[I - \frac{1}{a}, I + \frac{1}{a}]$) the more accurate the signal is.

To describe the optimal audit policy for this example, let's calculate the cutoff function by using (7) and (8). The cutoff function can be and expressed as

Figure 6: The cutoff function



$$\beta(s) = \begin{cases} 0, & \text{if } -\frac{1}{a} \leq s \leq -\frac{1}{a} + x \\ \min\{I, s + \frac{1}{a}\} - x, & \text{if } -\frac{1}{a} + x < s < I + \frac{1}{a} - x \\ s - \frac{1}{a}, & \text{if } I + \frac{1}{a} - x \leq s \leq I + \frac{1}{a}, \end{cases} \quad (20)$$

where $x \equiv \frac{I + \frac{2}{a} - \sqrt{(I + \frac{2}{a})^2 - \frac{8I}{a}(1-P)}}{2}$ is the solution to the equation $x^2 - x(I + \frac{2}{a}) + \frac{2I}{a}(1-P) = 0$ which satisfies $0 \leq x \leq \frac{2}{a}$, where again $P = \frac{1+\pi}{c}[B - K(a)]$ is the number of taxpayers who with probability $\frac{1}{1+\pi}$ are audited. For conciseness of notation I write x instead of $x(a, P(a))$, but it is important to remember that x is a function of a .

As shown in Figure 6, for low enough signals, $s \in [-\frac{1}{a}, -\frac{1}{a} + x]$, and high enough signals, $s \in [I + \frac{1}{a} - x, I + \frac{1}{a}]$ the cutoff income, $\beta(s)$ is equal to the lowest income $l(s)$. This means that taxpayers with those signals, $s \in [-\frac{1}{a}, -\frac{1}{a} + x]$ and $s \in [I + \frac{1}{a} - x, I + \frac{1}{a}]$, are not audited. The taxpayers with signal $s \in (-\frac{1}{a} + x; I + \frac{1}{a} - x)$ are audited if their reported income is below $\min\{I, s + \frac{1}{a}\} - x$. The value of x is defined so that for a given level of signal accuracy, there are enough resources for audits given the chosen cutoff. So, the optimal audit rule prescribes the tax authority not to audit those taxpayers about whom they have relatively more accurate signals and focus audits only on those about whom the tax authority receives relatively more accurate signals.

It is also of interest to consider how the cutoff income (20) depends on signal accuracy.

There are two channels through which a change in accuracy a influences the cutoff function. One effect is mechanical: an increase in a requires a higher investment in accuracy, i.e., a bigger $K(a)$, leaving fewer resources for audits, P , which results in a decrease of the cutoff income for each signal. The other effect is a direct consequence of a signal providing a more accurate prediction of income. This effect can be analyzed as an exogenous change in a , assuming the available number of audits, P , to be fixed; and it appears to be contingent on P . Roughly, when a converges to infinity, the cutoff income, $\beta(s)$, contracts to the conditional expectation of the income, which is s (or converges to s).

More specifically, this dependence for an interior signal $s \in [\frac{1}{a}; I - \frac{1}{a}]$ can be described as follows. Note first that $x > \frac{1}{a}$ when $\frac{1}{a} < I(1 - 2P)$, $\frac{\partial x}{\partial(1/a)} = \frac{\partial x}{\partial \sigma} > 1$ when $\frac{1}{a} < \frac{I(1-2P)}{2}$, and $\frac{\partial x}{\partial a} < 0$ always. So, when the number of audits is high enough, $P \geq \frac{1}{2}$, the cutoff income, $\beta(s)$, is always greater than s , and it decreases as a increases. When $\frac{1}{4} \leq P < \frac{1}{2}$, the cutoff income, $\beta(s)$, is greater than s and decreases as a increases until $\frac{1}{a} > I(1 - 2P)$. After this, $\beta(s)$ becomes less than s , but continues to decrease as a increases until $\frac{1}{a} > \frac{I(1-2P)}{2}$. Then $\beta(s)$ begins to increase as a increases staying below s . When $P < \frac{1}{4}$, the cutoff income, $\beta(s)$, is less than s , it decreases as a increases until $\frac{1}{a} > I(1 - 2P)$, and after this increases as a increases.

However, the described dependence of $\beta(s)$ on a does not in itself determine a change in the audit probability for each signal because the lowest (highest) conditional income $l(s)$ ($h(s)$) and the conditional density change with a as well. For this reason, let us consider how audits are reallocated between signals as a increases, assuming that the number of audits, P , is fixed. The number of taxpayers with a low and high enough signal who are left unaudited which is equal to $\int_{-\frac{1}{a}+x}^{-\frac{1}{a}} h(s)ds + \int_{I+\frac{1}{a}-x}^{I+\frac{1}{a}} h(s)ds = \frac{x^2}{2\sigma I} = \frac{ax(a)^2}{2I}$, decreases as a increases. Correspondingly, the number of signals (taxpayers), which may appear to be audited, increases. This can happen because for a given signal the probability of auditing decreases. Indeed, conditional on a signal s ($s \in [\sigma; I - \sigma]$), the probability of auditing is $\int_{s-\frac{1}{a}}^{\beta(a)} f(i|s)di = 1 - \frac{x}{2\sigma} = 1 - \frac{ax(a)}{2}$ which decreases as a increases³¹.

Let us refer to a change in the number of taxpayers subjected to audits as a change in the extensive margin of audits, while a change in the probability of auditing one taxpayer will be called a change in the intensive margin of audits. Given that the total number of audits is unchanged, the above observations demonstrate that when signal accuracy increases the extensive margin of audits increases, while the intensive margin of audits decreases.

The general logic for this result is the following. The optimal auditing policy requires that

³¹For $y \in [-\frac{1}{a} + x, \frac{1}{a}]$ and $y \in [I - \frac{1}{a}, I + \frac{1}{a} - x]$ this dependence is more complicated. However, the share of such signals goes to zero as a goes to infinity.

for all audited individuals the net marginal revenue from an audit is the same. If there exists a relatively more accurate set of signals, so that audits of the taxpayers with such signals cannot generate high enough extra tax charges due to high degree of voluntary compliance, than taxpayers with such signals are not audited. When accuracy of all signals uniformly increases the marginal revenue of an audit audited individuals decreases. As a result, those signals, which initially were excluded from audits due to relatively low potential marginal revenue of an audit, might be included in class of audited signals.

B Proofs

Proof of Proposition 1

As discussed, the tax authority maximization problem (5) can be solved in two steps. First, for any given amount of resources assigned to conduct audits of taxpayers with signal s , $B(s)$, the audit probability is chosen to maximize $\int T(i, s, p(\cdot))dF(i | s)$ s.t. $\int c \cdot p(r(i, s), s)dF(i | s) = B(s)$. Second, optimal allocation of resources, $B(s)$, is chosen to maximize the tax authority objective function s.t. $\int B(s)dH(s) \leq B - K(a)$.

To see this, let's denote $B(s) = \int c \cdot p(r(i, s), s)dF(i | s)$. Then, problem (5) can be rewritten as

$$\begin{aligned} & \max_{p(r(i, s))} \left\{ \int \int T(i, s, p(\cdot))dF(i | s)dH(s) \right\} \\ & \text{s.t. } \int c \cdot p(r(i, s), s)dF(i | s) = B(s) \forall s, \\ & \int B(s)dH(s) \leq B - K(a). \end{aligned} \tag{21}$$

The Lagrangian for this problem is

$$\begin{aligned} L = \int \left\{ \int T(i, s, p(\cdot))dF(i | s) + \mu(s) \left(B(s) - \int c \cdot p(r(i, s), s)dF(i | s) \right) \right\} dH(s) + \\ + \lambda \left[B - K(a) - \int B(s)dH(s) \right]. \end{aligned}$$

As can be seen, the maximization of the Lagrangian w.r.t. $B(s)$ requires $\mu(s) = \lambda$ for all s if $B(s) > 0$.

Now, let us consider the problem of choosing optimal audit probability $p(r(i), s)$ for a given signal s to maximize $\int T(i, s, p(\cdot))dF(i | s)$ s.t. $\int c \cdot p(r(i, s), s)dF(i | s) = B(s)$.

If the conditional distribution of income has a bounded support, and hence the lower bound, $l(s)$, is finite, then this problem is identical to the one solved by Sanchez and Sobel

(1993). It was shown that the optimal probability is $p^*(r, s) = \begin{cases} \frac{1}{1+\pi}, & \text{if } r < \beta(s) \\ 0, & \text{if } r \geq \beta(s) \end{cases}$, where

$\gamma(\beta(s) | s) = \frac{\mu(s)c}{t(1+\pi)}$. By combining this result with the requirement that $\mu(s) = \lambda$ for all s if $B(s) > 0$, we can express the optimal solution as described by (6), (7), and (8).

Now, let us consider the case when the support of the conditional distribution of income is unbounded. Let us divide the support of the income distribution, $F(i|s)$ into two intervals: $(-\infty, l_n)$ and $[l_n, \infty)$, where l_n is some number. The total tax revenue can be calculated as a sum of two parts - the tax revenues corresponding each of two intervals, that is,

$$TR(s) = \int_{-\infty}^{+\infty} T(i, s, p(\cdot))dF(i | s) = \int_{-\infty}^{l_n} T(i, s, p(\cdot))dF(i | s) + \int_{l_n}^{+\infty} T(i, s, p(\cdot))dF(i | s). \quad (22)$$

We can estimate the upper bound of the total tax revenue as follows. First, assume that taxpayers with income $i \in (-\infty, l_n)$ are honest and, therefore, report their income truthfully. So, the tax authority does not need to spend resources to conduct audits among those taxpayers, and at the same time can collect the maximal revenue, $\int_{-\infty}^{l_n} tidF(i | s)$. This gives an estimate of $\int_{-\infty}^{l_n} T(i, s, p(\cdot))dF(i | s)$ from above which is $\int_{-\infty}^{l_n} tidF(i | s)$.

Second, let assume that taxpayers with income $i \in [l_n, +\infty)$ cannot report income less than l_n . This assumption reduces the number of incentive compatibility constraints for the problem under consideration, and therefore, makes our estimate of the upper bound for the tax revenue only higher. Also, this assumption allows us to apply results from Sanchez and Sobel (1993) to calculate the maximum for $\int_{l_n}^{+\infty} T(i, s, p(\cdot))dF(i | s)$ given that budget constraint is $\int_{l_n}^{+\infty} c \cdot p(r(i, s), s)dF(i | s) = B(s)$. The optimal audit policy is characterized

by $p^*(r, s) = \begin{cases} \frac{1}{1+\pi}, & \text{if } r < \beta_n(s) \\ 0, & \text{if } r \geq \beta_n(s) \end{cases}$, $\gamma(\beta_n(s) | s) = \frac{\mu_n(s)c}{t(1+\pi)}$, and $\int_{l_n}^{b_n(s)} \frac{c}{1+\pi}dF(i | s) = B(s)$. The

last equation defines $b_n(s)$. The solution provides the maximum for $\int_{l_n}^{+\infty} T(i, s, p(\cdot))dF(i | s)$, which is equal to $\int_{l_n}^{b_n(s)} tidF(i | s) + \int_{b_n(s)}^{+\infty} tb_n(s)dF(i | s)$. Thus, an upper bound for the total tax revenue is

$$\begin{aligned} T\hat{R}_n(s) &= \int_{-\infty}^{l_n} tidF(i | s) + \int_{l_n}^{b_n(s)} tidF(i | s) + \int_{b_n(s)}^{+\infty} tb_n(s)dF(i | s) = \\ &= \int_{-\infty}^{b_n(s)} tidF(i | s) + b_n(s) \int_{b_n(s)}^{+\infty} tdF(i | s). \end{aligned}$$

Consider a sequence $\{l_n\}$ such that $\lim_{n \rightarrow \infty} l_n = -\infty$. Let us calculate the limit of the tax revenue upper bounds as l_n converge to $-\infty$. By the properties of limit

$$\lim_{l_n \rightarrow -\infty} T\hat{R}_n(s) = \int_{-\infty}^{b(s)} tidF(i | s) + b(s) \int_{b(s)}^{+\infty} tdF(i | s),$$

where $b(s)$ denotes $\lim_{l_n \rightarrow -\infty} b_n(s)$. It is easy to see that $b(s)$ exists and is the solution for $\int_{-\infty}^{b(s)} \frac{c}{1+\pi} dF(i | s) = B(s)$

Because $\lim_{l_n \rightarrow -\infty} T\hat{R}_n(s)$ is still an upper bound, it is true that

$$TR(s) = \int_{-\infty}^{+\infty} T(i, s, p(\cdot))dF(i | s) \leq \lim_{l_n \rightarrow -\infty} T\hat{R}_n(s)$$

If we verify that the policy function characterized by $p^*(r, s) = \begin{cases} \frac{1}{1+\pi}, & \text{if } r < \beta(s) \\ 0, & \text{if } r \geq \beta(s) \end{cases}$, and

$\int_{-\infty}^{b(s)} \frac{c}{1+\pi} dF(i | s) = B(s)$ allows the tax authority to obtain the amount of tax revenue equaled to $\lim_{l_n \rightarrow -\infty} T\hat{R}_n(s)$, then this will conclude our proof that this policy function is optimal. Indeed, this policy function provides the tax revenue equaled to $\int_{-\infty}^{b(s)} tidF(i | s) + \int_{b(s)}^{+\infty} tb(s)dF(i | s) = \lim_{l_n \rightarrow -\infty} T\hat{R}_n(s)$.

Therefore, even in the case when the support of income distribution is unbounded the optimal audit policy is described by (6), (7), and (8). **Q.E.D.**

Proof of Corollary

Based on condition (7) from Proposition 1, it should be that

$$\frac{1 - G(a_1(\beta_1 - s_1))}{a_1 g(a_1(\beta_1 - s_1))} = \frac{1 - G(a_2(\beta_2 - s_2))}{a_2 g(a_2(\beta_2 - s_2))}, \quad (23)$$

where β_1 and β_2 are the cutoff incomes in the first group with accuracy a_1 and in the second group with accuracy a_2 correspondingly.

Let's define $\Delta_1 = a_1(\beta_1 - s_1)$ and $\Delta_2 = a_2(\beta_2 - s_2)$. Let's denote by $\tilde{\gamma}(\cdot) = \frac{1-G(\cdot)}{g(\cdot)}$. Equation (23) can be then rewritten as

$$\frac{\tilde{\gamma}(\Delta_1)}{\tilde{\gamma}(\Delta_2)} = \frac{a_1}{a_2}$$

By assumption of Proposition 1, $\tilde{\gamma}(\cdot)$ is strictly decreasing function. Hence, because $\frac{a_1}{a_2} < 1$, $\Delta_1 > \Delta_2$. Therefore, $G(\Delta_1) > G(\Delta_2)$. **Q.E.D.**

Proof of Proposition 2

i) Recall that $R'(P) = \gamma(G^{-1}(P))$ where $G^{-1}(P)$ is strictly decreasing function. Therefore, the condition that inverse hazard rate $\gamma(z) = \frac{1-G(z)}{g(z)}$ is strictly decreasing implies that $R(P)$ is strictly concave. Moreover, strict concavity of $R(P)$ implies that $TR(a, P) = t(\mu + \frac{1}{a}R(P))$ is strictly quasi-concave.³²

When $K(a)$ is convex, the budget constraint set $\frac{c}{1+\pi}P + K(a) \leq B$ is convex. Thus, the conditions that $\gamma(z) = \frac{1-G(z)}{g(z)}$ is strictly decreasing and $K(a)$ is convex are sufficient to satisfy the assumptions the Theorem M.K.4 from Mas-Colell, Whinston, Green “Microeconomic Theory”, Oxford University Press, 1995, which guarantees that the accuracy a which satisfies to the condition (13) is the unique solution of (12).

ii) When $\frac{R(0)}{R'(0)} = 0$ ³³ and $K'(a_L) = 0$ a corner solution is impossible. Indeed, for a_H to be an optimum, it should be that $-\frac{c}{1+\pi} \frac{R(0)}{a_H R'(0)} - K'(a_H) > 0$, but because $\frac{R(0)}{R'(0)} = 0$ it follows that $-\frac{c}{1+\pi} \frac{R(0)}{a_H R'(0)} - K'(a_H) = -K'(a_H) < 0$. Therefore, a_H cannot be a solution.

For a_L to be an optimum, it should be that $-\frac{c}{1+\pi} \frac{R(\frac{1+\pi}{c}B)}{a_L R'(\frac{1+\pi}{c}B)} - K'(a_L) < 0$, but because $K'(a_L) = 0$ it follows that $-\frac{c}{1+\pi} \frac{R(\frac{1+\pi}{c}B)}{a_L R'(\frac{1+\pi}{c}B)} - K'(a_L) = -\frac{c}{1+\pi} \frac{R(\frac{1+\pi}{c}B)}{a_L R'(\frac{1+\pi}{c}B)} > 0$ for any $0 < \frac{1+\pi}{c}B < 1$. Therefore, a_L cannot be a solution.

Thus, the solution can be only interior. **Q.E.D.**

Proof of Proposition 3

Under the assumptions of Proposition 2 there is only the interior solution which is defined by $-\frac{c}{1+\pi} \frac{R(P)}{a^* R'(P)} = K'(a^*)$. By differentiating this expression w.r.t. B , we can obtain $\frac{da^*}{dB}$ which is equal to

$$\frac{da^*}{dB} = \frac{1}{[-SOC(a^*)]} \frac{1}{a^*} \left[\frac{R(P^*)R''(P^*)}{R'(P^*)^2} - 1 \right], \quad (24)$$

where $SOC(a^*) = -K''(a^*) + \frac{c}{1+\pi} \frac{R(P^*)^2 R''(P^*)}{a^{*2} R'(P^*)^3} < 0$ is negative due to the assumptions.

From this expression it follows that the dependance of a^* on the budget B is defined by the sign of $\frac{R(P^*)R''(P^*)}{R'(P^*)^2} - 1$.

Let us show that there exist the number of audits which we denote by \bar{P} such that $\frac{R(\bar{P})R''(\bar{P})}{R'(\bar{P})^2} = 1$.

In the case when the c.d.f. is the standard normal (i.e., $G(\cdot) = \Phi(\cdot)$), the value of $\frac{R(P)R''(P)}{R'(P)^2}$ varies from (positive) infinity to zero as P varies from 0 to 1. Define \bar{P} such that

³²Note that the better set $\{(a, P) : TR(a, P) \geq TR(a_0, P_0)\}$ is convex for any (a_0, P_0) .

³³This condition is true when the c.d.f. is the standard normal (i.e., $G(\cdot) = \Phi(\cdot)$)

$$\frac{R(\bar{P})R''(\bar{P})}{R'(\bar{P})^2} = 1.$$

By differentiating $P^* = \frac{1+\pi}{c} (B - K(a^*))$ w.r.t. B and substituting $\frac{da^*}{dB}$ into the formula, we can obtain $\frac{dP^*}{dB}$ which is equal to

$$\frac{dP^*}{dB} = \frac{1}{[-SOC(a^*)]} \left[\frac{1+\pi}{c} K''(a^*) - \frac{R(P^*)}{a^{*2} R'(P^*)} \right] > 0. \quad (25)$$

This derivative shows that optimal P^* increases with the budget. Moreover, because TR is continuous in a and B , and the set of compacts $a \in [a_L, a_H(B)]$ is continuous, the maximizer $a^*(B)$ is continuous³⁴ in B by the Theorem of the Maximum. Therefore, $P^* = \frac{1+\pi}{c} (B - K(a^*(B)))$ is continuous in B as well. Also P^* converges to 0 as B converges to 0 which follows from the fact that $P^* \leq \frac{1+\pi}{c} B$. And P^* converges to 1 as B converges to $\frac{c}{1+\pi}$, because when $B = \frac{c}{1+\pi}$ then $P^* = 1$ gives the maximum value.

These four facts insure that there exists a value of B , which we define as \bar{B} , such that $\frac{1+\pi}{c} (\bar{B} - K(a^*(\bar{B}))) = \bar{P}$. Hence, $\frac{da^*}{dB} > 0$ when $B < \bar{B}$ and $\frac{da^*}{dB} < 0$ when $B > \bar{B}$. **Q.E.D.**

Proof of Proposition 4

i) Under the assumptions of Proposition 2 there is only the interior solution which is defined by $-\frac{c}{1+\pi} \frac{R(P)}{\delta a^* R'(P)} = K'(a^*)$. Note that the FOC is adjusted to account for the inclusion of parameter δ . By differentiating this expression w.r.t. δ , we can obtain $\frac{da^*}{d\delta}$ which is equal to

$$\frac{da^*}{d\delta} = \frac{1}{[-SOC(a^*)]} \left\{ -\frac{1}{\delta} \left[K'(a^*) - \frac{K(a^*)}{a^*} \right] - \frac{K(a^*) R(P^*) R''(P^*)}{\delta a^* R'(P^*)^2} \right\} < 0, \quad (26)$$

where $K'(a^*) - \frac{K(a^*)}{a^*} > 0$ because $K(a)$ is convex, and, as before, $SOC(a^*) = -K''(a^*) + \frac{c}{1+\pi} \frac{R(P^*)^2 R''(P^*)}{a^{*2} R'(P^*)^3} < 0$. Therefore, $\frac{da^*}{d\delta}$ is negative.

An expression for $\frac{dP^*}{d\delta}$ can be obtained by differentiating $P^* = \frac{1+\pi}{c} (B - \delta K(a^*))$ w.r.t. δ and substituting $\frac{da^*}{d\delta}$ into the formula, which gives

$$\begin{aligned} \frac{dP^*}{d\delta} &= -\frac{1+\pi}{c} K(a^*) + \frac{1+\pi}{c} \delta K'(a^*) \left(-\frac{da^*}{d\delta} \right) = \\ &= \frac{1+\pi}{c [-SOC(a^*)]} \left\{ -K(a^*) K''(a^*) + K'(a^*) \left[K'(a^*) - \frac{K(a^*)}{a^*} \right] \right\}. \end{aligned} \quad (27)$$

The first term in (27) is negative, the second term in (27) is positive. As a result, the sign of $\frac{dP^*}{d\delta}$ is uncertain. Intuitively, there are two effects of an increase in δ which can be referred to as substitution and income effects by analogy with the effects of price change

³⁴Specifically, $a^*(B)$ is hemi-continuous, but because the maximizer a^* is unique for each B , it is continuous

on normal goods in standard consumption theory. An increase in δ can be interpreted as an increase in “price” of signal accuracy. First, it causes a decrease in the optimal signal accuracy, which leaves more resources for audits, and, therefore, increases P^* . This is the substitution effect which is positive. Second, it causes the income effect which is negative. Specifically, an increase in δ requires greater resource expenditure to provide the same level of signal accuracy. This leaves less resources for audits, and, therefore, decreases P^* .

ii) By differentiating the FOC $-\frac{c}{1+\pi} \frac{R(P)}{a^* R'(P)} = K'(a^*)$ w.r.t. $\frac{c}{1+\pi}$ we can obtain $\frac{da^*}{d\frac{c}{1+\pi}}$ which is equal to

$$\frac{da^*}{d\frac{c}{1+\pi}} = \frac{1}{[-SOC(a^*)]} \frac{1}{a^*} \left[\frac{-R(P^*)}{R'(P^*)} + P^* \left(1 - \frac{R(P^*)R''(P^*)}{R'(P^*)^2} \right) \right], \quad (28)$$

The first term in (28) is positive, the second term in (28) is negative if $B < \bar{B}$ and positive if $B > \bar{B}$. As a result, the sign of $\frac{da^*}{d\frac{c}{1+\pi}}$ is uncertain if $B < \bar{B}$ and positive if $B > \bar{B}$.

Considering substitution and income effects of increase in $\frac{c}{1+\pi}$ is helpful to provide an intuition for this result. First, the substitution effect of an increase in $\frac{c}{1+\pi}$ on the optimal signal accuracy is positive, because an increase in $\frac{c}{1+\pi}$ causes a decrease in P^* , which releases additional resources for signal accuracy improvement.

Second, because an increase in $\frac{c}{1+\pi}$ requires greater resources to conduct the same number of audits leaving less resources to invest in signal accuracy, it has the income effect on the optimal signal accuracy. The sign of the income effect depends on the size of the budget. If $B < \bar{B}$ the income effect is negative, if $B > \bar{B}$ the income effect is positive.

An expression for $\frac{dP^*}{d\frac{c}{1+\pi}}$ can be obtained by differentiating $P^* = \frac{1+\pi}{c} (B - K(a^*))$ w.r.t. $\frac{c}{1+\pi}$ and substituting $\frac{da^*}{d\frac{c}{1+\pi}}$ into the formula, which gives

$$\frac{dP^*}{d\frac{c}{1+\pi}} = \frac{1}{[-SOC(a^*)]} \frac{1+\pi}{c} \left\{ -PK''(a^*) - \frac{K'(a^*)}{a^*} \frac{P^* R'(P^*) - R(P^*)}{R'(P^*)} \right\} < 0 \quad (29)$$

Both terms in (29) are negative, and, therefore, $\frac{dP^*}{d\frac{c}{1+\pi}}$ is negative. So, an increase in direct cost of audits causes decrease in the optimal number of audits. In other words, both the income and substitution effects are negative. **Q.E.D.**

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