Absorbing Shocks: The Role of Rainy Day Funds and Transfers in a Fiscal Union

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Abstract: In this paper we investigate the role of rainy-day funds and transfers in a fiscal union. We begin by examining a country in autarky and considering its savings and funding decision for a rainy day fund. We then introduce a fiscal union and examine how the introduction of the union affects the rainy day fund. We find that contributions to a rainy day fund will be lower in a fiscal union, and further that the higher the fiscal transfer, the lower will be the contributions to the rainy-day fund. Is there an optimal size of the fiscal union in the sense of the size of the transfers involved? We find that there is. The trade-off is that members of the union benefit from the insurance provided by the union, but suffer because it reduces their contributions to their own rainy-day funds. We find that the optimal size of transfers in the fiscal union depend on a number of factors. In particular, member countries should have lower transfers if (1) own-fiscal policy is more effective; (2) rainy-day fund savings are highly (negatively) influenced by the presence of the transfers; (3) recession is a relatively lower probability event; and (4) redistribution yields a lower utility gain in the union. We also find that commitment to a transfer policy is an issue. A union that is prone to break the rules on transfers negatively impacts the contribution of individual member country’s rainy day funds.
I. Introduction

The European Union is engaged in a debate over whether a Monetary Union requires a Fiscal Union to function properly. A Monetary Union takes monetary policy out of the hands of individual countries and creates a fixed exchange rate system between them. In times of recession countries of the Union thus have only fiscal policy to rely upon for internal balance. If a country is not part of a fiscal union, it must rely on its own funding for fiscal policy in a recession. A fiscal union, on the other hand, allows countries that are not in a recession to provide transfer payments to those in recession; this is a type of insurance and presumably the transferring country would be the recipient should it be struck by a recession.

Over the long-run, budget balance requires that expenditures in recessionary periods be recouped in good times. If a country is not part of a fiscal union and must rely on its own funds, the budget will balance because of saving in good times. The savings can be set aside in a fund often termed a rainy-day fund for use during a recession. If a country is in a fiscal union it may still save in a rainy day fund and use these funds to fight a recession on its own, but the fiscal union implies that some transfers are also made to countries that are part of the union and in recession from countries that are part of the union but not in recession.

In this paper we investigate the role of rainy-day funds and transfers in a fiscal union. We begin by examining a country in autarky and considering its savings and funding decision for a rainy day fund. We then introduce a fiscal union and examine how the introduction of the union affects the rainy day fund. We find that contributions to a
rainy day fund will be lower in a fiscal union, and further that the higher the fiscal transfer, the lower will be the contributions to the rainy-day fund.

Is there an optimal size of the fiscal union in the sense of the size of the transfers involved? We find that there is. The trade-off is that members of the union benefit from the insurance provided by the union, but suffer because it reduces their contributions to their own rainy-day funds. We find that the optimal size of transfers in the fiscal union depend on a number of factors, and may differ between members of the union. In particular, member countries for which (1) own-fiscal policy is more effective should have smaller transfers; (2) savings is highly influenced by the presence of the transfers should have lower transfers; (3) recession is a higher probability event should, other things equal, obtain more transfers. We also find that when redistribution yields a lower utility gain in the union, transfers should optimally be lower. Finally, a union that is prone to break the ex-ante rules on transfers, even optimal ones, will negatively affect individual member countries’ contributions to their rainy day funds.

II. Precautionary Savings in Autarky

We start with a model of a single country and investigate its savings decisions. The country has certain income in period 1 and uncertain income in period 2 resulting from a business cycle shock. Uncertain income in period 2 can be high with probability $P$ (the country experiences a boom) or low with probability $(1 - P)$ (the country experiences a recession). A country can save some of its period 1 certain income $\bar{Y}$ leaving it with $Y = \bar{Y} - S$ period 1 income. This saving constitutes a rainy-day fund,
which can be used as a fiscal policy lever to combat a recession in period 2, thus increasing the probability of ending up with high income, so \( P \) is a function of \( S \). We assume that the first derivative of \( P(S) \) is positive, \( P'(S) > 0 \) and the second derivative is negative, \( P''(S) < 0 \), i.e. we assume that savings can help out in a recession but that fiscal policy is subject to diminishing returns. Utility in period 1 is denoted \( v(Y) \) and in period 2 is \( P(S)u(Y_H) + (1-P(S))u(Y_L) \).

A country’s optimization problem is

\[
\begin{align*}
\text{Max }_S v(\overline{Y} - S) + P(S)u(Y_H) + (1 - P(S))u(Y_L)
\end{align*}
\]

The first order condition is

\[
\begin{align*}
(2) \quad \frac{\partial P}{\partial S} \left[ u(Y_H) - u(Y_L) \right] = \frac{\partial v}{\partial Y}
\end{align*}
\]

The country balances the marginal benefit and cost of saving. The cost, on the right hand side, is the loss in current spending power resulting from saving. The marginal benefit on the left hand side results from the fact that increasing saving today allows for an expansionary fiscal policy should a recession occur in the uncertain future and so increases the probability of a high income outcome and decreases the probability of a low-income outcome.

III. Introducing a Fiscal Union as set of Transfers between States

We next introduce a fiscal union to the model. This is most easily described by considering the case of two countries. In this fiscal union each of the two countries has certain income in period 1 and uncertain income in period 2 as before. Each can be hit by
a recession and each is able to save in a rainy-day fund to lessen the severity of the recession, increasing the probability of a high income outcome and decreasing the probability of a low-income outcome. The shocks that hit each member of the union are i.i.d. The joint possibilities for income in the union are:

i. \((Y_H, Y*_{H})\) with probability \(P(S)P(S*)\)

ii. \((Y_H, Y*_{L})\) with probability \(P(S)(1-P(S*))\)

iii. \((Y_L, Y*_{H})\) with probability \((1-P(S))P(S*)\)

iv. \((Y_L, Y*_{L})\) with probability \((1-P(S))(1-P(S*))\)

where the asterisk differentiates the two countries. We will however begin by assuming symmetry across the two countries.

We interpret the fiscal part of the union as a way to transfer funds between parts of the union to balance risks – when one of the countries is hit by a recession and the other is not funds are transferred to the recession-hit country. Should both or neither country be hit by a recession, no transfer takes place. These transfers could be implemented in a number of ways. For instance, EU-wide automatic stabilizers such as an EU-level progressive income tax or common EU-level pension or unemployment policies is one type of fiscal union. We model a fiscal union in a simpler way that still captures the essence of the idea that transfers are made from countries in a boom to countries in a recession. In particular, we assume that the country that experiences the low-income recessionary shock receives a direct transfer from the country that experiences the high-income shock. We call this transfer \(T\).
A first question is what happens to each country’s rainy-day fund contributions when we introduce such a fiscal union. The problem of a representative country becomes:

\[
\max_S v(\bar{Y} - S) + \\
+ P(S) P(S^*) u(\bar{Y}_H) \\
+ P(S)(1 - P(S^*)) u(\bar{Y}_H - T) \\
+ (1 - P(S)) P(S^*) u(\bar{Y}_L + T) \\
+ (1 - P(S))(1 - P(S^*)) u(\bar{Y}_L) 
\]

and the first-order condition is:

\[
\frac{\partial P}{\partial S} P(S^*) [u(\bar{Y}_H) - u(\bar{Y}_L + T)] - \frac{\partial P}{\partial S} (1 - P(S^*)) [u(\bar{Y}_L) - u(\bar{Y}_H - T)] = \frac{\partial v}{\partial S} 
\]

Once again saving for precautionary reasons in a rainy-day fund has a cost: you divert money that could be used today to an uncertain future. This cost, the fall in utility in the present is shown on the right hand side of the first order condition.

The left hand side indicates the marginal benefit of saving the rainy-day fund. This is different when a fiscal union is introduced. In fact, the marginal benefit of savings is lower in a fiscal union, so contributions to a rainy-day fund will be lower.

**Proposition 1: Rainy-day fund savings are lower in a fiscal union than in autarky.**

**Proof:** The proof proceeds by showing that the marginal benefits of savings in a fiscal union are less than the savings in autarky; since marginal costs are the same, savings will be lower. To see this, rewrite the left-hand side of (3) as

\[
\left[ P\mu(\bar{Y}_H) + (1 - P)\mu(\bar{Y}_H - T) \right] - \left[ P\mu(\bar{Y}_L) + (1 - P)\mu(\bar{Y}_L + T) \right] 
\]

Recall that the marginal benefit of savings in autarky is
We assume that the transfer does not make income in the high income state lower than in the low income state. Since utility is concave in income the first bracketed term in (4) is less than the first term of (5) and the second bracketed term of (4) is greater than the second term of (5). Hence the positive term of (4) is smaller and the negative term of (4) is bigger so (4) must be smaller than (5).

This is not quite the end of the story however since there are two players in our simple characterization of the fiscal union and we need to think about the Nash equilibrium in this game. But given the assumed symmetry between the members of the union, each will have identical reaction functions as defined implicitly by (3). It is not too difficult to show that the reaction functions are downward sloping, that a Nash equilibrium exists, and (given symmetry) that the level of savings of each member will be identical. Each member of the union will save less than they would in autarky in equilibrium.

Moreover, the savings level will depend on the size of the transfer $T$.

\[ (5) \quad \frac{\partial P}{\partial S} u(Y_H) - \frac{\partial P}{\partial S} u(Y_L) \]

**Proposition 2: The greater is the transfer $T$ in the fiscal union the smaller will be the level of rainy-day fund savings.**

**Proof:** As the transfer $T$ rises, the first bracketed term of the marginal benefit of savings given by (4) falls and the second (negative) bracketed term rises. Thus the marginal benefit of savings falls leading to a lower contribution to the rainy-day fund.

To summarize, a fiscal union leads to lower contributions to a country’s rainy-day fund. The larger is the transfer, the lower will be members’ contributions to their own
rainy-day funds. Moving to the Nash equilibrium, each country’s reaction function is lower when transfers rise, so the Nash equilibrium level of savings is lower as well.

III. What is the Optimal Degree of Transfer in a Fiscal Union?

As we have seen, savings will be lower in a fiscal union. Consequently each member will be less equipped to use their own fiscal policy in times of recession, i.e. each will have to rely more on borrowing to solve a recession. Yet the fiscal union also has a benefit, namely that members of the union insure each other against a recessionary fall in income. As there are benefits and costs, the question naturally arises as to the optimal degree of transfer among members of the union.

To answer this question we consider the size of transfer that maximizes the union members’ well-being. This amounts to choosing the transfer size to maximize the sum of utilities across the union, taking into account the effect of the transfer size on savings. The problem is:

\[
\begin{align*}
\max_T & \quad v(\bar{Y} - S(T)) + v(\bar{Y} - S^*(T)) \\
& \quad + P(S(T))P(S^*(T))u(Y_{H}) \\
& \quad + P(S(T))(1 - P(S^*(T)))u(Y_{H} - T) \\
& \quad + (1 - P(S(T)))P(S^*(T))u(Y_{L} + T) \\
& \quad + (1 - P(S(T)))(1 - P(S^*(T)))u(Y_{L}) \\
& \quad + P(S(T))P(S^*(T))u(Y^*_H) \\
& \quad + P(S(T))(1 - P(S^*(T)))u(Y^*_L + T) \\
& \quad + (1 - P(S(T)))P(S^*(T))u(Y^*_H - T) \\
& \quad + (1 - P(S(T)))(1 - P(S^*(T)))u(Y^*_L)
\end{align*}
\]

where \(S(T) = S^*(T)\) are the Nash equilibrium saving levels and the FOC for \(T\) is:
\[ P(1 - P^*) \frac{\partial u(Y_H - T)}{\partial T} = (1 - P^*)P \frac{\partial u(Y^*_L + T)}{\partial T} + (1 - P)P^* \frac{\partial u(Y_L + T)}{\partial T} - P^*(1 - P) \frac{\partial u(Y^*_L - T)}{\partial T} \]
\[ + \left[u(Y_H - T) + u(Y^*_L + T) \right] \left[ \frac{\partial P}{\partial S} \frac{\partial S^*}{\partial T} (1 - P^*) - \frac{\partial P^*}{\partial S} \frac{\partial S}{\partial T} P^* \right] \]
\[ + \left[u(Y^*_H - T) + u(Y_L + T) \right] \left[ \frac{\partial P^*}{\partial S^*} \frac{\partial S^*}{\partial T^*} (1 - P) - \frac{\partial P}{\partial S} \frac{\partial S}{\partial T} P^* \right] \]
\[ - \left[u(Y_L) + u(Y^*_L) \right] \left[ \frac{\partial P}{\partial S} \frac{\partial S}{\partial T} (1 - P^*) + \frac{\partial P^*}{\partial S^*} \frac{\partial S^*}{\partial T^*} (1 - P) \right] \]
\[ + \left[ \frac{\partial v}{\partial S} \frac{\partial S}{\partial T} + \frac{\partial v}{\partial S^*} \frac{\partial S^*}{\partial T} \right] \]

We can simplify by using (i) the assumed symmetry and (ii) the envelope theorem, which implies that fiscal union members will pick a savings level that satisfies the reaction function (3). The first order condition for \( T \) then simplifies to:

\[ \frac{\partial u(Y_H - T)}{\partial T} = \frac{\partial u(Y_L + T)}{\partial T} + \left[ \frac{\partial P}{\partial S} \frac{\partial S}{\partial T} \right] \left[ P\{u(Y_H) - u(Y_H - T)\} + (1 - P)\{u(Y_L + T) - u(Y_L)\} \right] \]
\[ = \frac{\partial u(Y_L + T)}{\partial T} + \left[ \{1 - P\}(u(Y_L + T) - u(Y_L)) \right] \frac{\partial P}{\partial S} \frac{\partial S}{\partial T} \left[ \left( \frac{P}{1 - P} \right) \frac{u(Y_H) - u(Y_H - T)}{u(Y_L + T) - u(Y_L)} + 1 \right] \]

From this first-order condition, we can deduce the following proposition:

**Proposition 3:** The optimal transfer \( T \) in the fiscal union will be less than the transfer that equalizes the marginal utility of income.

**Proof:** The proof is simply to show that the second term on the right hand side is negative. To prove that the second term is negative simply note that all terms are positive except \( \partial S/\partial T \), which is negative by Proposition 2. Hence the second term is negative,
and this implies that the marginal utility income for the low-income state inclusive of the transfer is greater than it otherwise would be. This can only be the case if $T$ is smaller than it otherwise would be.

**Proposition 4: The optimal transfer $T$ in the fiscal union will be greater**

(i) the smaller is $\partial P/\partial S$,

(ii) the smaller is $|\partial S/\partial T|$,

(iii) the smaller is $P/(1-P)$,

(iv) and the smaller is $[U(Y_H) - U(Y_H - T)]/[U(Y_L + T) - U(Y_L)]$.

**Proof:** The optimal transfer is greater the smaller is the second term of the first order condition. Inspection of the first order condition indicates that smaller values of each of the above factors implies a smaller second term and hence a larger transfer.

This proposition deserves some elaboration. The first factor is the degree to which savings is effective in combatting a recession. The smaller is the internal impact of fiscal policy, the more need there is for outside transfers, and hence the greater is the optimal transfer. The second factor is the impact of the transfer on savings in the rainy day fund. The less the transfer decreases contributions to the rainy day fund, the more efficient are the transfers and the greater should be the optimal transfer.

The third and fourth factors relate to relative expected utility of a transfer. These can be interpreted as an insurance factor and a redistributive factor. The third factor is the inverse of the relative probability of a recession. If the relative odds of a recession in the future are higher, optimal transfers are higher in order to insure against a greater risk.
The fourth factor is the difference in utility before and after the transfer for the high income country relative to the difference before and after the transfer for the low income country suffering the recessionary shock. Given that utility is concave, the gain to the recession-hit country is greater than the loss to the recession-free country so that the ratio is less than one. The relative gain depends on the degree of concavity, that is, the third derivative of the utility function. The larger is the relative gain to the recession-hit country in utility terms, the smaller will be this term and the greater will be the optimal fiscal transfer. Another way of saying this is that the greater is the value of redistribution between members of the union, the greater is the transfer.

V. Breaking the rules: The problem and consequence of time-inconsistency in transfers

One of the difficulties with implementing transfers in a fiscal union is that the union might feel compelled to break the rules of its own transfer policies after the fact. As discussed above, optimal transfers will be lower the greater is the perceived impact of a transfer on ex-ante contributions to a rainy-day fund. This immediately raises a time consistency issue. The union is effectively committing ex-ante to a policy to treat unequally – meaning less than full insurance - countries that enter a recession in order to encourage more savings in a rainy day fund. But would the union have the will-power to stick to its ex-ante policy or would it bend the rules and rescue a member state? This may depend on a number of factors including the political clout of the particular member and the severity of the recession. Nevertheless, it is worth investigating the consequences of a possible break in the rules.
Suppose the union is not able to keep its commitment, for political or other reasons. What might it do?

Suppose that we continue to assume that the central government’s objective function is the sum of countries’ utilities, but we consider the problem from an ex-post perspective, after the recession has occurred. Ex-post, the central government chooses transfers to maximize

\[
\max_T u(Y_H - T) + u(Y_L^* + T)
\]

The first order conditions are:

\[
\frac{\partial u(Y_H - T)}{\partial T} = \frac{\partial u(Y_L^* + T)}{\partial T}
\]

Ex-post, the central government’s optimal policy is not to implement second-best transfers; rather, it will want to equate the marginal utility across countries, which implies that it wants to equalize incomes ex-post. Ex-post optimal transfers are thus

\[
T = \frac{Y_H - Y_L^*}{2}
\]

If a member state believes that the ex-post transfer will follow (14) instead of (7), it will lower its rainy day fund savings. In fact, members’ rainy day fund savings will be the minimum possible.

**Proposition 5:** Breaking the rules and following the ex-post optimum implies that rainy-day fund savings will be minimized.

**Proof:** The proof is straightforward. Assuming that the transfer does not make income in the high income state lower than in the low income state, the transfer in (14) is at its maximum value. From Proposition 2, a higher transfer lowers savings. Since the transfer is at its maximum value, savings must be at its minimum value.
VI. Conclusion

A monetary union takes monetary policy out of the hands of individual countries and creates a fixed exchange rate system between them. In times of recession countries of the Union thus have only fiscal policy to rely upon for internal balance. In the long-run, if a country is not part of a fiscal union, it must rely on its own funding to finance fiscal policy in a recession. If shorter term budget balance is required, such funding can be accumulated in a rainy-day fund during good times.

A fiscal union, on the other hand, allows countries that are not in a recession to provide transfer payments to those in recession; this is a type of insurance since presumably the transferring country would be the recipient should it be struck by a recession.

In this paper we have analyzed the role of rainy-day funds and transfers in a fiscal union. We find first that contributions to a rainy day fund will be lower in a fiscal union, and further that the higher the fiscal transfer, the lower will be the contributions to the rainy-day fund. We also investigate the optimal amount of transfers that should take place. Optimal transfers will balance the benefit from the insurance provided by the union against the cost of lower contributions to rainy-day funds.

We find that the optimal size of transfers in the fiscal union depend on a number of factors, and may differ between members of the union. In particular, member countries for which (1) own-fiscal policy is more effective should have smaller transfers; (2) savings is highly influenced by the presence of the transfers should have lower transfers; (3) recession is a higher probability event should, other things equal, obtain
more transfers. We also find that when redistribution yields a lower utility gain in the union, transfers should optimally be lower.

Finally, a worrisome factor is that a union that is prone to break the ex-ante rules on fiscal transfers, even optimal ones, will negatively affect individual member countries’ contributions to their rainy day funds. Thus, the design of fiscal transfers needs to be based on ex-ante criteria that are both time consistent and not easily manipulated by member countries.
References


