The Assignment and Division of the Tax Base in a System of Hierarchical Governments

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Abstract

Vertical externalities, changes in one level of government's policies that affect the budget of another level of government, may lead to non-optimal government policies. These externalities are associated with tax bases that are shared or "co-occupied" by two levels of government. Here we consider whether the co-occupancy of tax bases is desirable. We examine the optimal extent of the tax bases of a lower level of government (local) and a higher level (state). While we find that it is not optimal to have co-occupancy in the absence of other corrective policies, eliminating co-occupancy does not eliminate fiscal externalities, meaning that tax rates can still be above or now below the socially-optimal levels. Elimination of co-occupancy is not a substitute for a policy such as intergovernmental matching grants which directly eliminates fiscal externalities. We show that if policies are available that eliminate fiscal externalities, co-occupancy will, in fact, be desirable.

JEL H77 - Intergovernmental Relations; Federalism; Optimal Taxation

1. Introduction

While the concept of a "horizontal" fiscal externality arising from "tax competition" among governments at the same level has been the topic of numerous papers in the past twenty-five years[1] only relatively recently have "vertical" fiscal externalities come to attention of researchers beginning with Johnson (1988) and Flowers (1988) and continuing with Dahlby (1994, 1996, 2001), Boadway and Keen (1996), Boadway, et. al. (1998), Keen (1998), Hoyt (2001), Dahlby and Wilson (2003), Wrede (1996, 2000), Keen and Kotsogiannis (2002), and Wilson and Janeba (2005) among others.

As the name "vertical" implies, these externalities arise between governments at different levels, for example, between state and local governments or federal and state governments. In this case the focus is on the "overlap" in the tax bases of two levels of government. An example from Dahlby (1996) is the excise tax placed on cigarettes by both the federal and state governments in the United States. When choosing its tax rate, each state presumably only considers the tax's impact on its own revenues and ignores the impact on the revenues of other states and the federal government. As a result of an increase in the state's tax rate, other states' tax revenues will increase because of cross-border shopping (a horizontal externality) and federal tax revenues will be reduced because of the reduction in the cigarettes purchases, part of their tax base (a vertical externality). Because of these impacts on the revenues of other governments, the cost of funds perceived by the state differs from the social cost of the funds. While the horizontal fiscal externality is positive, the vertical externality is negative as increases in the state tax reduce federal revenues. Because the state government ignores this negative externality, it will overtax cigarettes.

A number of studies have considered policies by the higher level of government to correct for the vertical externalities created by taxes imposed by the lower level of government. Corrective policies include separating the tax bases of the two levels of government (Flowers, 1988); increasing the number of lower-level governments (Keen (1995); Keen and Kotsogiannis (2004)); and providing intergovernmental grants (Dahlby (1996); Boadway and Keen (1996); Boadway, et. al., (1998); and Flochel and Madies, (2002)).

With the exception of a few studies including Dahlby (1996), Keen (1998), Hoyt (2001), Dahlby et.

al. (2000), and Dahlby (2001) vertical fiscal externalities have been examined in the context of a single tax base, generally labor income, shared or "co-occupied" by two different levels of government. Of course, this is a simplification as in most countries governments rely on a number of different tax bases and instruments. While we also examine tax policies in a hierarchical system of governments, we depart from previous studies in a several respects. First, rather than considering vertical externalities when a single tax base serves as the source of revenue for both levels of government (state and local), we consider vertical externalities with multiple tax bases. Specifically, we consider a large number (a continuum) of commodities to be included in either or both of levels of governments' tax bases. The consideration of multiple commodities enables us to address the question of central interest to this paper -- how should the tax base be allocated between the two levels of government?

Vertical fiscal externalities act in both directions -- state taxes affect local revenues and local taxes affect state revenues. Flowers (1988), Keen (1995), Wrede (1996), and Keen and Kotsogiannis (2002), for example, assume that both levels of government ignore the vertical externality imposed on the other level of government when setting tax policies. This leads to excessive taxation at both levels of government.[2] Here, as we wish to consider what limitations state governments should impose on the commodities included in both state and local tax bases, we assume that the state government considers the impact of its tax policies on local revenues.

In addition to having multiple tax bases, different levels of governments rely on very different sources of revenue. While there is only limited overlap or "co-occupancy" in sources of revenue of state and local governments in the United States, for example, there is likely to be a strong link between their alternative tax bases. Changes in a major source of state revenue such as the personal income tax will undoubtedly affect revenues from the property tax, a major local source of revenue. In contrast, there is much more apparent co-occupancy of the federal and state tax bases primarily because the personal income tax and, to a lesser extent, the corporate income tax are major sources of tax revenue for both levels of

[1]See Wilson (1999) for an extensive review of the literature on tax competition.

government. Thus, while vertical fiscal externalities will almost certainly arise in a co-occupied tax base, it does not follow that eliminating co-occupancy eliminates fiscal externalities, an idea that may underlie the recommendation by some to eliminate co-occupancy. In fact, eliminating co-occupancy may change the fiscal externality from being negative to being positive if the commodities in the two tax bases are substitutes. This, in turn, would suggest under-taxation rather than over-taxation.

The issue addressed here, what level of government should tax what goods or services or inputs is referred to in the federalism literature as the "assignment" problem. In a surprisingly small literature, the best known discussion of the appropriate assignment of the tax base in a system of hierarchical governments is found in Musgrave (1983) with nice summaries in Musgrave and Musgrave (1989), Oates (1994), Keen (1998) and Dahlby (2001). While Musgrave provided some general guidelines for assigning tax bases based on the elasticity of alternative tax bases, he does not discuss how vertical fiscal externalities might affect assignment. Keen (1998) does devote some discussion (and analysis) to co-occupancy and assignment by addressing the question of whether it is better to co-occupy an inelastic tax base or a more elastic tax base. Dahlby (2001) raises several concerns with Musgrave's rules for assignment including the issue of co-occupancy. While not presenting any formal model, Dahlby (2001), by highlighting the general interdependency of tax bases, raises questions similar to those we address here.

Here we address the assignment question using a very different framework from those in either Musgrave (1983) or Keen (1998) but similar in many ways to the framework implicit in Dahlby (2001). Rather than considering the type of tax base that should be taxed by different levels of government, we consider how to divide a uniform tax base among two levels of government and whether co-occupancy is desirable or not. This framework, we believe, helps focus on the question of whether the existence of vertical fiscal externalities might, as suggested by Flowers (1988) and Dahlby (2001) among others, lead to the conclusion that there should be no or very limited co-occupancy among tax bases. Even when the elimination of co-occupancy may be optimal, it does not, in general, eliminate vertical fiscal externalities. As a

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[2]Overprovision need not be the result if governments provide public inputs in production as in Wrede (2000) and Dahlby and Wilson (2003).

consequence, even if co-occupancy is eliminated the tax rates of the two levels of government will not be optimally set. If the commodities in the tax base are gross substitutes, eliminating co-occupancy results in a positive fiscal externality, meaning that tax rates will become "too" low. Here, because both the tax rates and tax bases of governments are policy instruments, we need to distinguish between fiscal externalities associated with changes in a government's tax rate and one associated with changes in its tax base. While the division of the tax base obviously influences the vertical externalities associated with the tax rates, the extent and direction of the fiscal externalities associated with tax increases and those associated with increases in tax bases can be quite different. Elimination of co-occupancy is social-welfare improving because of the fiscal externalities associated with the tax base and not those associated with the tax rates. In fact, eliminating co-occupancy will not, in general, eliminate vertical fiscal externalities.

While we show that it is not desirable to allocate the tax base among the two governments to eliminate the fiscal externalities from taxes, elimination of these fiscal externalities using other instruments will dramatically change the optimal structure of the two governments' tax bases. If matching grants are used to eliminate the vertical fiscal externalities associated with the tax rates, social welfare is maximized only when both governments tax all commodities, that is, the entire tax base is co-occupied; if the state government can differentially tax (or subsidize) commodities, part of the tax base will be co-occupied and part is exclusively taxed by the state. With either of these policies the state government is able to equate both the marginal rates of substations for the public services of the two levels of government and optimally tax all commodities. Thus the policy of eliminating co-occupancy is only desirable if the fiscal externalities associated with tax rates cannot be eliminated using other instruments.

Our basic model is found in *Section 2*. In *Section* 3 we consider the tax rates and tax bases that would independently be chosen by both levels of government if they were provided the opportunity to do so. The optimal tax base for the different levels of government is considered in *Section 4*. In this section we first consider the question of how to divide the tax base between the two levels of government in the absence of any overlap. We then consider whether and under what conditions, would co-occupancy be socially optimal. In *Section 5*, we briefly discuss how the tax base should be allocated if the state government uses matching intergovernmental grants or differential taxation to eliminate the fiscal externalities associated with the taxes. *Section 6* considers extensions and concludes.

2. A Simple Model of Optimal Tax Base Division

We consider an economy with a single state government and n local governments. All localities have a single, identical resident. Each government provides public service to its residents with g_s being the level provided by the state government and g_j, $j = 1, \ldots, n$, the level provided by locality j. Both public services are produced with constant costs with the cost of providing g_s to the n localities equal to ng_s and the cost of providing the local public service in locality j equal to g_j, $j=1,\ldots,n$. While there are n independent localities, each local government has the same policy objectives and instruments as well as identical residents. Then, in equilibrium, all localities will independently choose the same policies. Given this symmetry, we denote local policies by the subscript l and suppress notation referring to specific localities as much as possible. To further simplify the analysis, we also assume that the number of localities is large enough so that no individual locality considers the impacts its policies have on state revenues.

In addition to the public services, residents also consume private commodities. Following Wilson (1989), we consider a continuum of these private commodities identified on the interval *[0,1]*. We denote the gross of tax price of commodity i, $x(i)$, by $q(i)$. The net price of all commodities is unity. Since our interest is in how to divide the tax base between the two levels of government, we assume identical demand functions over the set of commodities. By this we mean that when the prices of two commodities are identical, the quantity demanded is the same for both. In addition, all commodities have the same own price and cross-price elasticities with respect to all other commodities. More formally,

$$x(i) = x(j),\ \frac{\partial x(i)}{\partial q(i)} = \frac{\partial x(j)}{\partial q(j)},\ \frac{\partial x(i)}{\partial q(k)} = \frac{\partial x(j)}{\partial q(k)}, k \neq i, j \ \ if\ q(i) = q(j) \qquad (2.1)$$

where we denote these derivatives by $x_{11} = \frac{\partial x(i)}{\partial q(i)}$ and $x_{21} = \frac{\partial x(i)}{\partial q(j)}$, $j \neq i, \forall i, j \in [0,1]$.[3] In addition to this

[3]This should be viewed as an approximate since commodities facing different prices will not have the same demand. However, in the absence of any a priori information about the sign of $\frac{\partial^2 x(i)}{\partial q(i) q(j)}$, this approximation greatly simp-

continuum of commodities, there is a single untaxed commodity *z*.[4] For our purposes, an important implication of having identical commodities is that the optimal tax structure is extremely simple -- all commodities should be taxed equally.

As the local governments and the state government assess uniform commodity taxes to finance their public services, the gross price of each commodity depends on whether it is part of the local government's tax base and/or the state government's tax base. The assumption of a single, uniform tax rate applied by the state is relaxed in *Section 5*. Localities tax the set of commodities on the interval $[0, \bar{k}_l]$ while the set taxed by the state government is on the interval $[\bar{k}_s, 1]$. Let the length of the interval taxed only by the local government be denoted by k_l and the length of the interval taxed only by the state government be denoted by k_s. If $\bar{k}_l > \bar{k}_s$ the length of the interval taxed by both governments is k_{ls}. Then the gross of tax price for the commodities can be summarized by

$$q(i) = \begin{cases} 1+\tau_l,\ i \in \left[0, \min\left(\bar{k}_l, \bar{k}_s\right)\right] \\ 1+\tau_s,\ i \in \left[\max\left(\bar{k}_l, \bar{k}_s\right), 1\right] \\ 1+\tau_l+\tau_s,\ i \in \left[\bar{k}_s, \bar{k}_l\right], \bar{k}_l > \bar{k}_s \end{cases} \tag{2.2}$$

where τ_l and τ_s denote the local and state tax rates respectively.

As we assume the utility function is separable in private consumption and the two public services, the indirect utility function can be expressed as

$$V[q, g_l, g_s] = V^x(q) + V^s(g_s) + V^l(g_l) \tag{2.3}$$

where $V^x(q) = \int_0^1 V(q(k))dk$, the sub-utility function with respect to prices, and the argument for *z* is suppressed as it is untaxed.

The objective function for the local governments is given by

$$W^l[q, g_l] = V^x(q) + V^l(g_l). \tag{2.4}$$

As we assume that local governments ignore the impact of their policies on state revenues, we do not in-

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lifies the analysis. While the specific formulation of some of the results would be somewhat modified if we relaxed this assumption, the conclusions obtained, as least qualitatively, are unaffected.

clude state public services as an argument in the local government's welfare function. For the state government we consider the objective function,

$$W^s[q, g_l, g_s] = n[V^x(q) + V^s(g_s) + V^l(g_l)]. \tag{2.5}$$

The state tax base in a locality is denoted by $X_s \equiv k_s x_s + k_{ls} x_{ls}$ and the local base by $X_l \equiv k_l x_l + k_{ls} x_{ls}$ where x_s, x_l, and x_{ls} denote the demand for commodities subject to the state tax only, to the local tax only, and to both taxes, respectively. Then the government budget constraints for the state government and the local governments are given by

$$ng_s = n\tau_s X_s \text{ and } g_l = \tau_l X_l. \tag{2.6}$$

3. Externalities and the Endogenous Choice of Tax Base

In the United States, the choice of tax base, that is what local governments can tax, is not at the discretion of local government but instead determined by state governments.[5] While this may be the case, it is still useful to examine what tax base local governments would choose if given the option. Here we begin by considering the problem facing both the local governments and the state government when they can choose both their tax rate and tax base. Following this, we then consider the nature of fiscal externalities in this framework. [6] In addition to the externality generated by the local government's choice of tax rate, its choice of tax base generates an externality as well. Here, we assume that the local and state governments choose their tax rates and base in Nash equilibrium. This is done, in part, to emphasize the independence and lack of coordination between the two levels of government in this setting. In the next section, when we consider the outcome when the state government chooses both tax bases, we assume a Stackelberg equilibrium. Since all localities have the same objective and the state must set the same tax rates in all localities, in this equilibrium all localities will have the same tax rate, τ_l, and tax base, $\bar{k}_l$.

[4]The untaxed commodity *z* is necessary to ensure that there are distortions associated with taxes placed on the entire tax base.

[5] States can and do regulate tax rates as well. State restrictions on local tax rates and bases are both examples of Dillon's Rule, the 1868 Iowa State Supreme Court opinion of John M. Dillon who wrote that municipalities were "the mere tenants at the will of the legislature" (Dillon, 1911, p. 448).

3.1 The Governments' Objectives

Each locality maximizes its residents' welfare by choosing both its tax rate and tax base given the tax rates and bases of the other localities and the state government. Then formally the local government's problem is

$$\underset{\tau_j,\bar{k}_j}{Maximize}\, W^j\left[q_j, g_j, g_s\right] = V_x\left(q(\tau_j, \tau_s, \bar{k}_j, \bar{k}_s)\right) + V^l\left(g_j(\tau_j, \tau_s, \bar{k}_j, \bar{k}_s)\right),\ j = 1,..n \qquad (3.1)$$

where $g_j(\tau_j, \tau_s, \bar{k}_j, \bar{k}_s)$ is defined by the government budget constraint, *(2.6)*. In a symmetric equilibrium the welfare-maximizing local tax rate and base satisfy the conditions,

$$W_{\tau_l}^l = V_y\left[-\left(k_l x_l + k_{ls} x_{ls}\right) + MRS_l\left(\left(k_l x_l + k_{ls} x_{ls}\right) + \tau_l\left(k_l + k_{ls}\right)\left(x_{ll} + \left(k_l + k_{ls}\right)x_{21}\right)\right)\right] = 0 \qquad (3.2a)$$

and

$$\left(1 - \bar{k}_l\right)W_{\bar{k}_l}^l = \left(1 - \bar{k}_l\right)V_y \tau_l\left[\left(MRS_l - 1\right)x_z + MRS_l \tau_l\left(k_l + k_{ls}\right)x_{21}\right],\ \begin{matrix} z = ls\ if\ \bar{k}_l \geq \bar{k}_s \\ z = ls\ if\ \bar{k}_l < \bar{k}_s \end{matrix} \qquad (3.2b)$$

where $MRS_j = \dfrac{\partial V}{\partial g_j}\Big/\dfrac{\partial V}{\partial y},\ j = l, s$. The impact of an expansion of the tax base depends on whether the tax bases overlap ($\bar{k}_l > \bar{k}_s$) or not. In the absence of an overlap, the addition of commodity $\bar{k}_l$ adds x_l to the local base; when the bases overlap, the expansion of the base adds x_{ls} to the local base. Of course, adding commodity $x(\bar{k}_l)$ to the local tax base increases its price by τ_l. This price increase will affect the demands for the other $k_l + k_{ls}$ commodities in the local base and therefore the revenues collected from these commodities.

Unlike the local governments, the state government chooses its policies accounting for their impacts on local revenues. Formally, the problem the state government is facing is

$$\underset{\tau_s,\bar{k}_s}{Maximize}\, W^s = \sum_{j=1}^{n} V^x\left(q_j\right) + nV^s\left(g_s\left(\tau_1,..., \tau_n, \bar{k}_1,...\bar{k}_n, \tau_s, \overline{k_s}\right)\right) + \sum_{j=1}^{n} V^l\left(g_j\left(\tau_j, \bar{k}_{,}, \tau_s, \overline{k_s}\right)\right) (3.3)$$

Then in the Nash equilibrium, the state's tax rate and base must satisfy the first order conditions,

$$W_{\tau_s}^s = V_y\begin{bmatrix} -\left(k_s x_s + k_{ls} x_{ls}\right) + MRS_s\left(\left(k_s x_s + k_{ls} x_{ls}\right) + \tau_s\left(k_s + k_{ls}\right)\left(x_{ll} + \left(k_s + k_{ls}\right)x_{21}\right)\right) \\ + MRS_l \tau_l\left(k_{ls} x_{ll} + \left(k_s + k_{ls}\right)\left(k_l + k_{ls}\right)x_{21}\right) \end{bmatrix} = 0 \ (3.4a)$$

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[6] We ignore the possibility of *horizontal* fiscal externalities, that is, the possibility that changes in the tax rates in one locality affect the tax revenues collected in other localities. The implications of horizontal fiscal externalities are discussed in the concluding section.

and

$$\bar{k}_s W^s_{\bar{k}_s} = -\bar{k}_s \tau_s V_y \begin{bmatrix} (MRS_s - 1)x_z + MRS_s \tau_s (k_s + k_{ls}) x_{21} \\ + MRS_l \tau_l (D x_{11} + (k_l + k_{ls}) x_{21}) \end{bmatrix} = 0, \quad \begin{matrix} z = ls \text{ and } D = 1 \text{ if } \bar{k}_l \geq \bar{k}_s \\ z = s \text{ and } D = 0 \text{ if } \bar{k}_l < \bar{k}_s \end{matrix} \quad \textit{(3.4b)}$$

Using these first order conditions we can determine the tax bases chosen by the two levels of government. As we are considering the optimal tax base for a local government given it also chooses its optimal tax rate, we use *(3.2a)* with *(3.2b)* to obtain

$$W^l_{\bar{k}_l} = -V_y \tau_l^2 MRS_l x_{11} > 0 \text{ if } \bar{k}_l < \bar{k}_s \qquad \textit{(3.5a)}$$

$$W^l_{\bar{k}_l} = -V_y \tau_l^2 MRS_l x_{11} + \left[\frac{k_l (x_l - x_{ls})}{(k_l x_l + k_{ls} x_{ls})} \right] V_y \tau_l MRS_l (-x_{11} + (k_l + k_{ls}) x_{21}) > 0 \text{ if } \bar{k}_l \geq \bar{k}_s . \qquad \textit{(3.5b)}$$

Expression *(3.5a)* gives the marginal increase in local welfare from an expansion in its tax base when there is no overlap while expression *(3.5b)* gives it when the two governments' tax bases overlap. In both cases, local welfare always increases as the local tax base expands. This result should not be surprising as the local government ignores the impact expansions of its base have on state tax revenues.

Because the local government will tax the entire base if given this option, we need only evaluate the state government's choice of tax base when $\bar{k}_l = 1$ to determine the equilibrium tax bases. The increase in social welfare from an increase in the state tax base can be obtained by using the first order condition for the state tax rate, *(3.4a),* in the first order condition for the state tax base, *(3.4b).* This yields $-V_y \tau_s MRS_s x_{11}$ when $\bar{k}_l = 1$. That the impacts of state and local expansions are symmetric despite the differences in their objectives arise because the state government sets its tax rate to internalize its impact on local revenues. By doing this, the state ensures the impact of expanding its tax base on local government revenues is internalized as well. In *Proposition 1* we briefly summarize these results:

Proposition 1: Assume both the local and state governments independently choose their tax bases.

a) Then the equilibrium tax bases are such that both levels of government tax the entire tax base, that is, $\bar{k}_l = 1$ *and* $\bar{k}_s = 0$ *;*

b) In equilibrium, $MRS_s > MRS_l$.

That the marginal rate of substitution for the state public service will always exceed that for the local public service is a result of the fact that the state government considers the impact its taxing decision have on local revenues while local governments do not.

4. *Optimal Tax Base Division and Co-Occupancy*

As shown in the preceding section, both levels of government will tax the entire tax base if given the option. Changes in the local governments' tax bases as well as their tax rates will generate fiscal externalities. Here we address the question of the social-welfare maximizing division of the tax base between the two levels of government is optimal. We first address the question of how the tax base should be divided between the two levels of government if there is to be no co-occupancy. After deriving the optimal division of the tax base, the question of whether the state and local governments should share tax bases, that is, whether there should be any overlap in the two tax bases is then addressed. We examine the division of the tax base in the context of a Nash equilibrium in which the state chooses its tax rate and the division of the tax base and the local governments are choose their tax rates.

Before formally examining the problem of how to divide the tax base between the two levels of government in the absence of co-occupancy, consider what might characterize the "ideal" division of the tax base. This division would yield what might be considered the "unified" outcome, achieved when a single government (state) finances both services and can set any tax rate it desires on the commodities. Because all commodities have the same own-price and cross-price elasticities, the tax rates levied by the two levels of government should be equal to ensure equal taxation of all commodities. Since the marginal cost is the same for the two public services, their marginal rates of substitutions should also be equal. As we show, with the two governments independently setting taxes and public services, this "ideal" outcome having both $\tau_l = \tau_s$ and $MRS_l = MRS_s$ is generally not obtainable. Then with this ideal outcome generally unobtainable in the absence of co-occupancy, the possibility that co-occupancy might, in fact, be optimal should not be dismissed immediately.

4.1 *The Optimal Division of the Tax Base in the Absence of Co-occupancy*

Given the tax rates chosen by the local governments, the problem facing the state government determining the optimal division of the tax base as well as its tax rate is

$$\underset{\bar{k}_l,\tau_s}{Maximize}\, W \quad = \int_0^1 v(q(k))dk + V^s\left(g_s\left(\tau_s,\tau_l,\bar{k}_l\right)\right) + V^l\left(g_l\left(\tau_s,\tau_l,\bar{k}_l\right)\right) \qquad (4.1)$$

where the state government, by choosing the extent of the local tax base, determines the extent of its tax base as well. In the symmetric equilibrium, the first order condition for *(4.1)* with respect to the tax base can be expressed as

$$\begin{aligned} W_{\bar{k}_l} &= V_y\left[(MRS_l - 1)\tau_l x_l - (MRS_s - 1)\tau_s x_s + (MRS_l\tau_l k_l + MRS_s\tau_s k_s)x_{21}(\tau_l - \tau_s)\right] \\ &\quad + V_y MRS_s \tau_s k_l k_s x_{21} \left.\frac{d\tau_l}{d\bar{k}_l}\right|_{k_{ls}=0} = 0. \end{aligned} \qquad (4.2a)$$

and the first order condition with respect to the state tax rate can be expressed as

$$\begin{aligned} W^s_{\tau_s} &= V_y\left[-k_s x_s + MRS_s\left(k_s x_s + \tau_s k_s\left(x_{ll} + k_s x_{21}\right)\right) + MRS_l \tau_l k_s k_l x_{21}\right] \\ &\quad + V_y MRS_s \tau_s k_l k_s x_{21}\frac{d\tau_l}{d\tau_s} = 0 \end{aligned} \qquad (4.2b)$$

where the reaction functions $\frac{d\tau_l}{d\tau_s}$ and $\left.\frac{d\tau_l}{d\bar{k}_l}\right|_{k_{ls}=}$ are found by differentiating the local governments' first order condition with respect to its tax rate, *(3.2a)*, by the state's instruments *(*τ_s*,* $\bar{k}_l$*,* $\bar{k}_s$*),*

$$\frac{d\tau_l}{d\bar{k}_l} = \left[\frac{MRS_l\left(\varepsilon_{g_l} + \left(\varepsilon_{g_l} - 1\right)\tau_l\varepsilon_{ll}\right)}{-W^l_{\tau_l\tau_l}}\right]\left.\frac{\partial X_l}{\partial\bar{k}_l}\right|_{k_{ls}},$$

and

$$\frac{d\tau_l}{d\tau_s} = \left[\frac{MRS_l\left(\varepsilon_{g_l} + \left(\varepsilon_{g_l} - 1\right)\tau_l\varepsilon_{ll}\right)}{-W^l_{\tau_l\tau_l}}\right]\frac{\partial X_l}{\partial\tau_s}$$

where $\varepsilon_{g_l} = \frac{\partial MRS_l}{\partial g_l}\frac{g_l}{MRS_l} < 0$ is the elasticity of the marginal rate of substitution for the local public service and $\varepsilon_{ll} = \frac{k_l x_{11} + k_l^2 x_{21}}{X_l} < 0$ is the own-price elasticity of the local tax base.[7]

To better understand *(4.2a)* recall that the impact of an increase in the local tax base with an equal

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[7]To derive these reaction functions we continue to assume that $\frac{\partial x_{21}}{\partial q_i} = \frac{\partial x_{11}}{\partial q_i} = 0 \;\forall\; i \in [0,1]$. While this assumption may affect the sign of the reaction functions themselves we do not believe they affect our results regarding the optimality of co-occupancy.

reduction in the state tax base on local tax revenue is $\tau_l\left[x_l + k_l x_{21}(\tau_l - \tau_s)\right]$. Local tax revenues are affected by both the addition of $x(\bar{k}_l)$ to their tax bases and the impact of the change of the tax rate on $x(\bar{k}_l)$ has on the demand for the other commodities in the local tax base. Analogously, the decrease in state tax revenue is $\tau_s\left[x_s + k_s x_{21}(\tau_l - \tau_s)\right]$. In addition, there is the change in utility associated with the change in the price of commodity of $x(\bar{k}_l)$ for which we use the first order approximation of $-V_y(\tau_l x_l - \tau_s x_s)$.

Finally, the extent of the local tax base is influence by how the local tax rate responds to changes in the state and local tax bases as well as the state tax rate. The reaction functions can be interpreted as the product of a term that determines how the local tax responds to an increase in its tax base (the bracketed term) and the impact of the policy (τ_s, $\bar{k}_l$, or $\bar{k}_s$) on its tax base. If all commodities are substitutes, then with no overlap in the tax bases both an increase in the local tax base or an increase in the state tax rate will increase local revenues. This being the case, the response of the local tax rate depends on the sign of the bracketed term. If the elasticity of substitution for the public service, ε_{g_l}, is of greater magnitude than $\frac{1}{1+\frac{1}{\tau_l \varepsilon_{ll}}}$ the bracketed term is negative. For what seem to be plausible values for the three parameters this is the case and the local tax responses we intuitively expect, $\left.\frac{d\tau_l}{d\bar{k}_l}\right|_{k_{ls}=0} < 0$, and $\frac{d\tau_l}{d\tau_s} < 0$ are obtained.

Then using the first order conditions for state and local tax rates, *(3.2a)* and *(4.2a)* in *(4.2b)* gives

$$-MRS_l \tau_l^2 x_{11} + MRS_s \tau_s \left(\tau_s x_{11} + x_{21}\left(\tau_l k_s + k_l \left(k_s \left.\frac{d\tau_l}{d\bar{k}_l}\right|_{k_{ls}=0} + \tau_s \frac{d\tau_l}{d\tau_s} \right)\right)\right) = 0 \qquad (4.4)$$

If $x_{21} = 0$, any division of the tax base that leads to the state and local government setting the same tax rate *(* $\tau_l = \tau_s$ *)* will result in $MRS_l = MRS_s$. However, if $x_{21} > 0$, the division of the tax base will affect the marginal cost of funds and there is no division of the tax base for which both the tax rates and the marginal rates of substitution can be equated. Less obvious, however, is how the optimal division of the tax base might be characterized when $x_{21} > 0$. We begin by considering the relationship between the two tax rates and marginal rates of substitution when the tax base is evenly split *(* $\bar{k}_l = .5$ *)*. If $\tau_l > \tau_s$ at $\bar{k}_l = .5$ then $MRS_l > MRS_s$; however $\tau_s > \tau_l$ does not imply that $MRS_s > MRS_l$. Using *(4.3)* we can see that when $x_{21} > 0$ and $\tau_l > \tau_s$, $W_{\bar{k}_l}\Big|_{\bar{k}_l=.5} > 0$ while $W_{\bar{k}_l}\Big|_{\bar{k}_l=.5} < 0$ when $x_{21} < 0$ and $\tau_s > \tau_l$. Summarizing these results and some of their implications, we have

Proposition 3:

a) *If* $\tau_l(.5) > \tau_S(.5)$ *and* $x_{21} > 0$ *then* $\bar{k}_l > .5$, $\tau_l\left(\bar{k}_l\right) > \tau_s\left(\bar{k}_l\right)$ *, and* $MRS_l < MRS_s$*;*

b) *If* $\tau_l(.5) < \tau_S(.5)$ *and* $x_{21} < 0$ *then* $\bar{k}_l < .5$, $\tau_l\left(\bar{k}_l\right) < \tau_s\left(\bar{k}_l\right)$ *and* $MRS_l > MRS_s$*;*

c) *If* $x_{21} \neq 0$ *either* $\tau_l\left(\bar{k}_l\right) > \tau_s\left(\bar{k}_l\right)$, and $MRS_l < MRS_s$ or $\tau_l\left(\bar{k}_l\right) < \tau_s\left(\bar{k}_l\right)$, and $MRS_l > MRS_s$.

Proof of *Proposition 3* is found in the appendix. Essentially, when the tax base is evenly divided $\left(\bar{k}_l = .5\right)$ and we either have: a) $\tau_l(.5) > \tau_S(.5)$ and $MRS_l > MRS_s$ or b) $\tau_l(.5) < \tau_S(.5)$ and $MRS_l < MRS_s$, we increase the tax base for the level of government with the higher tax rates and marginal rate of substitution for its public service. Part *c)* of the *Proposition* states that with $x_{21} \neq 0$ it will never be optimal for one level of government to have both a higher tax rate and relative under-provision of a public service.

Finally, we might consider whether the elimination of co-occupancy of the tax base leads to social-welfare maximizing tax rates for the two levels of government. Differentiating the social welfare function with respect to the tax rate of a single locality gives

$$W_{\tau_l} = \frac{1}{n} V_{g_s} \tau_s \bar{k}_s \bar{k}_l x_{21}. \qquad (4.4)$$

The impact on social welfare depends on the change in the state tax revenue An increase in revenue increases social welfare; a decrease will decrease social welfare. Then it follows that:

Proposition 4: Assume that the tax base is optimally divided between the two governments. Then at the optimal division:

a) If $x_{21} = 0$, a (marginal) change in the local tax rate has no impact on social welfare;
b) if $x_{21} > 0$, an increase in the local tax rate will increase social welfare;
c) if $x_{21} < 0$, a decrease in the local tax rate will increase social welfare.

Only if the cross-price elasticities between commodities equal zero are the local tax rates welfare maximizing. This is because elimination of the co-occupancy eliminates the fiscal externality in this case. With non-zero cross-price elasticities, eliminating co-occupancy does not eliminate the fiscal externality. In the case of gross substitutes, it may change the fiscal externality from being negative with co-occupancy to being positive with no co-occupancy. This, in turn, means that taxes also change from being "too" high to being "too" low, below the welfare maximizing rates.

4.2 The Optimal Co-Occupancy of Tax Bases

That the tax rates and marginal rates of substitutions for the public services are not equal for the two levels of government and tax increases or decreases can enhance social welfare suggests the possibility that co-occupancy could be desirable. Formally, the state's problem is to choose the tax bases for both levels of government $(\bar{k}_l, \bar{k}_s)$ and its tax rate (τ_s) or

$$\underset{\bar{k}_l}{Maximize}\ V^P(\tau_s(k_s,k_l),\tau_s(k_s,k_l)) = \int_0^1 v(q(k))dk + V_s(g_s(\tau_s(k_s),k_s)) + V_l(g_l(\tau_l(k_s),k_s))\ . \quad (4.5)$$

Then the first order conditions with respect to the tax bases can be expressed as

$$W^s_{\bar{k}_l} = V_y\left[\left[(MRS_l - 1)\tau_l x_l + \underset{(a)}{\left(MRS_l\tau_l(k_l + k_{ls}) + MRS_S\tau_s(k_s + k_{ls})\right)}\tau_l x_{21}\right] + \underset{(b)}{\left[MRS_s + MRS_l\right]}\tau_l\tau_s x_{11}\right] = 0 \quad (4.6a)$$

and

$$-W_{\bar{k}_s} = V_y\left[\left[(MRS_s - 1)\tau_s x_s + \underset{(a)}{\left(MRS_l\tau_l(k_l + k_{ls}) + MRS_S\tau_s(k_s + k_{ls})\right)}\tau_s x_{21}\right] + \underset{(b)}{\left[MRS_s + MRS_l\right]}\tau_l\tau_s x_{11}\right] = 0 \quad (4.6b)$$

where we use $\tau_l x_{11} \approx x_s - x_{ls}$ and $x_l + \tau_s x_{ll} \approx x_{ls}$ to obtain *(4.6).* Term *(a)* in both *(4.6a)* and *(4.6b)* gives the increase in welfare from expanding the tax base to an untaxed commodity. Term *(b)* in both expressions gives the impact of expanding the overlap in tax bases on revenue. The first order condition with respect to the state's tax rate remains *(3.4a).*

Our interest is in the impact of an expansion of the tax base of either level of government when there is no co-occupancy *(k_{ls}=0)* and the tax base is optimal divided between the two governments. Given an optimal division of the tax base, term (a) in *(4.6a)* equals term (a) in *(4.6b)* since the condition describing the optimal division of the tax base, *(4.2),* is simply term *(a)* from *(4.6b)* subtracted from term *(a)* in *(4.6a)*. Then given term (b) is the same in both equations, it must be the case that if an increase in the local tax base will increase social welfare when the tax base is optimally divided, so must an increase in the state tax base. Using the first order condition for the local tax, *(3.2a),* in *(4.6a)* and evaluating at $k_{ls} = 0$ we can express $W_{\bar{k}_l}$ as:

$$W_{\bar{k}_l}\Big|_{k_{ls}=0} = \tau_l V_y \left[\underset{(a)}{MRS_s \tau_s (x_{11} + k_s x_{21})} + \underset{(b)}{MRS_l (\tau_s - \tau_l) x_{11}} \right]. \qquad (4.7a)$$

Using the first order conditions for the state taxes, *(3.4a),* in *(4.6b)* gives

$$-W_{\bar{k}_s}\Big|_{k_{ls}=0} = \tau_s V_y \left[\underset{(a)}{MRS_l \tau_l x_{11}} + \underset{(b)}{MRS_s (\tau_l - \tau_s) x_{11}} \right]. \qquad (4.7b)$$

Term *(a)* of both *(4.6a)* and *(4.6b)* are both negative. While term *(b)* of *(4.7a)* is positive if $\tau_l > \tau_s$ term *(b)* of *(4.7b)* will be negative in this case; if $\tau_s > \tau_l$, term *(b)* of *(4.7b)* will be positive but term *(b)* of *(4.7a)* must be negative. Then it follows that both *(4.7a)* and *(4.7b)* cannot both be positive and therefore co-occupancy, through increases in either tax base cannot be optimal when *(4.2)* is satisfied. Summarizing:

Proposition 5: Co-occupancy of the tax base is never optimal if the tax base is optimally divided in the absence of co-occupancy, that is, if the division of tax base satisfies (4.2).

Of course, if the tax base is not optimally divided, specifically if it is the case that $\tau_l > \tau_s$ and $MRS_l > MRS_s$ or $\tau_s > \tau_l$ and $MRS_s > MRS_l$, then co-occupancy could be welfare-improving. To better understand the proposition, first consider the case of a single public good with all commodities equally. In this case, if one of *K* commodities has an additional tax of $d\tau$ placed on it the deadweight loss (with *MRS* = 1) is $x_{11}(\Delta\tau)^2 + (K-1)x_{21}\tau\Delta\tau$. If, instead, a tax of $\frac{1}{K}\Delta\tau$ is place on all K commodities deadweight loss is $K\left[x_{11}\left(\frac{\Delta\tau}{K}\right)^2 + (K-1)x_{21}\tau\frac{\Delta\tau}{K}\right]$ which is less than the deadweight loss imposed by taxing one commodity at the higher rate. Of course, with two public services and two different taxes, we do not have equal tax rates or equal marginal rates of substitution but the same logic applies. Rather than subjecting some small share of the tax base to a very high tax rate *(τ_s+τ_l)* the state government, by choosing the division of the tax base between the two levels of government, can "spread" the tax increase (or decrease) on a much larger share of the tax base. Thus if increases in the local public service are needed *($MRS_l > MRS_s$)* rather than overlapping a share of the tax base, the state could expand the local base and contract the its base as well as to adjust its tax rate to get it closer to those of the local governments.

One of the most interesting aspects of this result is that it is true regardless of the cross-price elasticities of the commodities and, consequently, the extent and direction of the vertical fiscal externality arising from tax increases. Of course, if $x_{21} = 0$, there is no vertical fiscal externality in the absence of co-occupancy and if $x_{21} < 0$ having or increasing the overlap in the two tax bases only serves to increase the negative fiscal externality, so the result is not unexpected in either of these two cases. However, if $x_{21} > 0$, then in the absence of any overlap a positive fiscal externality exists. Then co-occupancy could, in fact, eliminate any fiscal externality. This, too, is not optimal. Thus while many arguments for eliminating co-occupancy generally seem premised on the notion of eliminating a fiscal externality associated with the tax rates, eliminating co-occupancy is still desirable even if it increases a (positive) fiscal externality.

5. Optimal Tax Bases with Alternative Instruments

That co-occupancy of the tax base is not optimal means that an optimally designed tax base will not eliminate fiscal externalities. This, in turn, suggests there may be a role for the state to use additional fiscal instruments to reduce or possibly eliminate these externalities. The two instruments we briefly discuss here are the use of an intergovernmental grant and the use of different state tax rates on the co-occupied and the exclusive tax base of state government. With these additional instruments, we show that exclusive tax bases are no longer optimal. We ask "If intergovernmental grants or differential taxation are used, then, what should the optimal division of the tax base be?" If both public services were provided by a single unified government the result would be equal tax rates on all commodities and $MRS_s = MRS_l$. We demonstrate that this outcome can be obtained with either the use of intergovernmental grants or differential state taxation. When intergovernmental grants are available this outcome is obtained only with full co-occupancy; when the state can use differential taxation, it is obtained with full co-occupancy of the local tax base but with the state exclusively taxing part of the base.

5.1 The Optimal Tax Base with Intergovernmental Grants

We follow Dahlby (1996) and Hoyt (2001) and consider the use of a revenue matching grant rather than a lump sum grant as used by Boadway and Keen (1996). Again following Dahlby (1996) and Hoyt (2001) we examine these policies in a Nash equilibrium. With the matching grant the local government's budget constraint is given by:

$$(1-m)\tau_l X_l = g_l \tag{5.1}$$

where m is the matching grant with $m > 0$ implying a transfer of funds from the local governments to the state government. The state budget constraint is given by

$$\tau_s X_s + m\tau_l X_l = g_s . \tag{5.2}$$

Following Dahlby (1996) and Hoyt (2001) we set the matching rate, m, so that the fiscal externality imposed by the local government is eliminated, that is,

$$\frac{\partial g_S}{\partial \tau_l} = \tau_s \frac{\partial X_s}{\partial \tau_l} + m\left(X_l + \frac{\partial X_l}{\partial \tau_l} \right) = 0 \text{ or } m = -\tau_s \frac{\partial X_s}{\partial \tau_l}\left(X_l + \frac{\partial X_l}{\partial \tau_l} \right)^{-1} . \tag{5.3}$$

The sign of the transfer, m, depends on the sign of $\frac{\partial X_s}{\partial \tau_l}$, that is, whether an increase in the local tax increases or decreases state revenues. If $\frac{\partial X_s}{\partial \tau_l} < 0$ then $m < 0$ and there is a *positive* fiscal gap with the state government transferring funds to the local government; if $m > 0$ there is a *negative* fiscal gap with the local government transferring funds to the state government. With this matching rate, the first order condition for local governments can be expressed by

$$MRS_l = \frac{X_l}{X_l + \tau_l \frac{\partial X_l}{\partial \tau_l} + \tau_s \frac{\partial X_s}{\partial \tau_l}} \tag{5.4}$$

and the first order condition for the state government is

$$-X_s + MRS_S \frac{\partial g_s}{\partial \tau_s} + MRS_l \frac{\partial g_l}{\partial \tau_s} = 0 . \tag{5.5}$$

If $MRS_l = MRS_s$ then *(5.5)* can be expressed as

$$MRS_s = MRS_l = \frac{X_s}{X_s + \tau_l \frac{\partial X_l}{\partial \tau_s} + \tau_s \frac{\partial X_s}{\partial \tau_s}} .^8 \qquad (5.6)$$

Then using the expressions for MRS_l from *(5.5)* and for MRS_s (and MRS_l) in *(5.6)* we can see that MRS_s = MRS_l only if

$$\frac{X_l}{X_l + \tau_l \frac{\partial X_l}{\partial \tau_l} + \tau_s \frac{\partial X_s}{\partial \tau_l}} = \frac{X_s}{X_s + \tau_l \frac{\partial X_l}{\partial \tau_s} + \tau_s \frac{\partial X_s}{\partial \tau_s}} . \qquad (5.7)$$

Equation *(5.7)* will only be satisfied when $x_{21} \neq 0$ if the two tax bases are the same, $X_l = X_S$ or,

Proposition 6: If the state government is able to transfer funds between itself and the local governments through the use of a matching grant, then it is optimal for both governments to tax the entire tax base.

5.2 The Optimal Tax Base with Differential State Taxation

Recall that with non-zero cross price elasticities it was not possible to optimally divide the tax base between the two levels of government and simultaneously obtain the conditions that $MRS_s = MRS_l$ and $\tau_s = \tau_l$. However, if the state government has the ability to differentially tax (or subsidize) the co-occupied tax base, then it will be able to allocate the tax base among the two governments to satisfy both of these conditions. To see this, let the τ_s^o be the tax rate for the part of the state tax base that is not co-occupied *(X_s)* and $\tau_s^{'}$ be the tax rate on the co-occupied part of the tax base, equal to the entire local tax base *(X_l)*. The state chooses its tax rates and public service level to maximize social welfare. Then treating the public service, $g_s(\tau_s^o, \tau_s^{'})$, as a function of the two tax rates allows us to characterize the first order conditions for the state government as

$$-X_s + MRS_s\left(X_s + \tau_s^o \frac{\partial X_s}{\partial \tau_s^o} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_s^o} \right) + MRS_l \tau_l \frac{\partial X_l}{\partial \tau_s^o} = 0 \qquad (5.8a)$$

and

$$-X_l + MRS_s\left(X_l + \tau_s^o \frac{\partial X_s}{\partial \tau_s^{'}} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_s^{'}} \right) + MRS_l \tau_l \frac{\partial X_l}{\partial \tau_s^{'}} = 0 . \qquad (5.8b)$$

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[8] Expression *(5.6)* is obtained by setting $MRS_l = MRS_s$ and substituting $\frac{\partial g_s}{\partial \tau_s} = X_s + \frac{\partial X_s}{\partial \tau_s} + m\tau_l \frac{\partial X_l}{\partial \tau_s}$ and $\frac{\partial g_l}{\partial \tau_s} = (1-m)\tau_l \frac{\partial X_l}{\partial \tau_s}$.

The first order condition for the local governments is given by

$$-X_l + MRS_l\left(X_l + \tau_l \frac{\partial X_l}{\partial \tau_l} \right) = 0 . \qquad (5.9)$$

Proposition 7. If the state government chooses its tax rates, τ_s^o *and* $\tau_s^{'}$*, and the division of the tax base such that in the Nash equilibrium with the local governments* $\tau_s^o \frac{\partial X_s}{\partial \tau_l} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_l} = 0$ *then in equilibrium it is also the case that:*

a) *$MRS_l = MRS_s$;*

b) $\tau_s^o = \tau_s^{'} + \tau_l$*;*

c) *If commodities are substitutes then* $\tau_s^{'} > 0$ *and if they are complements then* $\tau_s^{'} < 0$.

If the state sets its tax rates and the division of the tax base so that the local government's fiscal externality is eliminated, $\tau_s^o \frac{\partial X_s}{\partial \tau_l} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_l} = 0$, then for both *(5.8b)* and *(5.9)* to be satisfied it must be the case that $MRS_s = MRS_l$ since if $\tau_s^o \frac{\partial X_s}{\partial \tau_l} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_l} = 0$ it follows that $\tau_s^o \frac{\partial X_s}{\partial \tau_s^{'}} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_s^{'}} = 0$ as well. Then given that $MRS_s = MRS_l$ and the identical own-price and cross-price elasticities of demand for the commodities, the optimal state tax structure defined by *(5.8a)* and *(5.8b)* implies equal taxes on all commodities or $\tau_s^o = \tau_s^{'} + \tau_l$. Part *c)* of the *Proposition* follows from the fact that $\tau_s^o > 0$ to ensure equal tax rates and that, by assumption, taxes are set so that $\tau_s^o \frac{\partial X_s}{\partial \tau_l} + \tau_s^{'} \frac{\partial X_l}{\partial \tau_l}$ =0. Then if $\frac{\partial X_s}{\partial \tau_l}$ > (<) 0 for there to be no fiscal externality it must be the case that $\tau_s^{'}$ > (<) 0 given that $\frac{\partial X_l}{\partial \tau_l}$ < 0.

If the state has the additional instrument of matching grants or differential taxation of its base, it can replicate the policies obtained with a unified government. With both instruments co-occupancy is optimal though with the use of matching grants complete co-occupancy is desirable while with differential taxation some portion of the tax base is exclusively in the state tax base. With both instruments, state policies are structured so that the local governments' fiscal externalities are eliminated.

6. Extensions and Conclusion

While much of the most literature on vertical fiscal externalities is recent, some suggested policies to remedy inefficiencies associated with the existence of these vertical fiscal externalities have to emerge. The most frequent policy recommendation is the use of intergovernmental grants to correct any misallocation of funds between levels of government and to force governments to internalize the fiscal externality. Another suggested policy and our focus is to reduce fiscal externalities by limiting the co-occupancy of the tax base.

We find that complete elimination of co-occupancy is optimal if the tax base is optimally divided in the absence of co-occupancy and other corrective policies are not available. This result is true regardless of the cross-price elasticities among commodities and whether or not the vertical fiscal externality is positive or negative. However, this policy generally does not lead to the governments setting social-welfare maximizing tax rates, thus suggesting that other corrective policies are still desirable. Further if other corrective policies, specifically matching grants or differential taxation (and subsidies) by the state government, are used to eliminate fiscal externalities then co-occupancy is, in fact, socially optimal to ensure equal taxation of commodities (the optimal tax policy in this model) and that both governments face the same marginal cost of funds.

One extension to consider is the question of which commodities should be included in each tax base when commodities are not identical and they do not have identical cross-price elasticities. This extension would bring the analysis in closer alignment with the framework for addressing the assignment problem underlying Musgrave (1983). In a framework with collection costs, Wilson (1989) finds that when adding a commodity to the set of taxable commodities, it should be the strongest available substitute with the existing set of taxable commodities. Does a similar result apply here or does substitution with the tax base of the other government need to also be considered?

Other possible extensions merit further research. In the simple model presented here there were no horizontal fiscal externalities as there was no flow of tax base between different localities. Recent studies by Keen and Kotsogiannis (2002), Wilson and Janeba (2005) and Flochel and Madies (2002) consider both vertical and horizontal fiscal externalities though not with multiple tax bases. Relatively simple adjustments to the model would provide the opportunity to consider how local and state tax bases should be designed when these externalities also exist. It would seem likely that commodities or tax bases that flow between localities ideally would not be included in local tax bases.

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Appendix: Proof of Proposition 3

Parts a) and *b)*, the conditions under which $\bar{k}_l > (<).5$ follows from the assumption that $W\left(\bar{k}_l, \tau_l\left(\bar{k}_l\right), \tau_s\left(\bar{k}_l\right)\right)$ is strictly concave in $\bar{k}_l$.

a) Relative Tax Rates

Using the first order conditions for the taxes, *(3.2a)* and *(3.4a)*, we obtain

$$MRS_l = \frac{x_l}{x_l + \tau_l\left(x_{11} + k_l x_{21}\right)} = \frac{x_l}{D_l} \qquad (A.1a)$$

and

$$MRS_s = \frac{x_s}{x_s + \tau_s\left(x_{11} + k_s x_{21}\right)} - MRS_l \tau_l \frac{k_l x_{21}}{x_s + \tau_s\left(x_{11} + k_s x_{21}\right)} = \frac{x_s}{D_s} - MRS_l \tau_l \frac{k_l x_{21}}{D_s} \qquad (A.1b)$$

or

$$MRS_s = \frac{x_s}{D_s} - \tau_l \frac{k_l x_l x_{21}}{D_s D_l} \qquad (A.1b')$$

Then we can express the left side of *(4.3)* as

$$\frac{\tau_l^2 S_l x_l}{D_l} + \frac{\tau_l^2 k_l x_l x_{21}}{D_l} - \frac{\tau_l \tau_s k_l x_l x_{21}}{D_l} - \frac{\tau_s^2 S_s x_s}{D_s} + \frac{\tau_l \tau_s k_s x_s x_{21}}{D_l} - \frac{\tau_s^2 k_s x_s x_{21}}{D_l} \qquad (A.2)$$

where $Sj = -(x_{11} + k_j x_{21})$. Simplifying using the fact that $Sj = -(x_{11} + k_j x_{21})$ gives

$$\Bigg[\underbrace{\frac{-\tau_l^2 x_l}{D_l}+\frac{\tau_s^2 x_s}{D_s}}_{(a)}\Bigg]x_{11} + \tau_l\tau_s\Bigg[\underbrace{\frac{-x_l k_l}{D_l}+\frac{x_s k_s}{D_s}}_{(b)}\Bigg]x_{21} = 0 \qquad \textit{(A.2')}$$

If $x_{21} > 0$, then from *(A.2')* we have $k_l > (<)\ k_s \rightarrow \tau_l > (<)\ \tau_s$ with the reverse true if $x_{21} < 0$. If we express *(4.2)* as

$$MRS_s\tau_s\big[\underbrace{-x_s+k_s x_{21}(\tau_l-\tau_s)}_{(a)}\big]+MRS_l\tau_l\big[\underbrace{x_l+k_l x_{21}(\tau_l-\tau_s)}_{(b)}\big]+\big(\underbrace{\tau_l x_l-\tau_s x_s}_{(c)}\big)=0 \qquad .\textit{(A.3)}$$

Then if $x_{21} > 0$, we have $k_l > (<)\ k_s \rightarrow \tau_l > (<)\ \tau_s$ Then it follows that term *(c)* of *(A.3)>0* implying (a) + (b) < 0. Since term *(b>0)* then term *(a)<0*. Further, the bracketed term in *(a)* is of smaller absolute value then the bracketed term in *(b)* ($k_l^* > k_s^*$) meaning that for the sum of *(a)* and *(b)* to be negative that $MRS_s\tau_s > MRS_l\tau_l$. Then since $\tau_l > \tau_s$ it must be the case that $MRS_s > MRS_l$. An analogous (and reversed) argument applies for the case with $x_{21} < 0$ and $\tau_l(.5) > \tau_s(.5)$.