Jointly Optimal Income Taxes for Different Types of Income†

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Abstract
In this paper we develop and estimate a model of jointly optimal income taxes for different types of income, such as wage, capital or self-employment income taking into account fiscal externalities between tax bases. In the theoretical part, we derive the optimality conditions for income type dependent non-linear tax schedules and provide a closed-form solution for the case of a linear tax. In the empirical part, we calculate social marginal welfare weights implicit in the German personal income tax schedule accounting for differences in the responsiveness across income types. We show that average welfare weights differ significantly between income sources. Finally, we use the obtained values to simulate optimal linear taxes for the case of Germany. We find that optimal taxes for capital income are sizably smaller than for wage income or income from self-employment.

JEL Classification: H21, H24, H26
Keywords: Optimal Taxation, Income Sources, Multidimensional Heterogeneity, Social Marginal Welfare Weights, Administrative Data

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1 Introduction

Following the seminal studies by Diamond (1998) and Saez (2001), the literature on personal income taxation as first developed by Mirrlees (1971) has provided a variety of models incorporating socio-economic characteristics optimal tax schedules can depend on. Potential criteria for the application of different tax schedules on distinguishable groups of taxpayers have been analyzed in various contexts such as age (Blomquist and Micheletto 2008; Bastani et al. 2013; Weinzierl 2011; Best and Kleven 2013), height (Mankiw and Weinzierl 2010), gender (Cremer et al. 2010; Alesina et al. 2011) or marital status (Boskin and Sheshinski 1983; Kleven et al. 2009). The rationale behind levying different taxes from taxpayers with different socio-economic characteristics can be based on two conditions: (i) The characteristics of the considered types of taxpayers have to be observable and immutable to a sufficient degree. (ii) The different types have to vary in their responsiveness to taxes such that the government can exploit the differential in the efficiency costs of taxation, or the welfare weights of the types differ to make redistribution across groups desirable.

In this paper, we derive a model in which the government taxes different sources of income on separate schedules and simulate it for the case of Germany.\footnote{We consider the separate taxation of different income sources such as wage or capital income. Another special case of this exercise is the optimal taxation of couples (Mirrlees 1972; Boskin and Sheshinski 1983 for initial contributions; Cremer et al. 2003; Cremer et al. 2012; Schroyen 2003; Brett 2007; Kleven et al. 2009 and Immervoll et al. 2011 for a recent state-of-the-art) with different tax schedules for men and women (gender based taxation; Cremer et al. 2010; Alesina et al. 2011).} In light of the discussed requirements this attempt seems to be promising. First, the source of income is easy to observe for the government. In fact in most actual tax systems taxpayers have to assign the reported income to different categories when filing taxes such as wage income or capital gains.\footnote{Most countries' tax systems follow the Haig-Simons standard of comprehensive income taxation by defining a single measure of taxable income as the sum of incomes from all sources to which a single rate schedule is applied. Notable exceptions are dual income tax systems (mostly Nordic countries and since 2009 also Germany). In such a schedular tax system, capital income is taxed at a low flat marginal rate whereas labor income is taxed at a progressive schedule (see Boadway 2004 for an overview of dual income tax systems).} Second, it is a well documented observation that the responsiveness of reported income to taxation differs sizably across different types of income. In particular self-employed workers have a higher elasticity with respect to the net-of-tax rate than wage earners (Saez 2010; Kleven and Schultz 2013), indicating that the efficiency costs of taxation are higher in the first group. Third, the distribution of different income sources varies along the level of taxable income. In Figure 1a we display for Germany the fraction of taxpayers according to their main source of income (capital, wage, or self-employment) as a function of taxable income. While the major part of taxpayers in the range of €0 to €100,000 primarily receives wage income, the fractions of self-employed and capital income earners surge dramatically after €100,000, and stabilize at a higher level. Figures 1b-c show the average fractions of wage, capital and self-employment income as a function of taxable income, and the fractile in the income distribution respectively.
While at the bottom of the income distribution wage income is the predominant income source, income from self-employment gains importance at a higher level of taxable income.

In light of this, the desirability of conditioning taxes on legally defined and distinguishable sources of income seems to be high. The literature on the taxation of different sources of income under the condition of joint optimality is relatively scarce.\(^3\) Rothschild and Scheuer (2013) and Scheuer (2014) provide models of optimal income taxes for entrepreneurs in economies where individuals can have different skills in different occupations. Ooghe and Peichl (2014) study the optimal (linear) taxation of different characteristics (which could be different types of income) in the case unobserved ability and unobserved preference heterogeneity. A major challenge in modeling jointly optimal taxes is to incorporate cross-effects between tax bases which is not considered in the standard optimal income tax model. However, inter- and intra-temporal income shifting (Auerbach and Slemrod 1997; Slemrod 1998; Gordon and Slemrod 2002; Kreiner et al. 2013; Harju and Matikka 2013; Kreiner et al. 2014) has been identified as an important component of the elasticity of taxable income (Saiz et al. 2012). Piketty et al. (2014) provide a formula for the optimal top tax rate incorporating fiscal externalities due to income shifting.

In the first part of the paper, we set up an optimal tax model in the spirit of Diamond (1998) and Saez (2001) to derive jointly optimal tax schedules for different income sources taking into account fiscal externalities occurring through differentials in tax rates. In the second part, we empirically implement the optimal tax schedules using administrative tax return data for Germany for 2007. To do so, we estimate the distribution of welfare weights inherent in the current German income tax schedule. We then use these estimated marginal welfare weights to simulate jointly optimal linearized taxes for wage, capital and self-employment income. We find that for reasonable values for the responsiveness of different income sources the optimal tax rate for capital is much lower than for wage or self-employed income. Interestingly, the German government introduced a flat marginal tax rate on capital income of 25% in 2009.

The remainder of this paper is structured as follows. In section 2 we derive and characterize the conditions for jointly optimal income taxes for different types of income. Section 3 explains the institutional background of the German tax law as well as the data used in this paper. In section 4 we present an empirical investigation of (income type specific) social welfare weights for the case of Germany before we turn to the simulation results in section 5. We summarize the findings and conclude in section 6.

\(^3\) Of course, a long literature on the (separate) optimal taxation of labor income (see, e.g., Piketty and Saez 2013 for a recent survey) and capital income (see, e.g., Kopczuk 2013) exists.
2 Optimal Tax Schedules

2.1 Theoretical Framework

In the following, we derive jointly optimal tax schedules for different income sources taking into account fiscal externalities occurring through differentials in tax rates. After introducing the setting of the model, in section 2.2 we derive the optimality condition for the case of non-linear tax rates and characterize its properties. In section 2.3 we present a closed form solution for optimal linear tax rates which are simulated in the empirical part of this paper.

The underlying framework of the model is in the spirit of Diamond (1998) and Saez (2001). Behavioral responses are captured in terms of elasticities of the tax base with respect to the net-of-tax rate. To capture the social planner’s preferences for redistribution we make use of social welfare weights derived from a standard welfare function. Nevertheless, all results also hold for the case of generalized social welfare weights as proposed in Saez and Stantcheva (2013).

Individuals: In the following suppose \( n \) distinguishable types of income on which the government can levy a tax. An individual earns an amount of income of type \( i \) of \( \hat{z}_i \) and reports an amount of \( z_i \) to the government which is taxed according to the tax schedule \( T_i(\cdot) \), where we denote the marginal tax rate by \( \tau_i \). We assume that a taxpayer cannot evade taxes but is able to shift income across tax bases, we therefore require \( \sum_{i=1}^{n} z_i = \sum_{i=1}^{n} \hat{z}_i \). Individuals are heterogenous in their ability to generate and shift a certain type of income as well as in their consumption preferences which is captured by the vector \( k \).

Individual utility of an individual of type \( k \in K \) reads

\[
U(k) = u^k(c; \hat{z}_1, \ldots, \hat{z}_n; z_1, \ldots, z_n),
\]

where consumption \( c = \sum_{i=1}^{n} \hat{z}_i - T_i(z_i) \) and \( \frac{\partial U}{\partial c} > 0 \) and \( \frac{\partial U}{\partial \hat{z}_i} < 0 \). We further assume that income shifting is costly, and thus \( \frac{\partial U}{\partial z_i} < 0 \) if \( z_i > \hat{z}_i \) and \( \frac{\partial U}{\partial z_i} > 0 \) if \( z_i < \hat{z}_i \). Behavioral responses of reported income to marginal tax rates are captured by the elasticity of tax base \( i \) with respect to the net-of-tax rate of tax base \( j \) defined by \( \zeta_{ij}(z_i) = \frac{\partial z_i}{\partial (1-\tau_j)} \bigg|_{z_i}(1-\tau_j) \cdot z_i \). Further, as common in the literature, it is assumed that utility is such that income effects are zero. Thus, uncompensated and compensated elasticities coincide. For the derivation all extensive elasticities are assumed to be zero. We define by \( H(z_1, \ldots, z_n) \) the joint distribution of reported income, and by \( H_i(z_i) \) the marginal distribution of income type \( i \). The distribution of types \( k \) is denoted by \( F(k) \).

Government: The government facing a revenue requirement of \( E \) maximizes a social welfare function \( S(\cdot) \) according to

\[
\max_{\{T_i(\cdot)\}} \int_{k \in K} S(U(k))dF(k)
\]
\[ \{c(k); \hat{z}_1(k), \ldots, \hat{z}_n(k); z_1(k), \ldots, z_n(k)\} \in \arg \max U(k) \quad \text{[Incentive Constraint]} \]
\[ \int_k \sum_{i=1}^n T_i(z_i(k)) dF(k) \geq E \quad \text{[Budget Constraint]} \]

We define the income specific average marginal social welfare weight as a function of \( z_i \) as
\[ g_i(z_i') = \int_{k \in K: z_i = z_i'} S'(U(k))U'_c(k)dF(k)/\lambda, \]
where \( \lambda \) is the Lagrange-multiplier of the government’s budget constraint. Intuitively, \( g_i(z_i') \) measures the average value of giving one dollar to a person with \( z_i = z_i' \) in terms of public funds.

2.2 Optimal Non-Linear Tax Schedules

In this section we derive the optimal conditions for a jointly optimal non-linear income tax system for different sources of income. An optimal non-linear income tax system is defined as the ordered set of tax schedules \( \{T_i(z_i), T_i(\cdot): \mathbb{R}_+ \to \mathbb{R}\} \) maximizing (1) and fulfilling the Incentive and Budget Constraint.

**Proposition 1.** The optimality condition for the marginal tax rate of income type \( i \) at income level \( z_i \) for a non-linear tax schedule is given by:
\[ \tau_i(z_i) = \frac{1 - G_i(z_i) - \sum_{j \neq i} \int_0^\infty \alpha_{ji}(z_j'|z_i)\zeta_{ji}(z_j')\tau_j(z_j')dH_j(z_j')}{1 - G_i(z_i) + \alpha_i(z_i)\zeta_{ii}(z_i)}, \]
where
\[ G_i(z_i) = \int_{z_i}^\infty g(z_i)dH(z_i), \]
\[ \alpha_i(z_i) = \frac{z_i h_i(z_i)}{1 - H_i(z_i)}, \]
\[ \alpha_{ji}(z_j|z_i) = \frac{z_j h_{ij}(z_i, z_j)}{1 - H_i(z_i)}. \]

**Derivation:** We employ the perturbation approach to disentangle the effects of an infinitesimal change \( d\tau_i(z_i) \) in the marginal tax rate of income type \( i \) in the small income band \([z_i, z_i + dz_i] \).

As shown of the following the optimality condition captures a mixture of four effects: (i) a \textit{Mechanical Effect}, (ii) a \textit{Welfare Effect}, (iii) an \textit{Elasticity Effect}, and (iv) a \textit{Cross-Elasticity Effect}.

**Mechanical Effect:** All taxpayers with reported income of type \( i \) higher than \( z_i \) have to pay an additional amount of taxes \( d\tau_i d z_i \). Thus, it follows that absent behavioral responses the government collects additional taxes in an amount of
\[ \Delta_i^M = d\tau_i dz_i (1 - H_i(z_i)). \]
Welfare Effect: Recall that the income specific average marginal social welfare weight of a person with \( z_i' \) is given by \( g_i(z_i') \) which is normalized in units of government revenue. Thus, the welfare loss of a person with \( z_i' \) is equal to \( d\tau_i dz_i g_i(z_i') \). Integrating over the mass of individuals with reported income of type \( i \) higher than \( z_i \) yields
\[
\Delta_i^W = -d\tau_i dz_i \int_{z_i}^\infty g_i(z_i') dH_i(z_i').
\]

Elasticity Effect: The assumption of zero income effects implies that only taxpayers in the small income band change their income due to the substitution effect provoked by an infinitesimal change \( d\tau_i \). Note that the mass of individuals in the income band is equal to \( dz_i h_i(z_i) \). The average change in reported income of type \( i \) of a taxpayer with \( z_i \) is given by \( \zeta_{ii}(z_i) z_i 1 - \tau_i(z_i) \). Thus the loss of government revenue due to the elasticity effect is equal to
\[
\Delta_i^B = -d\tau_i dz_i z_i h_i(z_i) \zeta_{ii}(z_i) \frac{\tau_i(z_i)}{1 - \tau_i(z_i)}.
\]

Cross-Elasticity Effect: The change in the marginal tax rate \( \tau_i \) will trigger an effect in tax base \( j \). This can happen because of income shifting or the substitutability/complementarity of the two income sources. The mass of individuals in the small band who report an amount of \( z_j' \) of income type \( j \) is equal to \( dz_i h_{ij}(z_i, z_j') \). These taxpayers change their reported income of type \( j \) by \( \zeta_{ij}(z_j') z_j' 1 - \tau_i(z_i) \). Integrating over the distribution of \( z_j \) yields that the cross-elasticity effect is equal to
\[
\Delta_{ji}^{BC} = -d\tau_i dz_i \int_0^\infty z_j' h_{ij}(z_i, z_j') \zeta_{ij}(z_j') \frac{\tau_j(z_j')}{1 - \tau_i(z_i)} dH_j(z_j').
\]

Optimality condition: In an optimal state all effects have to offset each other. Thus, it follows that
\[
\Delta_i^M + \Delta_i^W + \Delta_i^B + \sum_{j \neq i} \Delta_{ji}^{BC} = 0.
\]
Rearranging yields proposition 1.

The formula derived differs from the standard non-linear optimal tax formula by the term
\[
\sum_{j \neq i} \int_0^\infty \alpha_{ji}(z_j'|z_i) \zeta_{ji}(z_j') \tau_i(z_j') dH_j(z_j')
\] which captures the fiscal externality from tax base \( i \) on tax base \( j \). The sign and magnitude of the effect is driven by \( \zeta_{ji}(z_j) \), the cross elasticity of income of type \( j \) with respect to the net-of-tax rate of tax base \( i \). From a theoretical perspective the sign of this parameter is ambiguous due to opposing drivers. First, certain income sources can be complements (substitutes). Thus, an increase of earned income of type \( i \) due to a rise in the net-of-tax rate \( i \) in turn raises (lowers) earned income of type \( j \), which results
in a positive (negative) \( \zeta_{ji}(z_j) \). Second, tax differentials tempt taxpayers to shift parts of their reported income across tax bases which is a form of semi-legal tax avoidance. This results in a negative \( \zeta_{ji}(z_j) \). Empirical investigation of this parameter is scarce. Studies as Kleven and Schultz (2013), Pirttilä and Selin (2011) and Jacob (2014) show that tax differentials provoke an increase in the tax base with a lower tax levied, and thus indicate that the empirically relevant assumption on the cross-elasticity is \( \zeta_{ji}(z_j) > 0 \). Given the empirical evidence, this will by tendency lead to higher tax rates as in the standard model.\(^4\)

2.3 Optimal Linear Tax Schedules

In this section we present a closed form solution for an optimal linear tax system for different types of income. For this purpose we assume that elasticities \( \zeta_{ij} \) are constant across types \( k \in K \). Given the restriction \( T_i(\cdot) = \tau_i \forall i \) the government’s maximization problem reads

\[
L = \int_{k \in K} S\left( u^k((1-\tau_1)z_1 + \cdots + (1-\tau_n)z_n; \hat{z}_1, \ldots, \hat{z}_n; z_1, \ldots, z_n) \right) dF(k) \\
+ \lambda \left( \int_{k \in K} \tau_1 z_1 + \cdots + \tau_n z_n dF(k) - E \right).
\]

Proposition 3. The optimality condition for the tax vector \( \tau = (\tau_1, \ldots, \tau_n)' \) in a linear income tax system is given by:

\[
\begin{pmatrix}
\mathbf{m}_1 \\
\vdots \\
\mathbf{m}_i \\
\vdots \\
\mathbf{m}_n
\end{pmatrix} \times \tau =
\begin{pmatrix}
(1 - g_1) \\
\vdots \\
(1 - g_i) \\
\vdots \\
(1 - g_n)
\end{pmatrix}
\]

where

\[
\begin{align*}
\mathbf{m}_i &= (-\beta_{ii} \zeta_{ii}, \ldots, -\beta_{i-1i} \zeta_{ii}, (1 + \zeta_{ii} - g_i), -\beta_{i+1i} \cdot \zeta_{ii}, \ldots, -\beta_{ni} \cdot \zeta_{ii}) \\
\beta_{ji} &= -\frac{\partial z_j}{\partial (1 - \tau_i)} \bigg| \frac{\partial z_i}{\partial (1 - \tau_i)} \\
g_i &= \int_{k \in K} z_i(k) S'(U(k)) U'(k) dF(k) / (\lambda Z_i), \text{ with } Z_i = \int_{k \in K} z_i(k) dF(k)
\end{align*}
\]

\textit{Derivation:} Applying the envelope theorem on the individual utility maximization problem the

\(^4\)Note that this property does not need to hold locally since the distribution of reported incomes \( H_i \) is endogenous to the tax schedules.
first-order condition of the government’s optimization problem (for tax base $i$) reads

$$\frac{\partial L}{\partial (1 - \tau_i)} = \int_{k \in K} z_i(k) S'(U(k)) U'(k) dF(k) + \lambda \left( \int_{k \in K} \tau_1 \frac{\partial z_1(k)}{\partial (1 - \tau_i)} + \cdots + \tau_n \frac{\partial z_n(k)}{\partial (1 - \tau_i)} - z_i(k) dF(k) \right) = 0.$$  

Now define $\beta_{ji} = -\frac{\partial z_j(k)}{\partial (1 - \tau_i)} \Big/ \frac{\partial z_i(k)}{\partial (1 - \tau_i)}$ which is independent of $k$. The parameter $\beta_{ji}$ can be interpreted as the share of the cross-elasticity effect of tax base $j$ in the total elasticity effect of tax base $i$. It follows

$$\int_{k \in K} z_i(k) S'(U(k)) U'(k) dF(k) + \lambda \left( \int_{k \in K} \tau_1 \frac{\partial z_1(k)}{\partial (1 - \tau_i)} + \cdots + \tau_n \frac{\partial z_n(k)}{\partial (1 - \tau_i)} - z_i(k) dF(k) \right) = 0.$$  

Assuming that the own-elasticity $\zeta_{ii}$ is constant across individuals we arrive at

$$(1 - \tau_i) \cdot \int_{k \in K} z_i(k) S'(U(k)) U'(k) dF(k) / \lambda + \left( \int_{k \in K} \tau_1 \beta_{i1} \zeta_{ii} z_i(k) + \cdots + \tau_n \beta_{ni} \zeta_{ii} z_i(k) - (1 - \tau_i) z_i(k) dF(k) \right) = 0.$$  

After dividing by the total income of type $i$ denoted by $Z_i = \int_{k \in K} z_i(k) dF(k)$ and introducing the average welfare weight of income type $i$ as $g_i = \int_{k \in K} z_i(k) S'(U(k)) U'(k) dF(k) / (\lambda Z_i)$ we arrive at the proposition since $\forall i$ the previous equation has to hold in an optimal linear tax equilibrium.

Contrary to the case of only one tax base, the optimal tax equilibrium consists of a system of conditions which captures the fiscal externalities arising from tax differentials. Assuming $\beta_{ji} > 0$, tax rates turn out to be higher than without considering the cross-effects. Note that if $\beta_{ji} = 0$, the standard formula for the optimal linear tax rate is nested in the system of equations as $\tau_i = \frac{1 - g_i}{1 + \zeta_{ii} - g_i}$.

Recall that the own-elasticity $\zeta_{ii}$ captures the change reported income of type $i$ with respect to changes in the net-of-tax rate $1 - \tau_i$ holding all other $1 - \tau_j$ constant. Thus, this parameter differs from the elasticity parameter $\zeta_i = \frac{\partial z_i}{\partial (1 - \tau_j)} \frac{1 - \tau_i}{z_i}$ which would be estimated in a tax system taxing all income according to the same schedule. Given $\beta_{ji}$ the elasticities can be related as $\zeta_{ii} = \zeta_i / (1 - \sum_{j=1,j\neq i}^n \beta_{ji})$.

Figures 2a-c show optimal linear tax rates for the case of three separate tax bases with different elasticities, which—for illustrative purposes—we label wage, self-employment, and capital income.

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5 By definition $\beta_{ii} = -1$.  

as a function of the share of the cross-elasticity effects (for further details see notes). As a first observation the differential in optimal tax rates decreases with the share of the cross-elasticity effect. Furthermore, this function is convex and approaches one when behavioral responses are fully offset by fiscal externalities. Interestingly, the degree of convexity is positively related to the level of average social welfare weights.

3 Institutional Background and Data

3.1 The personal income tax in Germany

All individuals in Germany are subject to personal income taxation. The first step is to determine a tax unit’s broad gross income from different sources and to allocate it to the seven forms of income the German tax law distinguishes between: income from agriculture and forestry, (non-corporate) business income, entrepreneurial income, salaries and wages from employment, investment income, rental income, and other income (including, for example, pensions, annuities and certain capital gains). For our empirical analysis, we group income from those seven sources into three categories: labor, capital and self-employment income. Labor income consists of salaries and wages from employment, self-employment income comprises income from agriculture and forestry, business income as well as entrepreneurial income, whereas capital income comprises investment income, rental income, and other income.

Second, for each type of income, the tax law allows for certain income-related expenses (Werbungskosten). In principle, all expenses that are necessary to obtain, maintain or preserve the income from a source are deductible. These include, for instance, commuting costs, expenses for work materials or costs of training. For non-itemizing taxpayers, there is an allowance for labor earnings (€920 in 2008) and capital income (€750 in 2008). The sum of broad gross income minus income-related deductions per income source yields the adjusted gross income. As a third step, deductions, including expenses for investment in human capital, child care costs, donations to charity or political parties and church tax payments, are taken into account and subtracted from adjusted gross income yielding taxable income.

Finally, the income tax is calculated by applying the rate schedule to taxable income. In contrast to most other countries who use a bracket system with constant marginal tax rates within a bracket, Germany uses a formula (which is quadratic in income) to compute the tax liability. As a consequence, marginal tax rates increase linearly in income (up to an top

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6 The following types of income are tax exempt: payments from health insurance, accident insurance and insurance for disability and old age, welfare benefits and scholarships.
marginal tax rate of 42%). The formula for the years 2007 and 2008 is defined as follows:

\[
T = \begin{cases} 
0 & \text{if } TI \leq 7,664 \\
(883.74 \frac{TI-7,664}{10,000} + 1,500) \frac{TI-7,664}{10,000} & \text{if } 7,664 < TI \leq 12,739 \\
(228.74 \frac{TI-12,739}{10,000} + 2,397) \frac{TI-12,739}{10,000} + 989 & \text{if } 12,739 < TI \leq 52,151 \\
0.42TI - 7,914 & \text{if } 52,151 < TI \leq 250,000 \\
0.45TI - 15,414 & \text{if } TI > 250,000.
\end{cases}
\]

In addition to the personal income tax, households additionally pay the “Solidaritätszuschlag”, a tax supplement originally introduced to finance the German reunification. During the period of interest, 2000 - 2008, the supplement amounts to 5.5% of the income tax liability.

3.2 Reforms 2001–2008

Figure 3 shows the marginal tax rate schedule for the years 2001-03, 2004 and 2005-08. Taxpayers with a high taxable income and those with a taxable income slightly exceeding the basic tax allowance experienced the largest marginal tax rate cuts. Between 2000 and 2005, a major reform of the German personal income tax took place. The basic tax allowance was increased in several steps from €6902 in 2000 to €7664 (2004–2008) with €7206 in 2001 and €7235 in 2002/03. The lowest marginal tax rate decreased from 22.9% in 2000 to 15% (2005–2008) with 19.9% (2001–03) and 16% (2004) in between. The top marginal tax rate was reduced from 51% in 2000 to 42% in 2005 with 48.5% (2001-03) and 45% (2004) in between. The threshold where the top marginal tax rate kicks in was reduced from €58,643 in 2000 to €52,151 in 2004 with values of €55,007 (2001-03) in between. In 2007, an additional tax bracket at the top (for taxable income above €250,000) was introduced with a top marginal tax rate of 45%.

Tax rates in the medium range of the schedule were lowered as well.

3.3 Data

Data set: We use the German Taxpayer Panel, which is an administrative data set collected by German tax authorities, provided and administered by the German Federal Statistical Office (Kriete-Dodds and Vorgrimler 2007). The unit of observation is the taxpayer, i.e., either a single individual or a couple filing jointly. The panel covers all German tax units in the period 2001 to 2008. We have access to a 5% random sample of the Taxpayer Panel and employ the respective weights provided by the Statistical Office. The dataset contains all information necessary to calculate a taxpayer’s annual income tax, this includes basic socio-demographic characteristics such as birth date, gender, family status, number of children as well as detailed information

\footnote{For married taxpayers filing jointly, the tax is twice the amount of applying the formula to half of the married couple’s joint taxable income: }
on income sources and tax base parameters such as work-related expenses and (claimed and realized) deductions.

**Sample Selection and Summary Statistics:** We consider all taxpayers who are taxed individually, i.e. singles. [NOTE: In the next version of the paper, we will conduct a separate analysis of singles and couples.] Table 1 shows summary statistics for the key variable of the analysis. The sample consists of 2,253,691 million individuals with mean taxable income of €17,854. Wage income is the predominant income source with roughly 70% percent of individuals reporting a non-zero amount and mean of €18,552. Capital income accounts only for a small share with a mean of €6,659 and is least unequally distributed. Income from self-employment is most unequally distributed with a 99% percentile of €213,052, which is more than twice as high as the corresponding value for wage income and four times for capital income.

### 4 Empirical Estimation of Social Welfare Weights

#### 4.1 Derivation

In this section we provide a derivation and estimation of implicit marginal social welfare weights for the case of Germany similar to Lockwood and Weinzierl (2014). In the derivation we decompose the elasticity of taxable income into income type specific elasticities assuming constant elasticities for each type independent of the income level. This implicitly endogenizes the elasticity with respect to the amount of taxable income since shares of income types change as depicted in Figures 1b-c. We further aggregate the overall estimates within the groups of distinguishable income sources to obtain an estimate of the income type specific average social welfare weight.

Consider a social planner with a standard welfare function \( S(\cdot) \) maximizing social welfare being restricted to levy the same tax from every taxpayer with the same amount of taxable income (as it applies to German tax law until 2008). Since the welfare implications of taxation are independent of the sources of income reported the use of a standard welfare function implies that the mechanical as well as the welfare effect of a marginal increase \( d\tau \) in a small band \( z + dz \) is the same as in Saez (2001). However, the social planner will take into account the differential in the elasticities across different types of income. For a taxpayer with total income of \( z \) the average behavioral response in reported income of type \( i \) is given by \( \left( \int_0^z h_i(z'|z)z'\zeta_{ii}dz' \right)/\left(1 - \tau(z)\right) \), and it follows that the elasticity effect is given by

\[
-d\tau dz \sum_{i=1}^n \left( \int_0^z h_i(z'|z)z'\zeta_{ii}dz' \right) h(z) \frac{\tau(z)}{1 - \tau(z)}.
\]
Optimality implies that the mechanical, welfare, and the elasticity effect offset each other, and thus

\[ \int_0^{\infty} g(z') dH(z') = 1 - H(z) - \sum_{i=1}^{n} \left( \int_0^{z} h_i(z'|z) z'_i \zeta_{ii} d z'_i \right) h(z) \frac{\tau(z)}{1 - \tau(z)}. \]

Taking the derivative with respect to \( z \) yields

\[ g(z) = -\frac{1}{h(z)} \frac{d}{dz} \left( 1 - H(z) - \sum_{i=1}^{n} \left( \int_0^{z} h_i(z'|z) z'_i \zeta_{ii} d z'_i \right) h(z) \frac{\tau(z)}{1 - \tau(z)} \right). \]

### 4.2 Empirical results

The estimated marginal welfare weights as implied by the German tax law between 2002 and 2008 are displayed in Figures 4a-h for different values of the own-elasticities \( \zeta_{ii} \) [NOTE: right now, we are assuming different values for the elasticities. In the next version, we plan to estimate those from the same data]. We consider three different scenarios: In the first scenario we assume a uniform elasticity across income types of 0.5. In the second scenario we use empirically plausible values of 0.25 for wage income, 0.75 for self-employment income, and 0.5 for capital income. The third scenario employs elasticities twice as high as in the second scenario.

The different scenarios yield qualitatively similar results with a more pronounce pattern for higher elasticities.\(^8\) By construction, the welfare weights equal one for the bottom 20% of individuals not paying income tax (i.e. with a marginal tax rate of zero). Intuitively, social marginal welfare weights are above 1 for medium income earners and fall below 1 for high income earners capturing the redistributive motive of the government. High marginal tax rates for top earners – under the condition of optimality – can only be justified by lower marginal welfare weights for taxpayers in the top of the income distribution. Surprisingly, the empirical pattern displays an increase in marginal welfare weights in the range from the 20% to the 60% percentile (more pronounced in earlier years), followed by a sharp drop. While from a normative standpoint this finding seems to be counter-intuitive, an explanation could be offered by a political economy interpretation (Bierbrauer and Boyer 2013): In the political competition parties put a higher weight on the mobile voters in the middle of the distribution (median voter).

Interestingly, the tax reforms in the 2000’s have changed the quantitative pattern only slightly: First, the decrease in the top tax rate until 2006 has increased social marginal welfare weights for high income earners. The empirical analysis suggests negative marginal welfare weights for this income group for the pre-reform period under two of the three scenarios, indicating a

\[^{8}\text{Note that the average elasticity is primarily driven by the value of the elasticity for wage income (see Figure 1c).}\]
non-Pareto efficient state of the tax system. Second, the increase from the 20% to the 60% percentile observed in the pre-reform period was moderated. Third, the spikes around the 60% percentile as well as the 75% have become more pronounced during the reform. Furthermore, the tax change introduced additional volatility in the marginal welfare weights in the top quartile of the income distribution.

Income type specific average welfare weights in Table 2 are calculated as described in section 2.3 where we use the empirical estimates of $g(z)$. Technically, each average income specific welfare weight is calculated by integrating the marginal welfare weights over all taxpayers weighted with the level of income of the respective type. Intuitively, the difference in welfare weights across income sources is generated by the different distributions of the income types. For example, consider the case of wage income which is predominantly concentrated in the bottom part of the income distribution. Thus, the average welfare weight for wage income majorly captures higher than average marginal welfare weights. On the contrary, income from self-employment is concentrated in the upper part of the income distribution, and thus the average welfare weight for this income source is predominantly driven by lower than average marginal welfare weights.

We present estimates of the income type specific average welfare weights for 4 scenarios of elasticities parallel to the analysis of marginal welfare weights. As expected higher elasticities yield a lower estimate of the average welfare weight due to higher efficiency costs of taxation. As a general pattern, the weight for income from self-employment is lowest, while the estimates for capital and wage income lie in a similar range with slightly higher values for capital income.

5 Simulation of Optimal Tax Rates

Figures 5a-d show estimates of jointly optimal linear income tax rates for wage income, income from self-employment, and capital income based on the previously estimated welfare weights and the assumed elasticities as a function of the share of the cross-elasticities. In all calculations we assume each tax base $j$ to exhibit the same share of cross-response in the total income type-specific elasticity, and further all income tax bases $i \neq j$ to account for the same cross-response in tax base $j$, i.e. $\beta_{ji} = \beta \forall i, j, i \neq j$.

In the benchmark specification in Figure 5a, we assume an income type independent elasticity of 0.5. In this case differences in tax rates for different income types can be fully attributed to the differences in average welfare weights. An assumption of no cross-elasticity effect yields estimates of $\tau_{Wage} = 0.31$, $\tau_{Self-Employed} = 0.51$, and $\tau_{Capital} = 0.22$, $\beta = 0$. As expected tax rates are inversely related to the income type specific average welfare weight. When increasing the share of the cross-effect $\beta$ the estimates rise due to fiscal externalities. Interestingly, the increase in tax rates is negatively related to the level of tax rates without cross-effects: A rise
of the share of the cross-elasticity from $\beta = 0$ to $\beta = 0.4$ induces an increase of $\tau_{\text{Capital}}$ by 16 percentage points, while the corresponding rise in $\tau_{\text{Self-Employed}}$ is only 9 percentage points. Intuitively, when cross-responses increase the potential revenue due to fiscal externalities is biggest for the tax base where the tax rate is lowest in the baseline scenario.

In Figure 4b we assume different elasticity parameters for each income source. We make use of empirical plausible values of 0.25 for wage income, 0.5 for capital income, and 0.75 for income from self-employment. In the specification without any cross-effect (column 1) $\tau_{\text{Wage}}$ is equal to 0.35, while $\tau_{\text{Self-Employed}} = 0.32$. The lower welfare weight for income from self-employment is offset by the higher elasticity such that $\tau_{\text{Wage}} > \tau_{\text{Self-Employed}}$. The tax rate on capital income in this specification is still significantly lower than the tax rates for wage income or income from self-employment which is caused by the higher welfare weight. Doubling the assumed elasticities in Figure 4c decreases the estimates only slightly and does not change the overall pattern. Since we have derived the average welfare weight given the elasticity estimates, there are two effects. First, the higher elasticity lowers optimal tax rates, and second, the implied lower average welfare weights increase optimal tax rates. Thus, the effects by tendency outweigh each other.

6 Conclusion

Current income tax systems partially assign different tax schedules to different income concepts. For example, in the US as well as Germany capital gains are taxed at a lower rate than wage income or income from self-employment. The rationale behind such a policy is, first, to account for differentials in the efficiency costs of taxation, and second, to utilize differences in income distributions for the purpose of redistribution. As a consequence of differentials in tax rates a major challenge is to account for fiscal externalities arising from cross-effects between tax bases, with semi-legal income shifting as the predominant form.

In this paper we provide a theoretical framework of jointly optimal income taxes for different sources of income taking into account fiscal externalities due to cross-effects between tax bases. We present a closed form solution for the case of linear tax schedules.

In the empirical analysis we estimate implied social marginal welfare weights of the German tax system. Accounting for differences in the income distributions of income types, we show that in the current tax system the average welfare weight for income from self-employment is low relative to wage and capital income. We estimate optimal linear tax rates accounting for differences in welfare weights and elasticities across income types. When assuming a uniform elasticity income from self-employment is taxed at a higher rate than wage and capital income.
Allowing for an empirically plausible scenario of higher elasticities among self-employed decreases the tax rate for income from self-employment below the one for wage income. In all scenarios the optimal tax rate for capital income is much lower than the estimates for wage income and income from self-employment. Incorporating reasonable magnitudes of fiscal externalities, leads to significant increases in optimal linear tax rates.

[more to come .... ]
Figures 1a-b: Distribution of Income Sources I

Note: Figures 1a-b provide graphical evidence on the income distribution conditional on the income type. Figure 1a displays local polynomial regressions with Epanechnikov kernel with first degree polynomial of the fraction of taxpayers according to their reported main income source on total taxable income. For this purpose the main income source is defined as the one with the highest value. The graph contains observations with taxable income in the range of €0 to €500,000. Figure 1b reports local polynomial regressions with Epanechnikov kernel with first degree polynomial of the fraction of a taxpayer’s type of income on taxable income. The graph contains observations with taxable income in the range of €5000 to €500,000.
Figure 1c: Distribution of Income Sources II

Note: Figure 1c provides additional graphical evidence on the income distribution conditional on the income type. The figure shows the average fraction of a certain income type conditional on the fractile of taxable income. For this purpose, we assign each taxpayer to the corresponding 0.001 fractile in the distribution of taxable income and compute the average fraction of a certain income type. All fractions are smoothed with a local polynomial regression with Epanechnikov kernel.
Figures 2a-b: Optimal Linear Income Tax Rates Dependent on the Degree of Cross-Elasticities and Average Welfare Weights

Note: Figures 2a-b display optimal linear income tax rates as a function of the share of the cross-elasticity in the total elasticity for each income type. Given a certain share $\beta$, the own-elasticity for each income-type can be decomposed in a real response of size $1 - \beta$ and a fiscal externality of size $\beta$. It is further assumed, that each income type exhibits the same $\beta$ and accounts for the same proportion in the cross-elasticity for the other income types. Figure 2a assumes an average welfare weight of 0 for each income type. Figure 2b assumes an average welfare weight of 0.7 for each income type. For illustrative purposes income types are labeled wage income (elasticity: 0.25), self-employment (elasticity: 0.5), and capital (elasticity: 0.75).
Figures 2c: Optimal Linear Income Tax Rates Dependent on the Degree of Cross-Elasticities and Average Welfare Weights

Note: Figures 2c displays optimal linear income tax rates as a function of the share of the cross-elasticity in the total elasticity for each income type. Given a certain share $\beta$, the own-elasticity for each income-type can be decomposed in a real response of size $1 - \beta$ and a fiscal externality of size $\beta$. It is further assumed, that each income type exhibits the same $\beta$ and accounts for the same proportion in the cross-elasticity for the other income types. Figure 2c assumes an average welfare weight of 0.9 for each income type.
Figure 3: Marginal Tax Rates

Note: Figure 3 shows the marginal tax rate as a function of taxable income for Germany from 2001 to 2008. Additional to the income tax the government levies a surcharge of 5.5%. Therefore, the effective marginal tax rate is given by $1.055 \cdot \tau$. 
Figures 4a-h: Social Marginal Welfare Weights

Note: Figures 4a-h display social marginal welfare weights as functions of the fractile in the distribution of taxable income for different scenarios of income-type specific elasticities as implied by the tax German tax system from 2002-2008. The dashed gray line indicates the effective marginal tax rate for a given fractile. Scenario 1: elasticities with respect to the net-of-tax rate of 0.5 (wage income), 0.5 (income from self-employment), 0.5 (capital income). Scenario 2: assumes elasticities with respect to the net-of-tax rate of 0.25 (wage income), 0.75 (income from self-employment), 0.5 (capital income). Scenario 3: assumes elasticities with respect to the net-of-tax rate of 0.5 (wage income), 1.5 (income from self-employment), 1.0 (capital income).
Figures 5a-b: Optimal Linear Income Tax Rates for Germany

Note: Figures 5a-b report optimal linear income tax rates for different income type specific elasticities, dependent on the share of the cross-elasticities. As in Figures 2a-c, it is implicitly assumed that $\beta_{ji} = \beta \forall i, j, i \neq j$. Assumed income type specific elasticities are listed in the footer of each panel.
Figures 5a-b: Optimal Linear Income Tax Rates for Germany

Note: Figures 5c-d report optimal linear income tax rates for different income type specific elasticities, dependent on the share of the cross-elasticities. As in Figures 2a-c, it is implicitly assumed that $\beta_{ji} = \beta \forall i, j; i \neq j$. Assumed income type specific elasticities are listed in the footer of each panel.
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Taxable Income</th>
<th>Wage Income</th>
<th>Capital Income</th>
<th>Income from Self-Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>17854</td>
<td>18552</td>
<td>6589</td>
<td>20559</td>
</tr>
<tr>
<td>p25</td>
<td>3460</td>
<td>3528</td>
<td>542</td>
<td>474</td>
</tr>
<tr>
<td>p50</td>
<td>11736</td>
<td>13873</td>
<td>3216</td>
<td>5741</td>
</tr>
<tr>
<td>p75</td>
<td>25541</td>
<td>27889</td>
<td>9531</td>
<td>20000</td>
</tr>
<tr>
<td>p90</td>
<td>38083</td>
<td>40563</td>
<td>15102</td>
<td>46415</td>
</tr>
<tr>
<td>p95</td>
<td>48831</td>
<td>50478</td>
<td>21693</td>
<td>75863</td>
</tr>
<tr>
<td>p99</td>
<td>90170</td>
<td>82283</td>
<td>56243</td>
<td>213052</td>
</tr>
<tr>
<td>N</td>
<td>2253691</td>
<td>1509876</td>
<td>802892</td>
<td>728996</td>
</tr>
</tbody>
</table>

Note: Table 1 reports summary statistics for the key variables in our analysis. Distribution parameters are weighted with the sampling weights provided by the German Statistical Office. All statistics are conditional on whether a non-missing amount was reported.

Table 2: Average Income Specific Social Welfare Weights

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{Wage}$</td>
<td>0.771</td>
<td>0.864</td>
<td>0.735</td>
<td>0.868</td>
</tr>
<tr>
<td>$g_{Self-Employed}$</td>
<td>0.487</td>
<td>0.644</td>
<td>0.375</td>
<td>0.661</td>
</tr>
<tr>
<td>$g_{Capital}$</td>
<td>0.859</td>
<td>0.897</td>
<td>0.831</td>
<td>0.903</td>
</tr>
</tbody>
</table>

Specification (1): Elasticities Assumed: Wage: 0.50 – Capital: 0.50 – Self-Employed: 0.50
Specification (2): Elasticities Assumed: Wage: 0.25 – Capital: 0.50 – Self-Employed: 0.75
Specification (3): Elasticities Assumed: Wage: 0.50 – Capital: 1.00 – Self-Employed: 1.50
Specification (4): Elasticities Assumed: Wage: 0.25 – Capital: 0.75 – Self-Employed: 0.50

Note: Table 2 shows average income specific social welfare weights as defined in section 2.3.
References


