Reference Dependence
and
The Social Security System

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Abstract

This paper studies an inter-temporal model to understand the role of social security in determining intergenerational distribution and welfare when consumers have reference-dependent preferences. To explore the effect of social security on consumer welfare, this paper introduces a parametric model of a unified social security system, by which different social security plans are represented via different degrees of fundedness. Using this unified social security system in an environment of OLG production economy, this paper analyzes the intergenerational distribution of consumption and welfare, and examines the effects of a transition from a less funded system to more funded one on savings, consumption, and capital accumulation for an economy populated by loss-averse agents. By deriving closed form solutions for the variables, this paper finds that an increase in the intensity of fundedness unambiguously increases capital accumulation and increase savings, but decreases consumption, when the population growth rate is greater than the net return on capital. This paper also finds that increases in tax rates increase savings unambiguously, whereas the effect on consumption is not conclusive.

JEL Classification: H55, D9

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1 Introduction

This paper aims to provide a theoretical development to understand the role of social security in determining intergenerational distribution and welfare when people have reference-dependent preferences. Firstly, this paper analyzes consumption and saving behavior for those whose preferences are reference-dependent. By reference dependence, I posit that a decision maker's utility

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depends not only on his actual consumption but also on comparisons to his beliefs about optimal consumption. A reference-dependent decision maker derives utility from comparison to the reference beliefs: it may be a gain to the reference utility or a loss to it. A loss is considered more important to decision maker than a gain of the same size.[1] It follows from the recent development in reference-dependent models that reference points may be constructed from consumers' beliefs about optimal consumption. In the model I specifically use Koszegi and Rabin (2006, 2007, 2009)'s equilibrium approach to determine the reference points. Given any information regarding future income stream, the decision maker forms beliefs on what should be the optimal consumption in the standard world.[2] With this ex ante optimal consumption profile on hand, the decision maker decides whether he should follow this rule or not. According to this assumption, the standard decision maker is loss averse with respect to the reference points of the optimal consumption. Since he is afraid of the prospective loss in future consumption due to high consumption today, he would refrain from over-consuming in the current period. However when his loss aversion is low, the decision maker can deviate from the standard consumption-saving behavior.

Secondly, this paper investigates the effect of social security on consumer welfare through intergenerational distribution. Social security can change consumers' saving behavior and can thus affect consumption and welfare via reductions in private savings. Both the direction and size of this effect depend on the type of social security system in place. To study the effect of social security plans on welfare thoroughly, I introduce a parametric model of a comprehensive social security system. Under this scheme, a given system of social security system can be represented by the degree of fundedness−the portion of payroll invested for future benefits in the form of social security income. This implies that for a given payroll tax rate, the funded portion is subject to investment for the retirement income of the currently young people, but the unfunded portion is to be transferred to the currently old. Zero-fundedness specifies the PAYG system, in which all taxed amounts are used to finance the pensions of the currently old. Full intensity, by contrast, specifies a fully-funded social security system. Between these two extremes, there are many partially funded systems. On this model, it is possible to evaluate many types of combined national social security plans.

According to Kaganovich and Zilcha (2012), and Kunze (2012), it may be necessary to see a transition from an unfunded social security system to a fully funded one as we move toward an ageing society. Thus, it may be useful to develop a model for a unified social security plan whose effect on welfare we can directly analyze as it evolves into a more funded one. Using this unified social security system, I analyze the intergenerational distribution of consumption and welfare. Specifically I compare savings, consumption, and capital accumulation under the three different social security systems and examine the effects of alternative systems on these variables. One contribution of this paper is the derivation of closed form solutions for these variables in such a way that direct comparisons among different systems are made possible. The solution is based

[1] This is called *loss aversion*, one of the properties of the Prospect Theory by Kahneman and Tversky (1979) .

[2] This implies the usual utility maximization procedure to get an optimal solution in standard models.

on the maximization problem for two-period intertemporal choice models in an OLG production economy environment populated by a younger generation who work, consume, and save, and an older generation who consume out of their personal savings and public pensions. It is well known that a social security system gives the older generation the opportunities to smooth out their consumption stream over time. However, there is a crowding-out effect between private savings and public savings, and the effect depends crucially on the size of fundedness. The result shows that an increase in the intensity of fundedness unambiguously increases capital accumulation, and increases savings but decreases consumption, when the population growth rate is greater than the net return on capital. This paper also finds that increases in tax rate increase savings unambiguously, whereas the effect on consumption is not conclusive.

The following section introduces a simple consumption-saving model with the reference-dependent preference. Section 3 introduces the model of social security for those consumers with reference dependence. Section 4 discusses the equilibrium in steady states and compares the welfare among the different systems. Then it concludes.

2 Reference-Dependence in Consumption-Saving

To explore the implication of social security for decision makers with reference-dependent preference, I first construct a simple consumption-saving model with the preference, but without any consideration regarding intergenerational transfer using a social security plan. To define the reference-dependent preference, I introduce several notations presented by Kőszegi and Rabin (2006, 2007, 2009), who construct reference-dependent preferences in a way different from prospect theory by Kahneman and Tversky (1979), by adding up consumption utility to the gain-loss utility. Thus, the reference-dependent utility function for one dimensional state space is defined by

$$V(c|r) = U(c) + GL(c|r)$$

where $U(c)$ is classical consumption utility which is increasing, concave or quasi-concave, and differentiable. $GL(c|r)$ is the gain-loss utility that satisfies the following:

- Continuous, differentiable except at $x = 0$: $GL(0) = 0$
- Strictly increasing
- If $y > z > 0$, then $GL(y) + GL(-y) < GL(z) + GL(-z)$
- $GL''(x) < 0$ for $x > 0$ and $GL''(x) > 0$ for $x < 0$
- $lim_{x \to 0+} \frac{GL'(-x)}{GL'(x)} \equiv \lambda_k > 1$

Then I consider a two-period consumption and saving model for a decision maker who decides how much to consume and save in the first period. Let the two periods be denoted by t and $t + 1$, and the time discount rate be β. The decision maker is endowed with an income stream

$\{y_t, y_{t+1}\}$, both non-negative, and his utility is given by CRRA specification.[3] Assume further that the decision maker can borrow and lend freely at the market determined gross interest rate $R = 1 + r$. In a standard two-period consumption-saving decision, the agent chooses the optimal consumption profile by solving the maximization problem.[4] Let this solution $\{c_t^*, c_{t+1}^*\}$ be the *optimal consumption plan*, which is obtained from the standard maximization procedure given an income profile. Following Koszegi and Rabin (2009), I set this solution the reference points in my model of reference dependence: the solution is constructed from the decision maker's belief about optimal consumption in a *standard* world. The next step is to determine whether the decision maker who has reference-dependent preference would choose this consumption bundle among all other choices available to him?

I build a reference-dependence version of the consumption-saving model to answer the question. Consider the following scenario: the decision maker is an over-consumer and may want to consume more in the first period. If he intends to consume more than the suggested optimal at t, and as a result he will end up with consuming less than the optimal at $t+1$, i.e. $\{c_t > c_t^*,\ c_{t+1} < c_{t+1}^*\}$, then the reference-dependent decision-maker's optimization problem is

$$U(c_t, c_{t+1}|c_t^*, c_{t+1}^*) = \tag{1}$$

$$\frac{c_t^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right) + \beta\left[\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right) + \eta\omega\lambda\left(\frac{c_{t+1}^{1-\gamma}}{1-\gamma} - \frac{c_{t+1}^{*1-\gamma}}{1-\gamma}\right)\right]$$

$$\textit{subject to}$$
$$c_t + b_{t+1} = y_t$$
$$c_{t+1} = y_{t+1} + Rb_{t+1}$$

where b_{t+1} is bond holdings (saving if positive; borrowing if negative) at t for the next period, $t+1$. The total utility comes from both the consumption utilities, which is represented by $U(c_t)$ and $\beta U(c_{t+1})$, and the gain-loss utilities. If the decision maker consumes more than the reference point in the first period, he should have a contemporaneous gain utility in that period, which is $[U(c_t) - U(c_t^*)]$ when $\eta = 1$.[5] It follows that his consumption is reduced in the next period and this yields a prospective loss utility relative to the reference point. This is given by $\beta\omega\lambda[U(c_{t+1}) - U(c_{t+1}^*)]$, where $\lambda > 1$ is the coefficient of loss aversion and ω represents psychological discounting for the feeling that occurs in the future. Can the decision maker's intention of over-consumption be rationalized? Is there a profitable deviation from the optimal solution, i.e. the reference point? If there is no gain-loss utility, $\eta = 0$, in the above equation, then this returns to the standard

[3] This utility specification satisfies: $U' > 0$, $U'' < 0$ and $lim_{c\to 0}U'(c) = \infty$.

[4] This is the ex-ante solution and is also optimal ex post in the standard model, because, if otherwise, we can always find a consumption point that can make the decision maker better off.

[5] η : the weight of gain-loss utility relative to consumption utility

model and the solution to the problem is $\{c_t^*, c_{t+1}^*\} = \left\{\frac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}},\ (\beta R)^{1/\gamma}\left(\frac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}}\right)\right\}$. From the life-time resource constraint $c_{t+1} = y_{t+1} + R(y_t - c_t)$, the change in utility from a change in consumption is represented by

$$\frac{dU}{dc_t} = (1+\eta)c_t^{-\gamma} - \beta R(1+\eta\omega\lambda)[y_t + R(y_t - c_t)]^{-\gamma} \tag{2}$$

If I evaluate the value at the optimal consumption point $c_t^* = \frac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}}$, then I obtain:

$$\frac{dU}{dc_t^*} = \eta(1-\omega\lambda)\left(\frac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}}\right)^{-\gamma} \tag{3}$$

$$= \eta(1-\omega\lambda)c_t^{*-\gamma}\left\{\begin{array}{ll} \leqslant 0 & if\ \ \omega\lambda \geqslant 1 \\ > 0 & if\ \ \omega\lambda < 1 \end{array}\right\} \tag{4}$$

The condition implies that if $\omega\lambda \geqslant 1$, there is no incentive for the decision maker to deviate from the consumption point c_t^* because choosing any other consumption point would not increase his utility ex post. When the agent has high loss aversion, then his choice is not different from the one of the standard decision makers. In other words, the standard agents are those who care more about the prospective loss utility than the contemporaneous gain utility.[6] However, for agents with $\omega\lambda < 1$, deviation from the ex ante optimal consumption is profitable because it increases the utility ex post. The next step is to determine the optimal consumption for the deviators. From the maximization equation (1), I obtain the optimality condition for those who have relatively low loss aversion:

$$(1+\eta)c_t^{-\gamma} = R\beta(1+\eta\omega\lambda)c_{t+1}^{-\gamma} \tag{5}$$

Thus, the consistent consumption profile for the agent who intends to over-consume in the first period, $\{c_t > c_t^*,\ c_{t+1} < c_{t+1}^*\}$, is

$$\left\{\begin{array}{c} c_t = \dfrac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \\ c_{t+1} = \ (\beta R)^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\dfrac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right) \end{array}\right\} \tag{6}$$

It is easy to see $c_t > c_t^*$ and $c_{t+1} < c_{t+1}^*$ because

[6] This is what Koszegi and Rabin (2009) noticed: prospective loss from lowering future consumption tends to act as internal commitment device. This implies that decision makers with reference dependent preferences would not over-consume even though they do not have any external commitment device like illiquid asset such as the golden eggs in Laibson (1999).

$$c_t = \frac{Ry_t + y_{t+1}}{R{+}(\beta R)^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} > \frac{Ry_t + y_{t+1}}{R{+}(\beta R)^{1/\gamma}} = c_t^* \tag{7}$$

and

$$c_{t+1} = (\beta R)^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\frac{Ry_t + y_{t+1}}{R{+}(\beta R)^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right) < (\beta R)^{1/\gamma}\left(\frac{Ry_t + y_{t+1}}{R{+}(\beta R)^{1/\gamma}}\right) = c_{t+1}^* \tag{8}$$

If the decision maker cares more about contemporaneous gain utility than about prospective loss utility, his over-consumption in the current period is rationalized and his consumption can be bigger than the usual one in the standard model shown above.[7] However, we can construct the reference-dependent preference model in a different way. If the decision maker intends to *consume less than that of the optimal plan* in the first period, such as for the case of a natural born saver, expecting more consumption than the optimal in the the future, i.e. $\{c_t < c_t^*,\ c_{t+1} > c_{t+1}^*\}$, then he solves the following:

$$U(c_t, c_{t+1}|c_t^*, c_{t+1}^*) = \tag{9}$$

$$\frac{c_t^{1-\gamma}}{1-\gamma} + \eta\left[\omega\lambda\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right) + \beta\left(\frac{c_{t+1}^{1-\gamma}}{1-\gamma} - \frac{c_{t+1}^{*1-\gamma}}{1-\gamma}\right)\right] + \beta\frac{c_{t+1}^{1-\gamma}}{1-\gamma}$$

subject to the same budget constraint as in the over-consumer's problem. Because the decision maker consumes less (and saves more) than the optimal in the first period, he has a contemporaneous feeling of loss relative to the reference consumption point. This loss utility is given by $\omega\lambda[U(c_t) - U(c_t^*)]$. Less consumption in the first period results in higher consumption in the next period and this gives him a prospective feeling of gain, $\beta[U(c_{t+1}) - U(c_{t+1}^*)]$. Like in the case of an over-consumer, when the agent has high loss aversion he would not deviate from the ex ante optimal solution. Thus, the solution to this problem is summarized by

$$\left\{\begin{array}{c} c_t = \dfrac{Ry_t + y_{t+1}}{R{+}(\beta R)^{1/\gamma}} = c_t^* \\ c_{t+1} = (\beta R)^{1/\gamma}\left(\dfrac{Ry_t + y_{t+1}}{R{+}(\beta R)^{1/\gamma}}\right) = c_{t+1}^* \end{array}\right\} \text{ if } \omega\lambda \geqslant 1 \tag{10}$$

and

[7] This implies that RDP model yields the similar result as in the model with present biased preference or hyperbolic discounting that is based on time inconsistent preferences. In fact, this result comes from the Hyperbolic discounting-like component in gain-loss utility.

$$
\left\{
\begin{array}{c}
c_t = \dfrac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}} \\
c_{t+1} = (\beta R)^{1/\gamma}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}\left(\dfrac{Ry_t + y_{t+1}}{R+(\beta R)^{1/\gamma}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}}\right)
\end{array}
\right\} \text{ if } \omega\lambda < 1 \tag{11}
$$

It is clear that for a miser, i.e. who has $\omega\lambda < 1$ for contemporaneous loss, we have $c_t < c_t^*$ and $c_{t+1} > c_{t+1}^*$. When the decision maker cares more about prospective gain utility than about contemporaneous loss utility, he would deviate from the standard prediction and consume less (save more) in the current period. This may rationalize the behavior of natural born saver: the savers save because their prospective gain feelings outweigh the contemporaneous loss feelings. Unlike these savers, the standard decision makers are those who don't want to deviate because of their greater feelings of loss. By the two scenarios of the two-period reference-dependence model, we know that the deviation is two-folds: over-consumption or under-consumption than what is considered optimal in standard models.

3 The Model

3.1 Demographics

Consider the following OLG economy. Time is discrete and it runs $t = 0, 1, 2, ...$ At each time, L_t individuals are born and they live for two periods, $s = 1(young)$ and $s = 2(old)$. The population is growing at a rate of n so that $L_t = (1 + n)L_{t-1}$. Thus, at specific time t there are L_t young individuals and $L_{t-1} = L_t/(1 + n)$ old individuals. Each young agent is endowed with e_t units of labor measured in efficiency. The young agents earn labor income of $w_t e_t$, in which w_t is the wage rate determined in the labor market. When they are old, the agents live on their private savings from young age and their social security benefit. This implies that the old generations do not have any labor income, but financial income and transfers via social security. When the market-determined net interest rate is r_t, the financial income from private savings will be given by $(1 + r_{t+1})S_{1t+1}$, where S_{1t+1} is the saving of a young generation made at time t for the time $t + 1$. Each agent who is born at t, consumes c_{1t} when young and c_{2t+1} when old. Therefore, the young generations decide how much they consume and save for the future out of their labor income in working years. We assume that there is no bequest motivation among the old generations. Finally, the economy is closed and I assume that the government who runs a social security plan is in balanced budget each time.

3.1.1 Partially-Funded Social Security

For the model we assume a unified social security system, by which different social security schemes are represented via different degrees of fundedness. Let the intensity of the fundedness be $\pi \in [0,1]$. Then a partially-funded social-security system implies that given a payroll tax rate τ_t, only a portion $\pi\tau_t$ is invested in social security pension-funds for the future benefit of *currently young* tax payers. The remaining portion $(1-\pi)\tau_t$ is to be transferred to the *current old* people for their retirement benefits. This implies that an old individual at time t receives social security income of $\pi(1+r_t)\tau_{t-1}w_{t-1}e_{t-1} + (1-\pi)(1+n)\tau_t w_t e_t$, where $w_t e_t$ is their labor income as explained above. The first term represents funded portion and the second the unfunded part. Thus, PAYG social security system is represented by $\pi = 0$ and a fully-funded one is given by $\pi = 1$.

3.2 Allocation Decision

Young individuals born in time t have time-separable reference-dependent preferences and maximize their lifetime utilities by choosing c_{1t} and c_{2t+1}. In this paper we focus on the first type of agents (over-consumers) so that they may intend to over-consume when they are young. The representative agent solves the following inter-temporal choice problem in which his periodic utility follows from the one in Section 2:

$$MaxU_t(c_{1t}, c_{2t+1}|c^*_{1t}, c^*_{2t+1}) \tag{12}$$

$$= \frac{c_{1t}^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_{1t}^{1-\gamma}}{1-\gamma} - \frac{c_{1t}^{*1-\gamma}}{1-\gamma}\right) + \beta\left[\frac{c_{2t+1}^{1-\gamma}}{1-\gamma} + \eta\omega\lambda\left(\frac{c_{2t+1}^{1-\gamma}}{1-\gamma} - \frac{c_{2t+1}^{*1-\gamma}}{1-\gamma}\right)\right]$$

subject to

$$c_{1t} + S_{1t+1} = (1-\tau_t)w_t e_t \tag{13}$$

$$c_{2t+1} = (1+r_{t+1})S_{1t+1} + T_{t+1} \tag{14}$$

in which $c_{1t} \geqslant 0$, $c_{2t+1} \geqslant 0$ and the social security income T is given by

$$T_{t+1} = \pi(1+r_{t+1})\tau_t w_t e_t + (1-\pi)(1+n)\tau_{t+1}w_{t+1}e_{t+1}. \tag{15}$$

The second line in the budget constraint describes that old agents live on both private savings and social security income (public savings plus transfer). As seen in the previous section, when an agent's loss aversion is high, he would not deviate from the standard optimization rule. However, if $\omega\lambda < 1$ then the optimality condition should be

$$(1+\eta)c_{1t}^{-\gamma} = \beta(1+r_{t+1})(1+\eta\omega\lambda)c_{2t+1}^{-\gamma}. \tag{16}$$

From the FOC and the two constraints, Eq. (17) and Eq. (18), we obtain the following optimal solution for the agent regarding consumption and saving:

$$c_{1t} = \frac{(1{+}r_{t+1})(1\text{-}\tau_t + \pi\tau_t)w_t e_t + (1\text{-}\pi)(1{+}n)\tau_{t+1}w_{t+1}e_{t+1}}{(1{+}r_{t+1}) + [\beta(1{+}r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \tag{17}$$

$$c_{2t+1} = [\beta(1{+}r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\frac{(1{+}r_{t+1})(1\text{-}\tau_t{+}\pi\tau_t)w_t e_t{+}(1\text{-}\pi)(1{+}n)\tau_{t+1}w_{t+1}e_{t+1}}{(1{+}r_{t+1}) + [\beta(1{+}r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right) \tag{18}$$

$$S_{1t+1} = \frac{[\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}(1-\tau_t)w_t e_t - T_{t+1}}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \tag{19}$$

with the social security income T_{t+1} as defined in Eq. (19). Using this partial equilibrium result, I analyze first the effect of tax on the variables of the reference-dependence model compare to the ones in the standard case ($\omega\lambda \geqslant 1$). To explore the effect of payroll tax change, I assume for now that the rate change is permanent: $\tau_t = \tau_{t+1} = \tau$. Then given a value of fundedness intensity $0 \leq \pi \leq 1$, the change in consumption with respect to the tax rate is

$$\frac{\partial c_{1t}}{\partial \tau} = (1-\pi)\frac{(1+n)w_{t+1}e_{t+1} - (1+r_{t+1})w_t e_t}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \tag{20}$$

$$\frac{\partial c_{2t+1}}{\partial \tau} = \left(\beta(1{+}r_{t+1})\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}(1-\pi)\frac{(1+n)w_{t+1}e_{t+1} - (1+r_{t+1})w_t e_t}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \tag{21}$$

It is clear that an increase in tax rate offers higher consumption for both periods if $(1{+}n)w_{t+1}e_{t+1} > (1+r_{t+1})w_t e_t$.This is because the lifecycle wealth is increased as the tax rate increases. Moreover, the size of the effect in the model is the same as in the standard one. That is, the tax elasticities of consumption in both models are equal to each other:

$$\frac{\frac{\partial c_{1t}}{\partial \tau}}{\frac{c_{1t}}{\tau}} = \frac{\frac{\partial c^*_{1t}}{\partial \tau}}{\frac{c^*_{1t}}{\tau}} \quad \text{and} \quad \frac{\frac{\partial c_{2t+1}}{\partial \tau}}{\frac{c_{2t+1}}{\tau}} = \frac{\frac{\partial c^*_{2t+1}}{\partial \tau}}{\frac{c^*_{2t+1}}{\tau}} \tag{22}$$

Also, under the fully funded system, i.e. $\pi = 1$, the effect of tax rate change is neutral on consumption[8], which fact is also true in the standard case : $\frac{\partial c_{1t}}{\partial \tau} = 0$, implying that when $\pi = 1$, the intertemporal budget-constraint is, in fact, $c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = w_t e_t$. Similarly, the change in saving with respect to tax is

[8] This is also true with the standard case $\omega\lambda \geqslant 1$

$$\frac{\partial S_{1t+1}}{\partial \tau} = -\frac{([\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} + \pi(1+r_{t+1}))w_t e_t + (1-\pi)(1+n)w_{t+1}e_{t+1}}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} < 0 \quad (23)$$

This implies that given a level of π, as public savings and transfer increase, the incentive for private savings decreases. Unlike the case of consumption, the tax elasticities of saving in our reference-dependence model has a smaller effect than that of the standard model.

$$\frac{\frac{\partial S_{1t+1}}{\partial \tau}}{\frac{S_{1t+1}}{\tau}} < \frac{\frac{\partial S^*_{1t+1}}{\partial \tau}}{\frac{S^*_{1t+1}}{\tau}} \quad (24)$$

Next, let us analyze the change in saving with respect to fundedness intensity. By this analysis, we may get an idea regarding what would happen as we move from a less funded social security system toward a more funded one. I do this first in partial equilibrium, in which interest rates and wage rates are given. Later I will look for the effect in general equilibrium assuming that the rates are not fixed but adjusted in the long run. If I obtain the partial derivative with respect to π,

$$\frac{\partial S_{1t+1}}{\partial \pi} = \frac{(1+n)\tau_{t+1}w_{t+1}e_{t+1} - (1+r_{t+1})\tau_t w_t e_t}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \quad (25)$$

First of all, it is easy to find that size of change is greater in the model than the case in the standard model: $\left|\frac{\partial S_{1t+1}}{\partial \pi}\right| > \left|\frac{\partial S^*_{1t+1}}{\partial \pi}\right|$. From the equation, we can also infer that under the condition of $(1+n)\tau_{t+1}w_{t+1}e_{t+1} > (1+r_{t+1})\tau_t w_t e_t$, private savings by the young generation increase as funding intensity increases. If we assume that all the rates are fixed as in steady states, then this returns to $1+n > 1+r$, implying that, when population growth rate is higher than the net interest rate, the private savings increase as we move toward more funded system. This can be explained by the social security income equation $T_{t+1} = \pi(1+r_{t+1})\tau_t w_t e_t + (1-\pi)(1+n)\tau_{t+1}w_{t+1}e_{t+1}$. In the equation, higher π implies a lower weight of the second term (transfer). When $1+n > 1+r$, the second term has a greater portion and thus, the lower weight would affect negatively on overall income.

Second, let us define the total savings by $Z_{1t+1} = S_{1t+1} + \pi(1+r_{t+1})\tau_t w_t e_t$. The total saving for the next period is the private savings by young individuals plus the amount invested at t for the $t+1$ social security income. Because among all the tax collection, only portion π is saved, this makes public savings. Then the total savings increase as we go for a more funded system. To see this, let us first define the total saving by $Z_{1t+1} = S_{1t+1} + \pi(1+r_{t+1})\tau_t w_t e_t$. Then

$$\frac{\partial Z_{1t+1}}{\partial \pi} = \theta(1+n)\tau_{t+1}w_{t+1}e_{t+1} + (1-\theta)(1+r_{t+1})\tau_t w_t e_t > 0 \quad (26)$$

with $\theta = \frac{1}{(1+r_{t+1})+[\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}$. Compared to the standard model, this equation implies that $\left|\frac{\partial Z_{1t+1}}{\partial \pi}\right| > \left|\frac{\partial Z^*_{1t+1}}{\partial \pi}\right|$ whenever $(1+n)\tau_{t+1}w_{t+1}e_{t+1} > (1+r_{t+1})\tau_t w_t e_t$. Similarly,

$$\frac{\partial c_{1t}}{\partial \pi} = \frac{(1+r_{t+1})\tau_t w_t e_t - (1+n)\tau_{t+1}w_{t+1}e_{t+1}}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \tag{27}$$

When $(1+n)\tau_{t+1}w_{t+1}e_{t+1} > (1+r_{t+1})\tau_t w_t e_t$, the consumption decreases as fundedness (π) increases. This is because consumption depends on lifecycle income, which will be reduced as π increases for $n > r$ as explained above. Since c_{2t+1} is a monotonic function of c_{1t}, the effect on c_{2t+1} Also, because consumption in the second period is a monotonic function of c_{1t}, the effect on c_{2t+1} follows the same direction as the one in the first period.

3.3 Technology and Equilibrium

I assume a competitive market equilibrium for a production economy. For simplicity, I assume that there is no technological progress and the production function be Cobb-Douglas such that output per capita is given by

$$y_t = f(k_t) = k_t^{\alpha} \tag{28}$$

where $k_t = K_t/L_t$ and $y_t = Y_t/L_t$. Let us also assume that there exist initial old agents at the beginning of the economy and they own capital stock K_0. This capital stock is distributed equally among old individuals. Then according to the competitive market equilibrium, the factor prices are paid out their marginal productivity.

$$r_t = f'(k_t) - \delta = \alpha k_t^{\alpha-1} - \delta \tag{29}$$

$$w_t = f(k_t) - k_t f'(k_t) = (1-\alpha)k_t^{\alpha} \tag{30}$$

At equilibrium, the capital demand should meet capital supply: nation-wide total savings Z_{1t+1} should be equal to the capital stock at $t+1$. Since $Z_{1t+1} = S_{1t+1} + \pi(1+r_{t+1})\tau_t w_t e_t$, the market clearing condition implies

$$(1+n)k_{t+1} = S_{t+1} + (1+r_{t+1})\pi\tau_t w_t e_t \tag{31}$$

where the saving S_{t+1} is a function of both w_t and r_{t+1}. Assume $e_t = 1$. Then by substituting r_{t+1} and w_t into the equation, I obtain the following law of motion:

$$(1+n)k_{t+1} = \frac{[\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}(1-\tau_t)w_t - T_{t+1}}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} + (1+r_{t+1})\pi\tau_t w_t \tag{32}$$

where $T_{t+1} = \pi(1 + r_{t+1})\tau_t w_t e_t + (1 - \pi)(1 + n)\tau_{t+1} w_{t+1} e_{t+1}$. Replacing all the variables into the law of motion generates a difference equation in the form of[9] $(1 + n)k_{t+1} = \Psi(k_{t+1}, k_t, \tau_{t+1}, \tau_t\ ; \pi)$. To simplify this, let us assume that $\gamma = 1$, implying $u = log(c)$, and $\tau_t = \tau_{t+1} = \tau$, as well as $\frac{1+\eta\omega\lambda}{1+\eta} = \mu$. Then I obtain

$$\frac{(1+n)}{(1-\alpha)} k_{t+1} = \tag{33}$$

$$\frac{\beta\mu(1-\tau) - \pi\tau}{(1+\beta\mu)} k_t^{\alpha} - \frac{(1-\pi)(1+n)\tau k_{t+1}^{\alpha}}{(1+\beta\mu)(1+\alpha k_{t+1}^{\alpha-1} - \delta)} + (1 + \alpha k_{t+1}^{\alpha-1} - \delta)\pi\tau k_t^{\alpha}.$$

Here I perform a comparative analysis in general equilibrium analysis, in which the variables are not fixed but adjusted through the market equilibrium. As in the previous section, I specifically study the effect of τ and π on the policy variables such as saving. From the market clearing condition (35), for $\tau_t = \tau_{t+1} = \tau$,

$$(1 + n)k_{t+1} = S_{t+1} + (1 + r_{t+1})\pi\tau w_t \equiv Z_{t+1} \tag{34}$$

where Z_{t+1} is the total savings defined above. Because $Z_{t+1} = f(r_{t+1}, w_t, w_{t+1}; \pi, \tau) = f(r(k_{t+1}), w(k_t), w(k_{t+1}); \pi,$ equation (38) determines k_{t+1} as an implicit function of k_t, π, and τ. To get the effect of π and τ on other variables, I need to derive all the partial derivatives from the equation. Thus, the total differential from equation (38) gives

$$(1 + n)dk_{t+1} = \tag{35}$$

$$\frac{\partial Z_{t+1}}{\partial r} r'(k_{t+1})dk_{t+1} + \frac{\partial Z_{t+1}}{\partial w} w'(k_t)dk_t + \frac{\partial Z_{t+1}}{\partial w} w'(k_{t+1})dk_{t+1} + \frac{\partial Z_{t+1}}{\partial \pi} d\pi + \frac{\partial Z_{t+1}}{\partial \tau} d\tau$$

To figure out the total effect, I first focus on each of the five partial derivatives in the equation. Among the five, we know $\frac{\partial Z_{t+1}}{\partial \pi} > 0$ as seen in Equation (42).[10] Again, using $\theta = \frac{1}{(1+r_{t+1}) + [\beta(1+r_{t+1})]^{1/\gamma} \left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}$ with $0 < \theta < 1$, it follows that

$$\frac{\partial Z_{t+1}}{\partial w_t} = \theta(1-\tau)[\beta(1+r_{t+1})]^{1/\gamma} \left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} + (1-\theta)(1+r_{t+1})\pi\tau > 0 \tag{36}$$

$$\frac{\partial Z_{t+1}}{\partial w_{t+1}} = -\theta(1-\pi)(1+n)\tau < 0 \tag{37}$$

Equation (40) and (41) show that given any fundedness intensity, the total saving increases when current income increases, whereas it decreases when future income increases. Unlike these three variables,[11] the other two derivatives, $\frac{\partial Z_{t+1}}{\partial \tau}$ and $\frac{\partial Z_{t+1}}{\partial r}$, are not determined.

[9] because $T_t = (1 + r_t)\pi\tau_{t-1}(1 - \alpha)k_{t-1}^{\alpha} + (1 + n)(1 - \pi)\tau_t(1 - \alpha)k_t^{\alpha}$

[10] $\frac{\partial Z_{1t+1}}{\partial \pi} = \theta(1 + n)\tau w_{t+1} + (1 - \theta)(1 + r_{t+1})\tau w_t > 0.$

[11] $\frac{\partial Z_{t+1}}{\partial \pi} > 0$, $\frac{\partial Z_{t+1}}{\partial w_t} < 0$, $\frac{\partial Z_{t+1}}{\partial w_{t+1}} < 0.$

$$\frac{\partial Z_{t+1}}{\partial \tau} = -\theta\{[\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} w_t + (1-\pi)(1+n)w_{t+1}\} + (1-\theta)(1+r_{t+1})\pi w_t \quad (38)$$

This explains that an increase in tax decreases private savings but increases public savings. Even more complicated effect comes from the interest rate change due to the trade-off between income and substitution effects. Thus $\frac{\partial Z_{t+1}}{\partial r}$ is not determined.[12] When the interest rate increases, given fundedness intensity, it increases publicly saved amounts through T_{t+1}. However, a higher T_{t+1} tends to decrease the incentive for private saving, while the private saving itself is subject to intertemporal income and substitution effects when there is an increase in factor price, i.e. $-\frac{1}{1+r_{t+1}}$. Then, the overall effects of π and τ on k_{t+1} are

$$\frac{\partial k_{t+1}}{\partial \pi} = \frac{\frac{\partial Z_{t+1}}{\partial \pi}}{(1+n) - \frac{\partial Z_{t+1}}{\partial r}r'(k_{t+1}) - \frac{\partial Z_{t+1}}{\partial w}w'(k_{t+1})} \quad (39)$$

$$\frac{\partial k_{t+1}}{\partial \tau} = \frac{\frac{\partial Z_{t+1}}{\partial \tau}}{(1+n) - \frac{\partial Z_{t+1}}{\partial r}r'(k_{t+1}) - \frac{\partial Z_{t+1}}{\partial w}w'(k_{t+1})} \quad (40)$$

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{\frac{\partial Z_{t+1}}{\partial w}w'(k_t)}{(1+n) - \frac{\partial Z_{t+1}}{\partial r}r'(k_{t+1}) - \frac{\partial Z_{t+1}}{\partial w}w'(k_{t+1})} \quad (41)$$

First, notice that $r_t'(k_t) = -\alpha(1-\alpha)k_t^{\alpha-2} < 0$ and $w_t'(k_t) = \alpha(1-\alpha)k_t^{\alpha-1} > 0$. Because it is true that $\frac{\partial Z_{t+1}}{\partial w} < 0$, if $\frac{\partial Z_{t+1}}{\partial r}$ is positive then all the denominators are positive.[13] If it is the case, the sign of numerators completely determines the direction of change. Thus, it is straight forward to have $\frac{\partial k_{t+1}}{\partial \pi} > 0$ and $\frac{\partial k_{t+1}}{\partial k_t} > 0$. For the case of $\frac{\partial k_{t+1}}{\partial \tau}$, because $\frac{\partial Z_{t+1}}{\partial \tau}$ is not deterministic, the overall result is ambiguous. This finding is not different from the one of the standard model.

4 Long Run Effects

This section analyzes the effect of parameters on the policy variables at steady states, by deriving steady state capital-output level. I still assume the same equation: $(1+n)k_{t+1} = \Psi(k_{t+1}, k_t, \tau_{t+1}, \tau_t\ ; \pi)$, which is

$$\frac{(1+n)}{(1-\alpha)}k_{t+1} = \quad (42)$$

$$\frac{\beta\mu(1-\tau)-\pi\tau}{(1+\beta\mu)}k_t^\alpha - \frac{(1-\pi)(1+n)\tau k_{t+1}^\alpha}{(1+\beta\mu)(1+\alpha k_{t+1}^{\alpha-1}-\delta)} + (1+\alpha k_{t+1}^{\alpha-1}-\delta)\pi\tau k_t^\alpha.$$

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[12] $\frac{\partial Z_{t+1}}{\partial r} = -\frac{\left([\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}(1-\tau)w_t + \pi(1+r_{t+1})\tau w_t + (1-\pi)(1+n)\tau w_{t+1}\right)\left(1+\beta/\gamma[\beta(1+r_{t+1})]^{1/\gamma-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\right)}{\left(1+r_{t+1}+[\beta(1+r_{t+1})]^{1/\gamma}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\right)^2} + \theta\left(\beta/\gamma[\beta(1+r_{t+1})]^{1/\gamma-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}(1-\tau)w_t - \pi\tau w_t\right) + \pi\tau w_t$

[13] The postive numerator can be obtained even with the negative sign as long as it is moderate so that we have $(1+n) - \frac{\partial Z_{t+1}}{\partial w}w'(k_{t+1}) > \frac{\partial Z_{t+1}}{\partial r}r'(k_{t+1})$

Although there are many interrelated variables in the equation, we are able to get a closed form solution if the value of π is given. Thus, by setting $\pi = 0$, we obtain the solution for PAYG social security system and by $\pi = 1$, the fully-funded system. Let the solution to this equation be $\overline{k}$. Then all other steady state variables including steady state savings and consumption are determined. They are

$$c_1 = \frac{\overline{w}}{1+\beta\mu}\left((1-\tau) + \pi\tau + (1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) \tag{43}$$

$$c_2 = \frac{\overline{w}\beta\mu(1+\overline{r})}{1+\beta\mu}\left((1-\tau) + \pi\tau + (1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) \tag{44}$$

where $\overline{r} = r(\overline{k}) = \alpha\overline{k}^{\alpha-1} - \delta$ and $\overline{w} = w(\overline{k}) = (1-\alpha)\overline{k}^{\alpha}$. It may be intuitive that the consumption in the reference dependence model is greater than the one in the standard model in steady states: $c_1 > c_1^*$ and $c_2 < c_2^*$ as in the partial equilibrium case. Similarly,

$$S_1 = \frac{\overline{w}}{1+\beta\mu}\left(\beta\mu(1-\tau) - \pi\tau - (1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) < S_1^* \tag{45}$$

$$Z_1 = \frac{\overline{w}}{1+\beta\mu}\left(\beta\mu(1-\tau) - \pi\tau - (1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) + \pi(1+\overline{r})\tau\overline{w} < Z_1^* \tag{46}$$

The next is to analyze the effect of tax on savings, consumption and capital accumulation in steady states. Given a value of fundedness, the long-run effect of an increase in tax rate is

$$\frac{\partial S_1}{\partial \tau} = -\frac{\overline{w}}{1+\beta\mu}\left(\beta\mu + \pi + (1-\pi)\frac{1+n}{1+\overline{r}}\right) < 0$$

Thus, a higher tax rate induces lower private savings regardless of intensity level. In case of consumption, we have

$$\frac{\partial c_1}{\partial \tau} = \frac{\overline{w}}{1+\beta\mu}\left((1-\pi)\frac{n-\overline{r}}{1+\overline{r}}\right) \tag{47}$$

which is positive if $n > \overline{r}$, but negative if $n < \overline{r}$ for all values of $0 \leq \pi < 1$. But at $\pi = 1$ (the fully funded system), we can say that tax is not distorting because $\frac{\partial c_1}{\partial \tau} = 0$. Overall, the effect on consumption is not conclusive. Finally, for the effect on capital accumulation, we again use implicit differentiation to get

$$\frac{\partial k}{\partial \tau} = \frac{(1+\alpha k^{\alpha-1}-\delta)\pi - \frac{(\beta\mu+\pi)}{(1+\beta)} - \frac{(1-\pi)(1+n)}{(1+\beta\mu)(1+\alpha k^{\alpha-1}-\delta)}}{\left(\frac{(1+n)}{(1-\alpha)}k^{-\alpha} - \frac{(1-\pi)(1+n)\tau\alpha k^{\alpha-2}}{(1+\beta\mu)(1+\alpha k^{\alpha-1}-\delta)^2} + \pi\tau\alpha k^{\alpha-2}\right)(1-\alpha)}. \tag{48}$$

From this, one may infer that, when $\pi = 0$, tax increases capital level for $0 < k < \left(\frac{\sqrt{\frac{(1-\alpha)\tau\alpha}{(1+\beta\mu)}} - \alpha}{1-\delta}\right)^{\frac{1}{1-\alpha}}$. However, when $\pi = 1$, higher tax implies higher capital level for $0 < k < \left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}$.

Finally, I analyze regarding what would happen to the policy variables when we have more the long run effect of the intensity of fundedness on other policy variables. From the steady state saving equation (48), the partial derivative w.r.t. π is

$$\frac{\partial S_1}{\partial \pi} = \frac{\overline{w}}{1+\beta\mu}\left(\frac{n-\overline{r}}{1+\overline{r}}\right)\tau \tag{49}$$

This means that given a steady state wage rate and interest rate, a higher fundedness intensity increases savings if the population growth rate is greater than the steady state net return to capital. If, however, population growth rate is lower than the net return to capital, then saving decreases $\frac{\partial S_1}{\partial \pi} < 0$. Regarding effect on consumption, the opposite result is obtained because

$$\frac{\partial c_1}{\partial \pi} = \frac{\overline{w}}{1+\beta\mu}\left(\frac{\overline{r}-n}{1+\overline{r}}\right)\tau. \tag{50}$$

To see the effect of π on capital accumulation, we need to get partial $\frac{\partial k}{\partial \pi}$. Because the law of motion is an implicit function of capital and fundedness intensity, we need to obtain this through implicit differentiation: from equation(44), the partial derivative w.r.t. π around the state state capital is

$$\frac{\partial k}{\partial \pi} = \tau \frac{\frac{(1+\alpha k^{\alpha-1}-\delta)^2(1+\beta)+(1+n)-(1+\alpha k^{\alpha-1}-\delta)}{(1+\alpha k^{\alpha-1}-\delta)(1+\beta\mu)}}{\left(\frac{(1+n)}{(1-\alpha)}k^{-\alpha} + \frac{(1+n)\tau(1-\phi)\alpha k^{\alpha-2}}{(1+\beta\mu)(1+\alpha k^{\alpha-1}-\delta)^2} + \pi\tau\alpha k^{\alpha-2}\right)(1-\alpha)} \tag{51}$$

If $(1+\alpha k^{\alpha-1}-\delta)^2(1+\beta\mu)+(1+n) > 1+\alpha k^{\alpha-1}-\delta$, then $\frac{\partial k}{\partial \pi} > 0$ because $1-\alpha > 0$ and $1-\pi > 0$. In fact, $(1+\alpha k^{\alpha-1}-\delta)^2(1+\beta\mu)+(1+n) > 1+\alpha k^{\alpha-1}-\delta$ and this implies that we have increased capital accumulation in the long run as the funded intensity increases.

4.1 Steady State Welfare

First of all, I want to compare the steady state consumption in reference dependence with the case in the standard model. From the consumption function,

$$c_1 = \frac{\overline{w}}{1+\beta\mu}\left((1-\tau)+\pi\tau+(1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) \tag{52}$$

$$c_2 = \beta\mu(1+\overline{r})\frac{\overline{w}}{1+\beta\mu}\left((1-\tau)+\pi\tau+(1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) \tag{53}$$

It is clear that $c_1 > c_1^*$ and $c_2 < c_2^*$. Likewise, the reference-dependence saving is smaller than that of the standard consumer, $S_1 < S_1^*$ because

$$S_1 = \frac{\overline{w}}{1+\beta\mu}\left(\beta\mu(1-\tau)-\pi\tau-(1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) < \frac{\overline{w}}{1+\beta}\left(\beta(1-\tau)-\pi\tau-(1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) = S_1^* \tag{54}$$

Next I analyze the steady state movement under different social security plans. I specifically compare the three types of social security system: PAYG, partially funded and fully funded social security. Let us set the funded intensity be $\{0, \pi, 1\}$, where $0 < \pi < 1$. Each of the these values specifies social security system of PAYG, partially funded, or fully funded. Based on the result in Section 4, we are able to determine the relative size of consumption in steady states. For this analysis, it is necessary to set the initial steady state at the same level: $\overline{k}_{PAYG} = \overline{k}_{PF} = \overline{k}_{FF}$. Then from the consumption function we identify the consumption under each system in terms of one of them. Thus,

$$c_1^{PAYG} = c_1^{FF} + \frac{\tau w(n-r)}{(1+\beta\mu)(1+r)} \tag{55}$$

$$c_1^{PF} = c_1^{FF} + \frac{(1-\pi)\tau w(n-r)}{(1+\beta\mu)(1+r)} \tag{56}$$

This implies, it is true that $c_1^{PAYG} > c_1^{PF} > c_1^{FF}$ at steady states if $n > r$, whereas $c_1^{PAYG} < c_1^{PF} < c_1^{FF}$ if $n < r$. Likewise, because c_2 is a monotonic function of c_1, we obtain

$$c_2^{PAYG} = c_2^{FF} + \frac{\beta\mu}{(1+\beta\mu)}\tau w(n-r) \tag{57}$$

$$c_2^{PF} = c_2^{FF} + \frac{\beta\mu(1-\pi)\tau w(n-r)}{(1+\beta\mu)} \tag{58}$$

This implies that if $n > r$, we have the same ranking regarding second-period consumption so that $c_2^{PAYG} > c_2^{PF} > c_2^{FF}$. Under the condition, because the consumption in both periods have the same inequality, the welfare from a social-security plan for reference-dependent consumers has the following order:

$$U^{PAYG}(c_1, c_2) > U^{PF}(c_1, c_2) > U^{FF}(c_1, c_2) \tag{59}$$

However, for the economy with low population growth rate, the ranking will be reversed. Regarding savings, consider the saving functions under three types of social security:

$$S_1^{PAYG} = \frac{\overline{w}}{1+\beta\mu}\left(\beta\mu\,(1-\tau) - \tau\frac{1+n}{1+\overline{r}}\right) \tag{60}$$

$$S_1^{PF} = \frac{\overline{w}}{1+\beta\mu}\left(\beta\mu(1-\tau) - \pi\tau - (1-\pi)\tau\frac{1+n}{1+\overline{r}}\right) \tag{61}$$

$$S_1^{FF} = \frac{\overline{w}}{1+\beta\mu}\left(\beta\mu(1-\tau) - \tau\right) \tag{62}$$

From this, it is clear that $S_1^{PAYG} < S_1^{PF} < S_1^{FF}$ if $1 + n > 1 + \overline{r}$, because

$$-\tau\frac{1+n}{1+\overline{r}} < -\pi\tau - (1-\pi)\tau\frac{1+n}{1+\overline{r}} < -\tau \tag{63}$$

If the steady-state population growth rate is greater than the interest rate, then the consumers under fully-funded system save most. Likewise, if the opposite is true, $1 + \overline{r} > 1 + n$, then

$$-\tau\frac{1+n}{1+\overline{r}} > -\pi\tau-(1-\pi)\tau\frac{1+n}{1+\overline{r}} > -\tau \qquad (64)$$

implying $S_1^{PAYG} > S_1^{PF} > S_1^{FF}$: consumers under PAYG system save most. In a special case of $1+n = 1+\overline{r}$, all of the three system give the same saving rate for all $0 < \pi < 1$. Related to this, we have an interesting question regarding capital stock in a dynamic system. It is about what would be the steady state capital stock with which all the social security plans give rise to the same saving rate regardless of intensity of fundedness. To see this it is necessary to obtain the level of steady state capital stock that makes the three saving functions equal to each other. Because $\overline{r} = \alpha k^{\alpha-1} - \delta$, the identity condition returns

$$1+n = 1+\alpha\overline{k}^{(\alpha-1)} - \delta \qquad (65)$$

Therefore, the steady state capital stock that generates the same saving rate is

$$\overline{k} = \left(\frac{\alpha}{n+\delta}\right)^{\alpha-1} \qquad (66)$$

5 Numerical Exercise

This section demonstrates the implication of the RDP model by simulating the model to search for long-run general equilibrium and comparing the result with the one of the standard model. Specifically, I perform numerical exercises to derive steady state consumption and savings under the three plans of a pension system: {zero funded, partially funded, fully funded}. By this exercise, one may get an intuition regarding the intergenerational distribution and capital accumulation when an economy evolves from a less funded system to a more funded one. I set each period of the model 30 years. Then I use the following parameter values specified at annual values for both RDP and standard models:

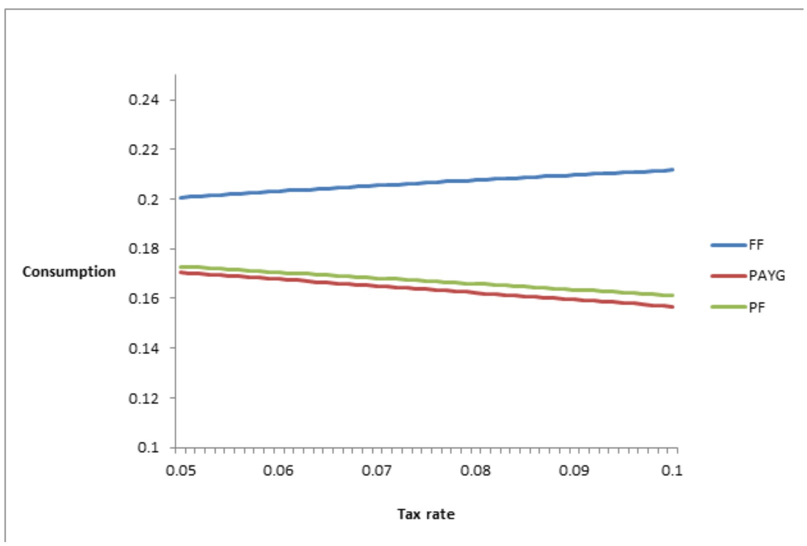
parameter	α	β	δ	τ	n	γ	η	$\omega\lambda$
annual value	0.33	0.98	0.05	0.05	0.01	1	1	0.8 *or* 1*

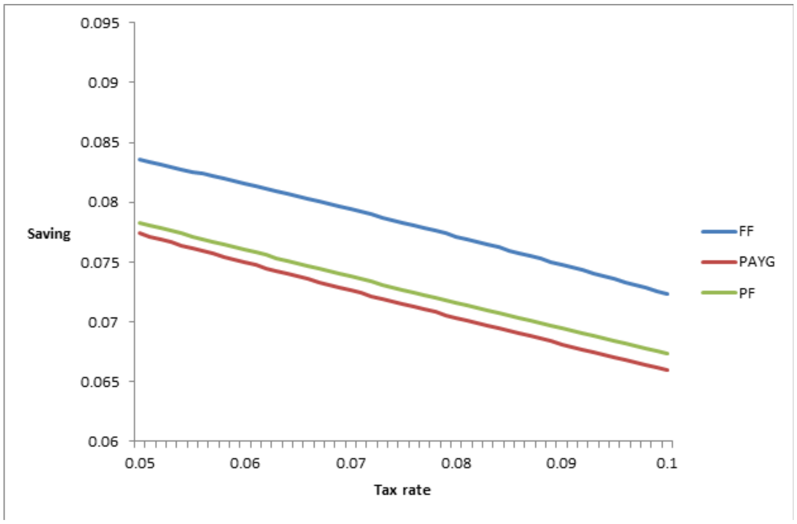
*: $\omega\lambda = 1$ implies the standard model.

The following table shows the *steady state* profile of capital level, consumption, and saving for the two models regarding the three plans of social security system.

	Standard Model	RDP Model
PAYG $\pi = 0$	$k = 0.063933$	$k^{RDP} = 0.057420$
	$c_1 = 0.170683$	$c_1^{RDP} = 0.170514$
	$S_1 = 0.086172$	$S_1^{RDP} = 0.077394$
	$c_2 = 0.244719$	$c_2^{RDP} = 0.233043$
Partial $\pi = 1/2$	$k = 0.066382$	$k^{RDP} = 0.059731$
	$c_1 = 0.172904$	$c_1^{RDP} = 0.172840$
	$S_1 = 0.087157$	$S_1^{RDP} = 0.078316$
	$c_2 = 0.243018$	$c_2^{RDP} = 0.231268$
FF $\pi = 1$	$k = 0.093443$	$k^{RDP} = 0.086863$
	$c_1 = 0.198285$	$c_1^{RDP} = 0.200646$
	$S_1 = 0.092839$	$S_1^{RDP} = 0.083547$
	$c_2 = 0.233711$	$c_2^{RDP} = 0.220823$

Next, I simulate the model to demonstrate the effect of tax on various policy variables.





From both of the figures I find that increases in tax rates decrease savings unambiguously for all plans considered, whereas the effect on consumption is not conclusive.

6 Conclusion

This paper introduces a parametric model of a unified social security system, by which different social security plan is represented via degrees of fundedness. With a unified social security system in an environment of OLG production economy, I analyze intergenerational distribution regarding consumption and welfare for consumers with reference dependence, and examine the effects of transition from a less funded system to more funded one on savings, consumption, and capital accumulation. By deriving closed form solutions for the variables, I find that increase in intensity of fundedness unambiguously increases capital accumulation, and increase saving but decreases consumption, when population growth rate is greater than the net return to capital. I also find that increase in tax rate increases savings unambiguously, while the effect on consumption is not conclusive.

With the unified social security system, this paper analyzes intergenerational distribution regarding consumption and welfare. This paper specifically compares the saving, consumption, and capital accumulation under the three different social security systems and examine the effects of alternative system on these variables. One contribution of this paper is to derive closed form solutions for these variables so that direct comparison among different systems is made possible. The solution is based on the maximization problem of two-period intertemporal choice model in an environment of OLG production economy populated by young generations who work, consume and save and the old who consume out of their personal savings and/or public pensions. It is well know that a social security system gives the old generation opportunities to smooth out consumption stream over time. However there is crowding out effect between private saving and public saving and the effect crucially depends on the size of fundedness. The analytic result shows that increase in intensity of fundedness unambiguously increases capital accumulation, and increase saving but decreases consumption, when population growth rate is greater than the net return to capital. This paper also finds that increase in tax rate increases savings unambiguously, while the effect on consumption is not conclusive.

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