

Optimal taxation, credit constraints and the timing of income-tested transfers

Robin Boadway¹, Jean-Denis Garon², and Louis Perrault³

¹Queen's University

²Université du Québec à Montréal

³Georgia State University

November 2, 2017

EARLY DRAFT. PLEASE DO NOT CITE.

Abstract

We study optimal income and commodity tax policy with credit-constrained low-income households. Workers are assumed to receive an even flow of income during the tax year, but make tax payments or receive transfers at the end of the year. They use their disposable income to purchase multiple commodities over the year. We show that differentiated subsidies on commodities can be welfare-improving even if the Atkinson-Stiglitz Theorem conditions apply. The extent of the differentiation is shown to be a function of the cost of transferring resources between periods and the ratio of costs coming from income effects of the subsidies through different levels of consumption made by constrained individuals of each commodity.

1 Introduction

Many government transfer programs are income-tested and delivered through the tax system. Examples include refundable tax credits that decline in income, such as employment tax credits (the Earned Income Tax Credit in the U.S.), tax credits for children and childcare (the Canada Child Benefit), value-added tax credits (the Goods and Services Tax Credit in Canada), disability tax credits, and pension tax credits. A key feature of these transfer programs is that entitlements cannot be fully determined until the taxpayer's income tax form has been filed and approved by the tax authorities. In the above examples, transfer payments are paid periodically in a given year based on taxable income (or family income) of the previous year. In some cases, adjustments can occur while the transfers are being received if the taxpayer's circumstances change in a way that can be verified by the government, such as childbirth or change in employment or disability status.

The consequence is that transfer recipients' income flow is lumpy. Those with low enough income to be eligible for a transfer from the government will have low—possibly zero—income during the year and a large transfer starting after the year ends. Individuals who anticipate a transfer would like to smooth their consumption stream over the year by borrowing. However, they may be precluded from doing so by a credit constraint. Financial institutions may be unwilling to lend to them except at exorbitant interest rates, especially if they do not have a credit rating or if the financial institution cannot verify the expected transfer.

We adopt an optimal income and commodity tax perspective to study policy responses to this issue. The informational assumptions of optimal taxation accord well with the problem. The model we use is stylized and meant to capture the essential features of the information constraint faced by the government and the credit constraint faced by transfer recipients. Unlike in the standard optimal income tax setting, we assume that individuals receive an even flow of income during the tax year, but make tax payments or receive transfers at the end of

the year.¹ Individuals use their disposable income to purchase a flow of multiple commodities over the tax year. The government knows only the workers' labor incomes at the end of the year. However, following [Guesnerie \(1995\)](#), we assume that the government observes all anonymous transactions on commodity markets and can impose a set of linear commodity taxes or subsidies at the time the purchases occur. Therefore, if the government wants to undertake some redistribution during the fiscal year, implicit transfers can be made through commodity subsidies and could be targeted to the intended individuals by a differential rate structure.

Our main focus is on the case where individuals are credit constrained which can prevent them from smoothing their consumption over the fiscal year. The credit constraint becomes especially relevant when the government's redistribution scheme implies paying transfers at the end of the year. With perfectly functioning credit markets, those anticipating transfers would borrow throughout the year to smooth the consumption financed by their future transfer. Then, the standard results of optimal tax theory would hold, including the well-known [Atkinson & Stiglitz \(1976\)](#) theorem when labor and consumption are separable. However, when transfer recipients face a binding credit constraint that precludes them from smoothing their consumption, giving transfers at the end of the year does not achieve the government's redistributive objectives earlier in the year. We show that differentiated subsidies on commodities can be welfare-improving even if the Atkinson-Stiglitz Theorem conditions apply.

The idea that consumption tracks income due to credit constraints is well established. For example, in the buffer-stock model of [Deaton \(1991\)](#), consumers' inability to borrow and impatience predicts that consumption will track income and that credit constraints can be binding. Using evidence on caloric intake of food stamp recipients, [Shapiro \(2005\)](#) finds that the short-term discount rate of these individuals is very high and hardly reconcilable with geometric discounting. Studying the effect of stimulus payments from the 2001 tax cut

¹In practice, tax remittance are often made throughout the year by employers through payroll deductions, but this only applies for taxpayers and not transfer recipients. Ignoring these remittances will have no effect on our analysis since those who pay taxes face no credit constraint.

episode to explain the phenomenon of ‘wealthy hand-to-mouth’ who mostly own illiquid assets, [Gruber \(1997\)](#) finds evidence that unemployment insurance, which is paid on a frequent basis, significantly smooths household consumption. [Parker \(1999\)](#) finds that consumers do not perfectly smooth their demand for goods when they expect a change in their income (although, in their case, the complexity in the tax code may be at stake). More recently, [Aguila et al. \(2017\)](#) found in a natural experiment that smoothing cash-transfers over the year facilitates consumption smoothing. In particular, they find that more frequent cash-transfer programs are associated with more consistent spending on basic needs, such as food and doctor appointments.

Another source of evidence comes from household behavior during the months when the Earned Income Tax Credit (EITC) is received. [McGranahan & Schanzenbach \(2013\)](#) find that households who are eligible for the EITC spend relatively more on healthy items during the months when most refunds are paid. Among these healthy items one finds vegetables, meat, poultry and dairy products. In a recent survey paper, [Nichols & Rothstein \(2016\)](#) stress that “[households] are often unable to borrow at reasonable interest rates (as evidenced by the high take-up of extremely high interest refund anticipation loans). If credit constraints are binding, a lump-sum payment has a smaller effect on the household’s utility than would a series of smaller payments throughout the year.” They also note that until 2010, EITC recipients could apply for a partial advance payment throughout the year. Although a small proportion of individuals opted-in, the most plausible explanation for taking up the credit would be that individuals are severely credit constrained.

In a recent work, [Baker \(2017\)](#) finds that the income elasticity of consumption is significantly higher for highly indebted households (after controlling for net assets). He concludes that “credit constraints play a dominant role in driving differential household consumption responses across households with varying levels of debt.” Also, using data from households who experienced a temporary income reduction during the U.S. federal government shutdown in 2013, [Baker & Yannelis \(2015\)](#) find indications that households who have a better

access to credit or who have more accumulated savings exhibit significantly smaller spending reductions during the transitional shock.

In the following sections, we study optimal income and commodity tax policy with credit-constrained low-income households in a multi-type nonlinear income taxation setting. The model features several skill-types of households who supply labor and consume two commodities. To simplify matters, we assume that transactions can occur at two discrete points: in the middle of the period and at the end. Preferences are weakly separable so in the absence of credit constraints optimal commodity taxes will be uniform at indeterminate rates given that proportional commodity taxation is equivalent to proportional income taxation. The two commodities are not consumed in the same proportions by different skill-types, and this will lead to differential commodity subsidization in the presence of credit constraints. The credit constraint will take the simplest of forms. As well, for reasons to be explained, we assume that it is costly for the government to make budgetary expenditures during the period. Doing so requires it to borrow against its end-of-period tax revenues.

In our analysis, we make the simplifying assumption that the government cannot use universal lump-sum payments to all individuals as frequently as transactions on consumption goods occur. To fully exhibit the policy consequences of this in a two-period model, we simply assume away lump-sum transfers. Lately, there have been debates in several countries about the introduction of a universal basic income. However, nothing guarantees that such transfer payment will be paid in a timely enough manner so that the most vulnerable agents in society can fully benefit from them. From a more technical standpoint, introducing the possibility of subsidizing consumption goods and the payment of lump-sum transfers with the exact same timing would be, into our model, similar to allowing the government to run a distinct linear-progressive tax scheme in every period at high frequency. We briefly explore the consequence of this later.

2 Model

Consider N types of individuals who are indexed by $i \in \{1, \dots, N\}$. There are n_i type- i s, each of whom has exogenous productivity w^i . The whole population is normalized to one so that $\sum_{i=1}^N n_i = 1$. We divide the year into two sub-periods $t = 1, 2$, and assume that each individual works with the same intensity in both sub-periods and earns a gross income $Y^i/2$ in each. At the end of $t = 2$, a type- i agent pays an income tax T^i (or receives a transfer if it takes a negative value). When agents choose their labor supplies ex-ante, they know their end-of-period income tax liability and therefore their disposable income over both sub-periods.

We use the methodology of [Christiansen \(1984\)](#) to introduce consumption of commodities in the model. In each sub-period t , agents choose a consumption bundle consisting of two goods (c_t^i, d_t^i) . The producer prices of goods c and d are set to unity, and the consumer prices can include a commodity tax, which can equivalently be either per unit or ad valorem: $q_c \equiv 1 + t_c$ and $q_d \equiv 1 + t_d$. Commodity taxes (per unit rates) t_c and t_d are the same for both sub-periods and for all households since otherwise arbitrage opportunities would exist. The agent's utility function is

$$U^i(c_t^i, d_t^i, Y_i) = \sum_t u(c_t^i - \bar{c}, d_t^i) - h\left(\frac{Y^i}{w^i}\right) \quad (1)$$

where Y^i/w^i is total labor supply over the two sub-periods and $h(\cdot)$ is a strictly convex cost or disutility function. The function $u(\cdot, \cdot)$ is the per-period utility of consuming the bundle of goods. To ensure that commodity tax differentiation is not a by-product of nonlinear Engel curves, we sometimes assume that u is quasi-homothetic by introducing a basic need \bar{c} on good c . This would stand for a minimal quantity of food, or dwelling. Individuals do not discount their utility across periods, which does not restrict our results and seems reasonable since we consider rational expectations over a short period of time (a fiscal year). Note that although the agent supplies labor in both sub-periods, the disutility of labor supply is defined

over total (annual) labor supply. Since commodities are separable from labor or leisure in the utility function (1), the Atkinson-Stiglitz Theorem would apply in this model in the absence of a credit constraint.²

We introduce imperfections in the credit market in the form of a credit constraint. The credit constraint applying in the first sub-period is

$$q_c c_1^i + q_d d_1^i \leq \frac{Y^i}{2} + \tau + \phi, \quad (2)$$

where ϕ is exogenously given. Individuals have access to a competitive credit market if they want to borrow or save. Those who save do so at rate \underline{r} and those who borrow do so at rate \bar{r} , with $\underline{r} \leq \bar{r}$. This reflects the cost of financial intermediation. For an individual i , we denote by $r_i \in \{\underline{r}, \bar{r}\}$ depending on whether, in the optimum, he is respectively a net saver or borrower at $t = 1$. If the government borrows, it can do so at rate $r_g > \underline{r}$, meaning that it borrows at a higher rate than the risk-free rate at which individuals can invest their short-term savings.³

Under these assumptions, we shall see that the two sub-periods setting gives the same solution as a standard Mirrlees problem when there is no credit spread, that is, when $\underline{r} = \bar{r} = r_g$. This is our benchmark case. Then, we introduce a borrowing constraint that prevents individuals from using more than ϕ dollars of their end-of-year transfers as a collateral when applying for a loan. The simplest case is when $\phi = 0$, which mimics the corner solution one would obtain if borrowers faced an interest rate that is prohibitively high. Given that our model abstracts from solvency issues and financial risks related to lending to individuals, this is a simple way to introduce credit market frictions without explicitly modeling solvency risks.⁴

²The model assumes that individuals commit to their labor supply and that labor supply is the same across periods. This does not drive the results and simplifies the analysis.

³In particular, this prevents the fiscal policy from being a Ponzi scheme and evacuate arbitrage opportunities.

⁴A more complex model would involve risk. Then, it would be costlier to banks to lend to individuals and the interest rate for borrowers would be high. This would give us the same intuition, but would significantly

To be precise, in the case where there would be no credit constraint, yearly budget constraints are (from an end-of-year standpoint)

$$(q_c c_1^i + q_d d_1^i)(1 + r_i) + q_c c_2^i + q_d d_2^i \leq \frac{Y^i}{2}(1 + r_i) + \frac{Y^i}{2} - T^i.$$

Since individuals earn $Y^i/2$ every sub-period and only pay their taxes (get their transfers) T^i at the end of the year and that individuals can make transactions in the financial markets, the nonlinear tax problem amounts to choosing $I^i \equiv \left(\frac{2 + r_i}{2}\right) Y^i - T^i$. Therefore, one can rewrite the constraint as

$$(q_c c_1^i + q_d d_1^i)(1 + r_i) + q_c c_2^i + q_d d_2^i \leq I^i. \tag{3}$$

2.1 Borrowing constraints and the role of commodity taxes: intuition

Now that the main parts of the model are set up, let us provide the intuition behind our results. Since sub-utility functions $u(c_t^i - \bar{c}, d_t^i)$ are the same across periods and there is no within-period discounting, without a binding credit constraint and if $\underline{r} = \bar{r} = 0$, individuals will perfectly smooth their consumption over time and consume half of their net yearly income in each period. Then, if the government imposes undifferentiated commodity taxes (or subsidies) $t_c = t_d$, it can reach the same allocation by taxing everyone's yearly income at rate $t_Y = t_c/(1 + t_c) = t_d/(1 + t_d)$. In this case, we can normalize one consumption tax to zero and let the flat revenue-collection component be captured by the proportional tax on income (leisure). Recall, however, that income taxes are collected at the end of the period, while commodity taxes apply in each sub-period. Thus, to replicate the effect of a uniform consumption tax that applies each sub-period on the budget stream of taxpayers, the government would have to impose a equal lump-sum tax or subsidy to all individuals in

complicate the problem.

both sub-periods.

However, when an individual's borrowing constraint binds, this equivalence does not hold. Since individuals will consume a different mix of the two goods in the two sub-period, the revenue-raising (subsidy) component in t_c and t_d cannot be imitated by a proportional tax (subsidy) on income. The income tax is not paid in the first period, so the binding credit constraint is, with $\phi = 0$,

$$(1 + t_c)c_1^i + (1 + t_d)d_1^i = \frac{Y^i}{2}. \quad (4)$$

Note that it does not contain a tax on its right-hand side. Therefore, if the government want to tax income specifically in the first period, it has to do it through the taxation of goods. Similarly, if he wants to redistribute in the first period, it has to do it either through a subsidy on goods or through a uniform lump-sum subsidy to all individual in the first sub-period (since it cannot identify individuals by type then).

Suppose we divide both sides of the credit constraint by $1 + t_c$:

$$c_1^i + \alpha d_1^i = \frac{1}{1 + t_c} \left(\frac{Y^i}{2} \right), \quad (5)$$

where $\alpha \equiv q_d/q_c$ is the price ratio chosen by the government, and $1/(1 + t_c)$ is the change in purchasing power of first-period disposable income induced by taxation or subsidization of t_c . Although the optimal price ratio, which tells us whether taxes should be differentiated in the optimum, may be unique, the optimal inter-temporal transfer of purchasing power on the right-hand side can be achieved through taxation or subsidization of either good. Let us denote by α^* the optimal price ratio chosen by the government.

Suppose instead we divide the credit constraint by $1 + t_d$:

$$\beta c_1^i + d_1^i = \frac{1}{1 + t_d} \left(\frac{Y^i}{2} + \phi \right), \quad (6)$$

where the optimal price ratio is now $\beta \equiv q_c/q_d$. The optimal price ratio that will be chosen

by the government will be $\beta^* = 1/\alpha^*$, but the transfer of purchasing power during the year will be performed by a tax or a subsidy on good d , t_d .

2.2 Government's budget constraint

The government's budget constraint in absolute terms is

$$\begin{aligned} \sum_i \left(\left(\frac{2+r_i}{2} \right) Y^i - I^i \right) + (1+r_g)(q_c-1) \sum_i c_1^i + (q_c-1) \sum_i c_2^i + (1+r_g)(q_d-1) \sum_i d_1^i \\ + (q_d-1) \sum_i d_1^i = R. \end{aligned}$$

Normalizing it by q_c , setting $\alpha \equiv q_d/q_c$, we get it in terms of relative prices:

$$\begin{aligned} \sum_i \left(\left(\frac{2+r_i}{2} \right) \frac{Y^i}{q_c} - \frac{I^i}{q_c} \right) + (1+r_g) \left(1 - \frac{1}{q_c} \right) \sum_i c_1^i + \left(1 - \frac{1}{q_c} \right) \sum_i c_2^i + (1+r_g) \left(\alpha - \frac{1}{q_c} \right) \sum_i d_1^i \\ + \left(\alpha - \frac{1}{q_c} \right) \sum_i d_1^i = \frac{R}{q_c} \end{aligned}$$

The fact that $r_g > \underline{r}$ makes it socially costly to transfer resources in $t = 1$.

2.3 Optimal tax mix

We derive the governments optimal tax structure using a standard mechanism design problem for income taxes augmented by a choice of commodity tax rates. The government offers bundles of income (Y^i, I^i) intended for types i . Then, taxes paid at the end of the period are residually given by $T^i = Y^i - I^i$, where T^i can be negative for the low-productivity type. The government also chooses t_c , or equivalently q_c , and the price ratio $\alpha = q_d/q_c$. When an individual is credit constrained in the optimum, α and q_c will generally differ from unity. In

the absence of a binding credit constraint, $\alpha = q_c = 1$, since the government relies entirely on income taxation. We solve the individual's problem in two steps in reverse order. In the second step, knowing Y^i, I^i, q_c and q_d , he chooses bundles (c_t^i, d_t^i) for $t = 1, 2$. In the first step and anticipating the outcomes of the second step, the individual chooses from the bundles of income and disposable income (Y^i, I^i) offered by the government.⁵

2.3.1 Step 2: Choice of commodity bundles

Given Y^i, I^i, q_c, α , individuals of the two types choose bundles (c_t^i, d_t^i) to maximize utility (1) subject to the annual budget constraint (3) and the credit constraint (2). The value function for this problem is:

$$\begin{aligned} \psi^i(Y^i, I^i, q_c, \alpha, \tau) = \max_{c_t^i, d_t^i} \sum_t u(c_t^i - \bar{c}, d_t^i) + \theta^i \left[\frac{I^i}{q_c} + (1 + r_i) \frac{\tau}{q_c} - \sum_t (1 + r_i)^{t-1} (c_t^i + \alpha d_t^i) \right] \\ - \mu^i \left[c_1^i + \alpha d_1^i - \frac{1}{q_c} \left(\frac{Y^i}{2} + \tau + \phi \right) \right], \end{aligned} \quad (7)$$

where $\phi \in \{0, \infty\}$, depending on the specific case that is under study.

The solution to the problem of a type- i individual is characterized by the following first-order conditions for consumption in the two sub-periods:

$$u_c(c_1^i, d_1^i) - \theta^i(1 + r_i) - \mu^i = 0, \quad u_d(c_1^i, d_1^i) - \theta^i(1 + r_i)\alpha - \mu^i\alpha \leq 0 \quad (8)$$

$$u_c(c_2^i, d_2^i) - \theta^i = 0, \quad u_d(c_2^i, d_2^i) - \theta^i\alpha \leq 0 \quad (9)$$

By the envelope theorem,

$$\psi_I^i = \frac{\theta^i}{q_c}, \quad \psi_Y^i = \frac{\mu^i}{2q_c}, \quad \psi_\alpha^i = -\theta^i[(1 + r_i)d_1^i + d_2^i] - \mu^i d_1^i. \quad (10)$$

⁵For a similar approach, see [Edwards et al. \(1994\)](#)

Note also that $d\psi^i/d\phi = \mu^i$. Since consumer utility is non-decreasing in the size of the credit constraint ϕ , that implies $\mu^i \geq 0$ with the inequality applying when the constraint is binding. Note also that, by definition, $\mu^i = 0 \forall i$ when $\phi \rightarrow \infty$.

2.3.2 Step 1: Choice of income and net income bundles

Given commodity tax rates (t_c, α) and anticipating step 2 above, the government on behalf of individuals of the two types offers consumption bundles (Y^i, I^i) . This yields total utility for a type- i person of:

$$V^i(Y^i, I^i, q_c, \alpha, \tau) = \psi^i(Y^i, I^i, q_c, \alpha, \tau) - h\left(\frac{Y^i}{w^i}\right). \quad (11)$$

Using the envelope results (10) on ψ^i , $V^i(\cdot)$ satisfies the following properties:

$$V_Y^i = \frac{\mu^i}{2q_c} - \frac{1}{w^i}h'\left(\frac{Y^i}{w^i}\right), \quad V_I^i = \psi_I^i, \quad V_\tau^i = \psi_\tau^i, \quad V_{q_c}^i = \psi_{q_c}^i, \quad V_\alpha^i = \psi_\alpha^i. \quad (12)$$

Preferences of an individual of type i in (Y, I) -space have a slope of:

$$\frac{dI^i}{dY^i} = -\frac{V_Y^i}{V_I^i} = \frac{q_c}{\theta^i} \left[\frac{1}{w^i}h'\left(\frac{Y^i}{w^i}\right) - \frac{\mu^i}{2q_c} \right]$$

Finally, denote \widehat{V}^i as the total indirect utility of a type i who mimics a type $-i$. The mimicker will have the same income stream so will face the same credit constraint as the individual being mimicked. Analogously to V^i in (11), indirect utility is given by:

$$\widehat{V}^i(q_c, \alpha, Y^{-i}, I^{-i}) = \psi^{-i} - h\left(\frac{Y^{-i}}{w^i}\right). \quad (13)$$

Similar envelope properties to (12) apply, and the slope of the mimicker's indifference curves

will be:

$$\frac{d\widehat{I}^i}{d\widehat{Y}^i} = -\frac{\widehat{V}_Y^i}{\widehat{V}_I^i} = \frac{q_c}{\theta^{-i}} \left[\frac{1}{w^i} h' \left(\frac{\widehat{Y}^i}{w^i} \right) - \frac{\mu^{-i}}{2q_c} \right]$$

2.3.3 Compensated demands

For further use, we present the expressions used for compensated demands. We compensate demands by varying I^i but taking Y^i as given. When an individual is unconstrained, they satisfy

$$\frac{\partial c_t^i}{\partial \alpha} = \frac{\partial \tilde{c}_t^i}{\partial \alpha} - q_c \frac{\partial c_t^i}{\partial I^i} \sum_t (1+r_i)^{2-t} d_t^i, \quad \frac{\partial d_t^i}{\partial \alpha} = \frac{\partial \tilde{d}_t^i}{\partial \alpha} - q_c \frac{\partial d_t^i}{\partial I^i} \sum_t (1+r_i)^{2-t} d_t^i.$$

If an individual is constrained, then in the first period $\partial c_1^i / \partial I^i = \partial d_1^i / \partial I^i = 0$. However, given time-separability we can make use of the fact that the first-period expenditure satisfies $q_c c_1^i + \alpha q_d d_1^i = Y^i / 2 + \phi$, evaluated locally at $\phi = 0$, to derive the compensated demands. At $t = 1$ they are,

$$\frac{\partial c_1^i}{\partial \alpha} = \frac{\partial \tilde{c}_1^i}{\partial \alpha} - q_c d_1^i \frac{\partial c_1^i}{\partial \phi}, \quad \frac{\partial d_1^i}{\partial \alpha} = \frac{\partial \tilde{d}_1^i}{\partial \alpha} - q_c d_1^i \frac{\partial d_1^i}{\partial \phi}.$$

Therefore, following a marginal change in the price ratio, keeping labor effort constant, income compensation can be achieved through a marginal increase in gross labor income or, equivalently, by allowing the constrained individual to borrow marginally more. In the second period,

$$\frac{\partial c_2^i}{\partial \alpha} = \frac{\partial \tilde{c}_2^i}{\partial \alpha} - q_c \frac{\partial c_2^i}{\partial I^i} d_2^i, \quad \frac{\partial d_2^i}{\partial \alpha} = \frac{\partial \tilde{d}_2^i}{\partial \alpha} - q_c \frac{\partial d_2^i}{\partial I^i} d_2^i.$$

2.4 Tax implementation

Let us find the formulas for tax implementation. The government implements a nonlinear tax function $T(Y^i)$. Let us denote indirect utility in the following way:

$$U(Y^i, I^i) = \psi^i(I^i) - h(Y^i/w^i).$$

Since this is implemented by making individual free choose his income Y^i , we rewrite it explicitly as

$$U^i = \psi^i \left(\left(\frac{2+r_i}{2} \right) Y^i - T(Y^i) \right) - h \left(\frac{Y^i}{w^i} \right).$$

Using the envelope conditions, this imply that the first-order condition for this individual is

$$\psi_I^i \frac{\partial I^i}{\partial Y^i} + \psi_Y^i - h_Y \left(\frac{Y^i}{w^i} \right) = \frac{\theta^i}{q_c} \cdot \left(\frac{2+r_i}{2} - T'(Y) \right) + \frac{1}{2} \frac{\mu^i}{q_c} - \frac{1}{w^i} h' \left(\frac{Y^i}{w^i} \right) = 0.$$

Isolating the marginal tax rate gives

$$T'(Y^i) = \frac{2+r_i}{2} - \frac{q_c}{\theta^i w^i} h' \left(\frac{Y^i}{w^i} \right) + \frac{1}{2} \frac{\mu^i}{\theta^i}.$$

2.5 Government's problem

In our problem, the government redistribute from more productive to less productive individuals. We use the methodology developed by [Hellwig \(2007\)](#) — also applied by [Bastani \(2015\)](#) — to derive optimal tax schedules with a finitely large number of types. The government maximizes social welfare:

$$W = \sum_i n^i \Phi(V^i)$$

subject to $N - 1$ incentive compatibility (IC) constraints that take the form of downward adjacent constraints,

$$V^i(Y^i, I^i, q_c, \alpha, \tau; w^i) \geq \widehat{V}^i(Y^{i-1}, I^{i-1}, q_c, \tau, \alpha; w^i), \forall i \quad (\gamma^i)$$

and to the budget constraint:

$$\sum_{i=1}^N n_i \left(\left(\frac{2+r_i}{2} \right) \frac{Y^i}{q_c} - \frac{I^i}{q_c} \right) + \left(1 - \frac{1}{q_c} \right) \sum_{i=1}^N n_i \sum_t (1+r_g)^{2-t} c_t^i + \left(\alpha - \frac{1}{q_c} \right) \sum_{i=1}^N n_i \sum_t (1+r_g)^{2-t} d_t^i - \frac{R}{q_c} = 0 \quad (\lambda)$$

where $\Phi(V^i)$ is a concave social utility function, with $\Phi'(V^i) > 0$ and $\Phi''(V^i) \leq 0$, and R is the exogenous revenue requirement of the government. The equation indicators γ^i and λ represent the Lagrangian multipliers of these constraints in the government's problem. Note that for R small enough, at least one type (the lowest) receives a transfer. The function $\widehat{V}^i(Y^{i-1}, I^{i-1}, q_c, \alpha, \tau; w^i)$ in the IC constraints is the indirect utility obtained by i if he mimics the adjacent lower type $i - 1$, so

$$\widehat{V}^i = \psi^{i-1}(\cdot) - h \left(\frac{Y^{i-1}}{w^i} \right).$$

Given our assumption about preferences, all individuals would perfectly smooth their consumption across sub-periods 1 and 2 in the absence of credit constraints. If credit is constrained, it can only be binding for those expecting a transfer at the end of the period since then they will want to borrow in period 1. Those who pay positive taxes will save at $t = 1$ to spread their tax liabilities across sub-periods.

We consider the government problem in three successive setting of increasing complexity. We begin with the benchmark case in which no one is credit-constrained. We then assume a credit constraint is binding on at least one type (the lowest), but restrict the government to

using a nonlinear income tax. Finally, we let the government choose differentiated commodity taxes or subsidies alongside the nonlinear income tax. We denote by \mathcal{L} the Lagrangian function of the government. For clarity, we explicitly retain all multipliers on the first-order conditions of the government's problem. The first-order conditions for the government's problem in the third, most general, setting where the credit constraint is binding for at least one type and the government chooses commodity tax rates are listed in the Appendix.

2.5.1 Benchmark: unconstrained individuals and no credit spread

This case corresponds to the standard optimal nonlinear income tax problem with linear commodity taxes. All individuals and the government can borrow at the unique interest rate r . First, we can establish whether the government will use commodity taxation at all. It can directly optimize on relative commodity prices $\alpha = q_d/q_c$ and sets $q_c = 1$ since we can normalize one tax to zero. Therefore, choosing α is equivalent to choosing $q_d = 1 + t_d$. Since individuals are not credit-constrained, $\mu^i = 0, \forall i$, in the individual's value function (7).

Using the envelope properties for the individuals in (10) and (12), the government's first-order conditions shown in the Appendix lead to the standard Atkinson-Stiglitz theorem:

Proposition 1. *When $i = 1, \dots, N$ are unconstrained and there is no credit spread, then the Atkinson-Stiglitz Theorem holds and commodity taxes are undifferentiated.*

Proof: See appendix.

The Atkinson-Stiglitz Theorem stipulates only that commodity taxes should be uniform if used, but redundant and thus unnecessary. Incentive-compatibility implies that, in this case, the optimal tax schedule, using (12), satisfies

$$T'(Y^i) = \left(\frac{2+r}{2}\right) + \frac{V_Y^i}{V_I^i} = \left(\frac{2+r}{2}\right) - \frac{q_c}{\theta^i w^i} h'\left(\frac{Y^i}{w^i}\right). \quad (14)$$

The first-order conditions give us the following marginal tax formulas:

$$T'(Y^i) = \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right). \quad (15)$$

As clarified by Hellwig (2007), the rightmost term in parentheses is always positive when the single-crossing condition is satisfied and when leisure is a normal good. Therefore, marginal tax rates are everywhere positive except for w^N , for which there is no distortion and $T'(w^N) = 0$.

We now turn to cases with either or both binding credit constraints or credit spreads.

2.5.2 Binding credit constraints; no commodity taxes

Suppose that transfers to the lowest type is sufficiently large that the credit constraint on at least one type is binding, so $\mu^i > 0$ for at least some i . Those whose credit constraint does not bind pay taxes at the end of the period and save for it, whereas the poorest ones would have liked to borrow using future transfers as collateral but they cannot. There are no commodity taxes in this specific case, so $q_c = \alpha = 1$. As a consequence, the government has no revenues and no expenses in the first sub-period, and r_g does not need to be specified. Those who save do it at rate $r_i = \underline{r}$. The first-order conditions of the government's problem are presented in the appendix.

First, notice that the first-order conditions of a constrained individual give the following wedge between disutility of labor and labor income:

$$T'(Y^i) = \left(\frac{2 + \underline{r}}{2} \right) - \frac{q_c}{\theta^i w^i} h' \left(\frac{Y^i}{w^i} \right) + \frac{1}{2} \frac{\mu^i}{\theta^i}.$$

Thus, because of the binding credit constraint, an individual will be more inclined to work more to generate income in the first-period. Not looking at the tax formula, which gives us

the optimal wedge, we get the standard expression

$$T'(Y^i) = \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right), \quad (16)$$

which will tend to give lower marginal tax rates for the credit constrained for two main reasons. First, binding credit constraints lower utility, and therefore reduces the incentive of higher types to mimick, which has an effect on γ^{i+1} . Also, the government has an interest in making constrained individuals work more, which is the only way to increase their (otherwise suboptimal) consumption in the first period.

2.5.3 Credit constraints not binding in the optimum; use of commodity taxes

We now discuss the case in which individuals face credit constraints, and where the government can simultaneously use commodity and income taxation. Here, we need to describe two types of optimal policy.

In the first one, which happens when $r_g - \underline{r}$ is small, the government can use commodity taxes to unconstrain all individuals. In this case, both goods are subsidized at the same rate. Since the first-period of funding subsidies on commodities is low enough, the government subsidize them enough, thereby increasing first-period purchasing power, so that everyone is finally unconstrained. In this case, the commodity tax (subsidy) system acts as a proportional subsidy on income. Since credit constraints are not binding, the equivalent of a second-best (although with borrowing interest rates higher than saving return rates) is recovered. To ensure that the whole tax system maximizes social welfare in an incentive-compatible way, income tax rates (both average and marginal) are adjusted, so that the effective marginal tax rate of individual i is identical to the one we would have obtained without credit constraints and without commodity subsidies.

Proposition 2: If in the optimum $\mu^i = 0, \forall i$, meaning that the policy can relax all credit

constraints in the economy, then $t_c^* = t_d^* < 0$.

Proof: See Appendix.

In this particular case, the marginal tax rate faced by individual i is

$$T'(Y^i) = \frac{\theta^i \gamma^{i+1} q_c}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right) - t^* \sum_t (1+r_g)^{2-t} (z_i - T'(Y^i)) \left(\frac{\partial c_t^i}{\partial I^i} + \frac{\partial d_t^i}{\partial I^i} \right), \quad (17)$$

where $z_i = (2 + r_i)/2$. The tax formula, analogous to that derived by (Edwards et al. (1994)), shows that when the government subsidizes consumption proportionally, this creates purchasing power that is identical to an increase in net income. Therefore, a share $(z - T'(Y^i))$ of the “income value” of the subsidy has to be left in the individual’s pockets, adjusted for the funding cost of the subsidies r_g . The tax formula reflects the fact that the wedge between labor and consumption must encompass the marginal incentives and disincentives generated by all tax instruments.

In particular, one can use (17) and rewrite it in terms of marginal effective tax rate:

$$T'(Y^i) + t^* \sum_t (1+r_g)^{2-t} (z_i - T'(Y^i)) \left(\frac{\partial c_t^i}{\partial I^i} + \frac{\partial d_t^i}{\partial I^i} \right) = \frac{\theta^i \gamma^{i+1} q_c}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right), \quad (18)$$

which shows that subsidies on consumption goods must be clawed back by increases in marginal income tax rates. It also shows that the standard properties of the optimal tax systems are not violated, including positive marginal tax rates at all income levels and a zero effective marginal tax rate at the top.

Finally, one should note that the most extreme case of such an optimum would be when subsidies can be funded at no opportunity cost for the government, or when $r_g = \underline{r}$. Then, transferring purchasing power from the second to the first period is done at no cost for the government, who can always lower the prices of commodities so as to slack all credit constraints in the economy. In this unrealistic example, the timing of income-tested payments

has no effective consequence on the overall tax policy and social optimum. As soon as subsidies become costly, for instance when they have to be (at least partially) funded with debt, there is a threshold level of the cost from which the government will leave some individuals credit constrained.

2.5.4 Credit constraints still binding in the optimum; use of commodity taxes

When the cost of funding commodity subsidies in the first period are high enough, the government may decide to not slack some individual's credit constraints. When this happens, it may be optimal to either differentiate the subsidies, or to fund one subsidy with a positive tax on another good. The intuition is the following. Commodity taxation generates both income and substitution effects. When there were no binding constraints, the nonlinear tax system could adjust to offset income effects, and it ensued that it was optimal to not create substitution effects.

With binding credit constraints, for those who are constrained in the optimum some income effects cannot be clawed back by proportional adjustments in the income tax schedule. These income effects are costly for the government because they induce an increase in consumption in the first period which is subsidized and must be financed through short-term debt. On the other hand, subsidizing at least one commodity is beneficial because it helps reducing the pressure of credit constraints. Thus, the government needs to compromise and “spend” its resources on the good that is proportionately more consumed by the constrained.

As it turns out, this may happen even when utility functions u feature linear Engel curves. Our simple case in which there is a basic need \bar{c} on one good justifies either subsidizing good c at a higher rate, or simply subsidizing c and taxing d if $r_g - \underline{r}$ is very high. These results are illustrated in the numerical section, and proposition 3 gives the general condition under which differentiation happens under linear Engel curves.

Proposition 3: Assuming linear Engel curves, when the optimal tax system involves $\mu^i > 0$ for at least some i , differentiation happens when, in the optimum,

$$\left(\frac{q_d^*}{q_c^*}\right) \equiv \alpha^* = \frac{E \left[c_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) \middle| \mu^i > 0 \right]}{E \left[d_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) \middle| \mu^i > 0 \right]} \bigg/ \frac{\sum_i \pi^i c_1^i}{\sum_i \pi^i d_1^i} \quad (19)$$

or $t_c^* < t_d^*$ when

$$\frac{E \left[c_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) \middle| \mu^i > 0 \right]}{E \left[d_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) \middle| \mu^i > 0 \right]} > \frac{\sum_i \pi^i c_1^i}{\sum_i \pi^i d_1^i}, \quad (20)$$

where $\pi^i = (\mu^i/\lambda^i)[n_i\Phi'(V^i) + \gamma^i - \gamma^{i+1}] - n^i(r_g - \underline{r})$ is a weight that is strictly negative for unconstrained individuals and which is increasing with first-period total expense.

We interpret the first term in the right-hand side of (19) as the cost ratio of remaining income effects through good c over those of good d , for all constrained individuals. We interpret the second term in the right-hand side of (19) as a ratio of net benefits of slacking constraints through a reduction in taxes (or an increase in subsidies) on good c versus on good d . When the whole term in the right (a cost-benefit ratio of differentiating t_c downwards) hand side of (19) is greater than one, we have either $t_c^* < t_d^* < 0$ or $t_c^* < 0 < t_d^*$, depending on $(r_g - \underline{r})$ and on the various parameters, giving (20).

Here again, the government must adjust the marginal and average income tax rates to include the effects of indirect taxes. The marginal income tax rates are

$$\begin{aligned} T'(Y^i) &= \frac{\theta^i \gamma^{i+1} q_c}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right) \\ &- t_c^* \left[\sum_t (1+r_g)^{2-t} \left(\frac{\partial c_t^i}{\partial I^i} (z_i - T'(Y^i)) + \frac{\partial c_t^i}{\partial Y^i} \right) \right] - t_d^* \left[\sum_t (1+r_g)^{2-t} \left(\frac{\partial d_t^i}{\partial I^i} (z_i - T'(Y^i)) + \frac{\partial d_t^i}{\partial Y^i} \right) \right], \end{aligned} \quad (21)$$

where $z_i = (2 + r_i)/2$. In this particular case, unconstrained individuals' consumption of commodities does not react to a change in Y^i since the government may adjust the income

tax schedule following a change in labor effort. For individuals who are still constrained, first-period consumptions do not react to I^i , and second period consumptions do not react to Y^i .

Example of differentiation with Cobb-Douglas and basic need

Let us give an example with a per-period utility function $u(c, d) = (c - \bar{c})^\alpha d^{1-\alpha}$. Start with the case where $\bar{c} = 0$. Using the well-known marshallian demands associated with these preferences, one can verify that there is no differentiation since $q_d^*/q_c^* = \alpha(1-\alpha)\alpha^{-1}(1-\alpha)^{-1}$.

With the basic need, denoting $R^i = Y^i/2$ for the constrained and $R^i = I^i/2$ for the unconstrained, $t_c^* < t_d^* < 0$ if

$$\frac{\alpha \sum_i \pi^i [R^i/q_c + (1-\alpha)\bar{c}]}{1-\alpha \sum_i \pi^i [R^i/q_d - \bar{c}q_c/q_d]} > \frac{\alpha \sum_i n_i z_i q_c \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) [\pi^i R^i/q_c + (1-\alpha)\bar{c}]}{1-\alpha \sum_i n_i z_i q_d \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) [R^i/q_d - \bar{c}q_c/q_d]}$$

Calculating functional forms for the derivatives of demands with respect to income, evaluated at $q_c = q_d$, it is welfare-improving to differentiate q_c when

$$\frac{\sum_i \pi^i [R^i + (1-\alpha)q_c \bar{c}]}{\sum_i \pi^i [R^i - q_c \bar{c}]} > \frac{\sum_i n_i z_i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) [R^i + (1-\alpha)q_c \bar{c}]}{\sum_i n_i z_i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) [R^i - \bar{c}q_c]}.$$

Defining N^C the number of constrained, and $E(R)^C$ their average first-period income, this is only verified if

$$\sum_i \pi^i [R^i - E[R^C]] < 0.$$

This result depends on the parameters of the model, but for having no differentiation, we should have to observe no correlation between the weights π^i and first-period expense, which

is not a feature of the second-best tax system.

3 Numerical examples

To illustrate our results we undertake a series of numerical simulations. We use the following quasi-homothetic per period consumption utility function

$$u(c_t^i, d_t^i) = \kappa \frac{(c_t^i - \bar{c})^{1-\rho}}{1-\rho} + \kappa \frac{d_t^{i1-\rho}}{1-\rho},$$

in addition to the following labor disutility function

$$h(l) = \frac{l^{1+\sigma}}{1+\sigma}.$$

We set $\rho = 0.9$ and $\sigma = 2$. The basic level of consumption of good, \bar{c} , is set to 5 which roughly represents a \$5.75 a day spending or \$2100 annually in our model.⁶ The constant κ is set to $(1/4)^\rho$.⁷ For the number of workers at each wage level i (type) n_i , we use a lognormal distribution with parameters $(\mu, \sigma) = (2.757, 0.5611)$ taken from Mankiw et al. (2009). The authors estimate these parameters from the 2007 March wave of the Current Population Survey (CPS). We then discretize the distribution to obtain 100 wage levels with the distance between each wage being fixed. The probability mass function is rescaled so that $\sum_i n_i = 1$. To create an interest rate spread large enough to highlight our main results, we use $\underline{r} = 0$ and $r_g = 0.15$.

⁶This level of minimum consumption is fairly conservative. The average Supplemental Nutrition Assistance Program (SNAP) benefit is about \$4 a day and if we consider c_t^i to include other basic necessities in addition of food, \$5.75 is relatively low. In addition the 2007 Federal poverty line is set at \$10,210 (\$28 a day) for a single person household, and thus our level of \bar{c} is roughly 20% of that threshold.

⁷This is chosen so the solution of the optimal tax problem would give the same utility level and optimal tax function as in the case where $\bar{c} = 0$ with $\underline{r} = 0$ and the yearly utility of an individual would be

$$u(c, l) = \frac{c^{1-\rho}}{1-\rho} - \frac{l^{1+\sigma}}{1+\sigma}$$

In our simulations, due to the introduction of commodity taxes and subsidies, we also calculate the effective tax burden paid at a given level of income and the effective marginal tax rates as in [Edwards et al. \(1994\)](#). The total tax burden of a worker at labor income Y^i is

$$T^E(Y^i) = T(Y^i) + t_c \left[\sum_t (1 + r_i)^{2-t} c_t^i \right] + t_d \left[\sum_t (1 + r_i)^{2-t} d_t^i \right]$$

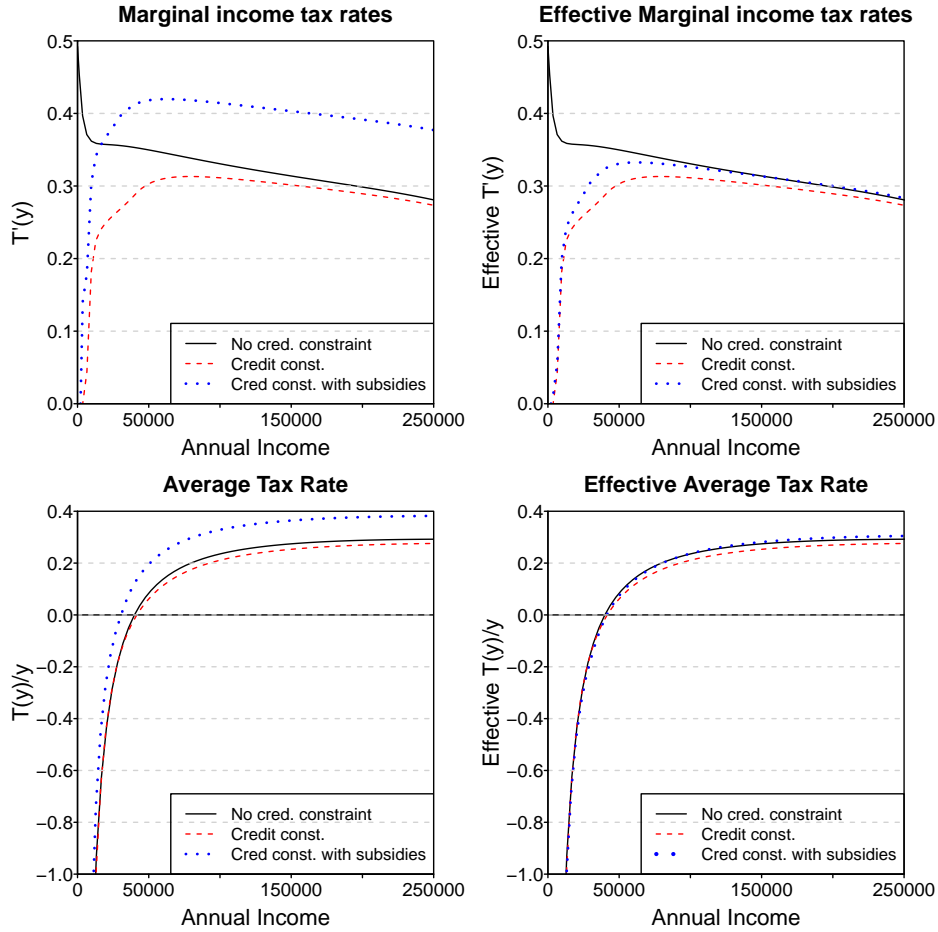
and the marginal effective tax rate at Y^i is

$$T'^E(Y^i) = T' + t_c \left[\sum_t (1 + r_i)^{2-t} \left(\frac{\partial c_t^i}{\partial I^i} (z_i - T') + \frac{\partial c_t^i}{\partial Y^i} \right) \right] + t_d \left[\sum_t (1 + r_i)^{2-t} \left(\frac{\partial d_t^i}{\partial I^i} (z_i - T') + \frac{\partial d_t^i}{\partial Y^i} \right) \right],$$

where $T' \equiv T'(Y^i)$ and $z_i \equiv (2 + r_i)/2$ for notational convenience.

To start, we consider three scenarios. The first scenario is the one called *No credit constraint*, this scenario encompasses many other possible cases. The first one is the case where there are no credit constraints and the individuals can both save and borrow at the same rate. The no credit constraint scenario will also give the same allocation and same effective marginal tax rates as the case where the planner can costlessly transfer money in the first period through subsidies or any other scheme. The second scenario that we consider is the *Credit constraint* where the individual's borrowing limit, i.e. ϕ , is set to 0. The planner in this case only has access to nonlinear taxation on labor income to achieve his goals. Due to the fact that we have set $\underline{r} = 0$, individuals are constrained whenever $T(Y^i) < 0$. The third scenario, *Credit constraint with subsidies* is similar to the case where we impose a credit constraint but now the planner is able to use taxation on commodities in addition to nonlinear labor income taxation to achieve his goals. Under all three scenarios the planner

Figure 1: Optimal Labor Income Tax Rates under Different Scenarios



has utilitarian social preferences, i.e. the social welfare function is

$$W = \sum_i n^i V^i.$$

The characteristics of the optimal solutions to all three cases are shown in Figure 1 and Table 1. From Figure 1, it is possible to see that the scenario with the credit constraint and subsidies features the highest marginal tax rates except at the very bottom of the income distribution and the highest average tax rates. As argued above, the optimal solution to this scenario where the government has access to commodity taxation will feature subsidies to

Table 1: Characteristics of Optimal Allocation under Different Scenarios

Scenario	q_c	q_d	q_d/q_c	% Constrained	$\sum_i n^i V^i$	$\% \Delta^{LF}$
No Credit Constraint	1	1	1	0%	16.399	10.22%
Credit Constraint	1	1	1	62.23%	16.346	7.01%
Cred. Const. and Subsidies	0.858	0.881	1.028	42.96%	16.35	7.28%

Source: Authors' calculations. Numbers rounded.

consumption goods. To finance these subsidies the planner must raise revenues through the use of labor income taxes.

Looking at the optimal effective marginal tax rates it is possible to see that the credit constraint and subsidies scenario is in fact an in-between case of the no credit constraint case and the credit constraint case with only nonlinear labor income taxation available to the planner. The no credit constraint scenario features the highest effective marginal tax rates because it is able to transfer money to the individual either through its own savings or other schemes, this implies that the planner is able to distort labor more at the bottom ensuring a higher level of redistribution. Whereas the credit constraint case without commodity taxes will feature the lowest effective marginal tax rates. In this scenario, due to the basic need \bar{c} , low skill workers must supply much more labor to meet their needs and since the planner is unable to transfer resources to them in the first period, the only thing he can do is to not discourage work all together. In addition, because of the credit constraint and the inability of low skilled workers to smooth consumption, transferring utility to them through transfers in the second period is more difficult and thus lowers the value of redistributing to them. This reduces the need for the planner to tax workers to raise revenue for redistribution.

The scenario with credit constraint and commodity taxation permits the planner to transfer some resources, albeit at a cost, to the first period through subsidies on goods. The same pattern from the credit constraint case emerges at the bottom of the income distribution since the planner does not want to discourage work. But now labor income is

slightly more valuable since the subsidies on goods increase the purchasing power of workers and thus allows for more redistribution and lower number of constrained workers.

Table 1 shows that, through subsidies, less workers are constrained in the case where commodity taxation is available to the planner. In the same Table, the utilitarian social welfare of all three cases is reported. Using the social welfare numbers we see that the case with credit constraint and commodity taxes is preferred to the case with the credit constraint but without the additional taxation tool. To give another measure of welfare to compare scenarios, we measure welfare gains of the optimal allocations from the *laissez-faire* allocation. This gain is calculated by the constant percentage increase in consumption ($\% \Delta^{LF}$) offered to both goods in both periods to all types to the *laissez-faire* economy to reach the same level of utilitarian social welfare in each scenario.⁸ Again, we are able to see that the case with commodity taxes is preferred to the case without. The small differences are most likely due to the utilitarian social welfare preference and the limited curvature of the consumption utility function which puts less weight on the lower skilled workers. Therefore any social welfare that would put more weight on the lower skilled workers would also put more value on policies that reduces the effect of the credit constraint on the workers.

We finish this section by considering two sets of simulations that explore changes in the cost of providing subsidies and changes to the level of basic needs. The first set of simulations that we consider changes the spread of interest rates, $r_g - \underline{r}$. To focus on the changes in the costs and not on increases in the amount of resources available to the economy due to higher interest rates on savings, we only change the level of r_g . The results of these simulations are shown in Figure 2 and Figure 3. Figure 2 presents the same information in three different manners. The main point is that the cheaper it is for the government to borrow the more it subsidizes good and the less it differentiate commodity taxes. In the case where costs are high enough, it is possible to obtain a tax on good d instead of a subsidy. The goal is for the planner to give every costly resource available to subsidize the good with a basic level

⁸This is the same as expected lifetime utility of a worker from behind the veil of ignorance.

Figure 2: Prices and Optimal Commodity Taxes under Different Interest Rate Spreads

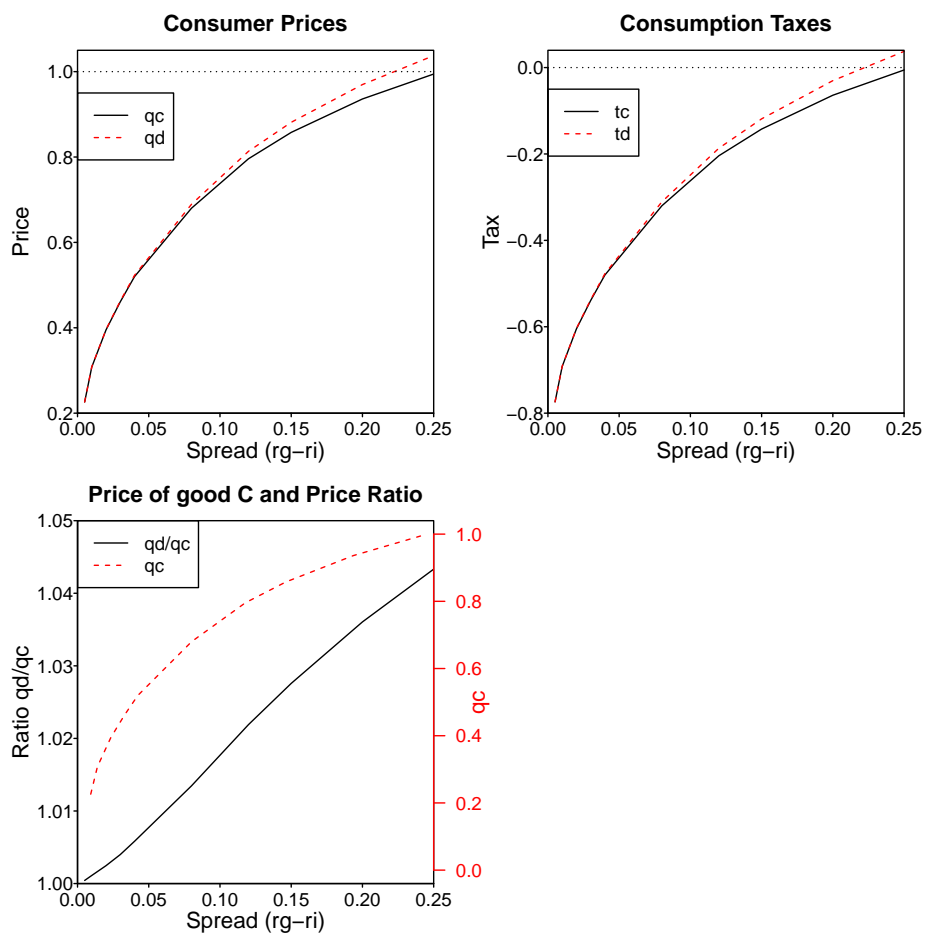
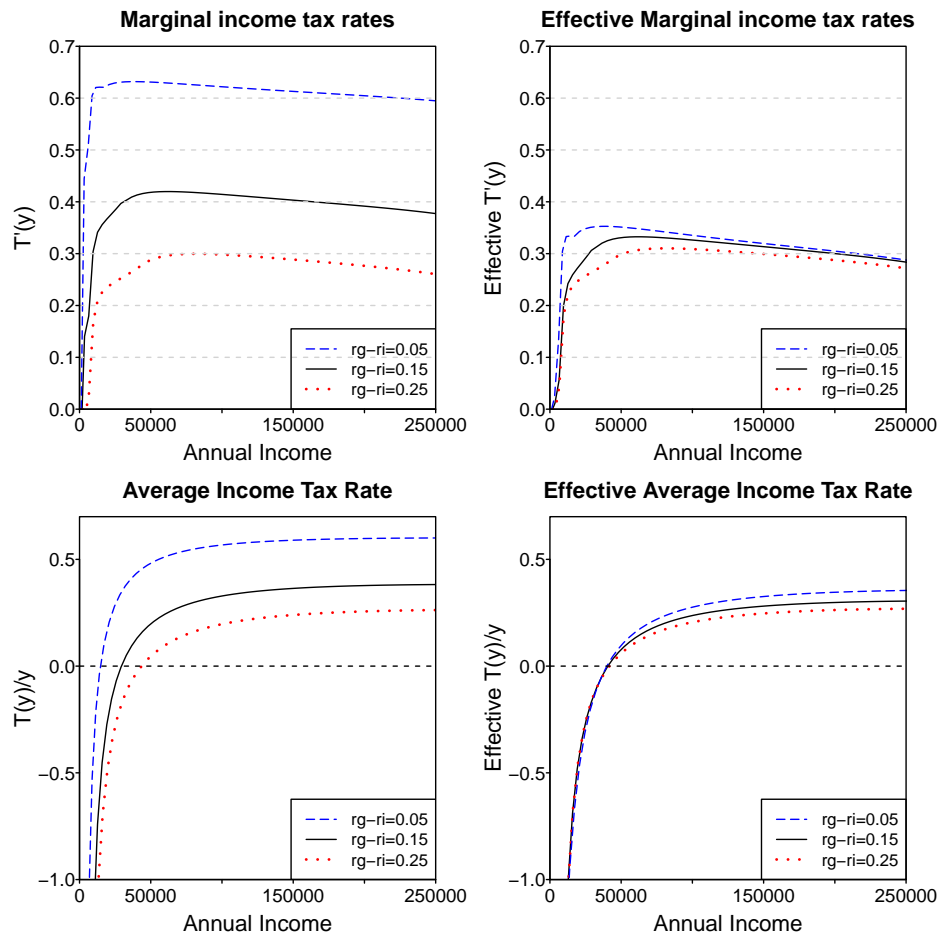


Figure 3: Optimal Labor Income Tax Rates under Different Interest Rate Spreads



of consumption that must be met.

Figure 3 shows the effect of different costs on the optimal labor income tax schedule. Whenever costs to subsidize are low, the optimal tax systems features large subsidies and requires higher labor income taxation as can be seen by both the marginal tax rates and average tax rates. As above, when the planner is able to transfer more money in the first period, he is also able to distort the labor decision of lower skilled workers a bit more and attempt more redistribution. This can be seen by looking at the effective marginal tax rates. The case with the highest spread, and thus the highest costs, also features the lowest effective marginal income tax rates since the planner must encourage work especially at the bottom of the distribution.

Simulations found in Figure 4 and Figure 5 consider changes in the level of basic need \bar{c} . From Figure 4, we see that the higher the level of basic need the more the planner will wish to subsidize consumption but also the more he will want to differentiate the commodity taxes. We can see that with $\bar{c} = 0$ the planner will still wish to subsidize consumption. This is due to the credit constraint and the cost of transferring resources to the first period. The greater the basic need the harder and more costly it is in terms of utility to achieve this level of consumption through work effort, hence the government wants to use as much resources as it can to help low wage workers by increasing the value of their labor earnings.

Figure 5 illustrates the effect on labor income tax rates of an increase in the basic need. To finance the increase in subsidies the planner requires higher levels of labor income taxation. This is evident from the plots showing marginal income tax rates and average tax rates. The higher level of basic need also appears to increase the level of redistribution which is indicated by higher effective marginal tax rates at the bottom of the income distribution but also for all workers. Average effective tax rates are also higher for higher levels of basic need. A bigger basic need leads to higher marginal utility of consumption of the necessity good for a given level of consumption, since it reduces the distance between that level of consumption and the basic need. This would increase the willingness of a utilitarian planner, who seeks

Figure 4: Prices and Optimal Commodity Taxes under Different levels of Basic Need

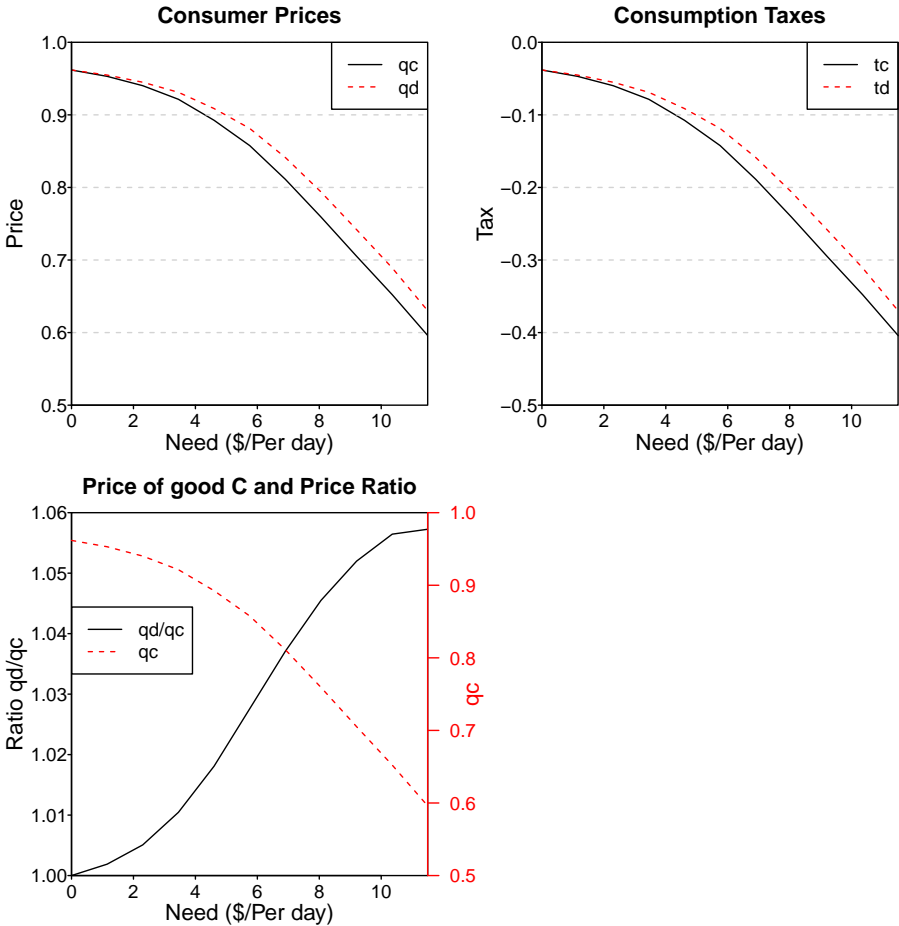
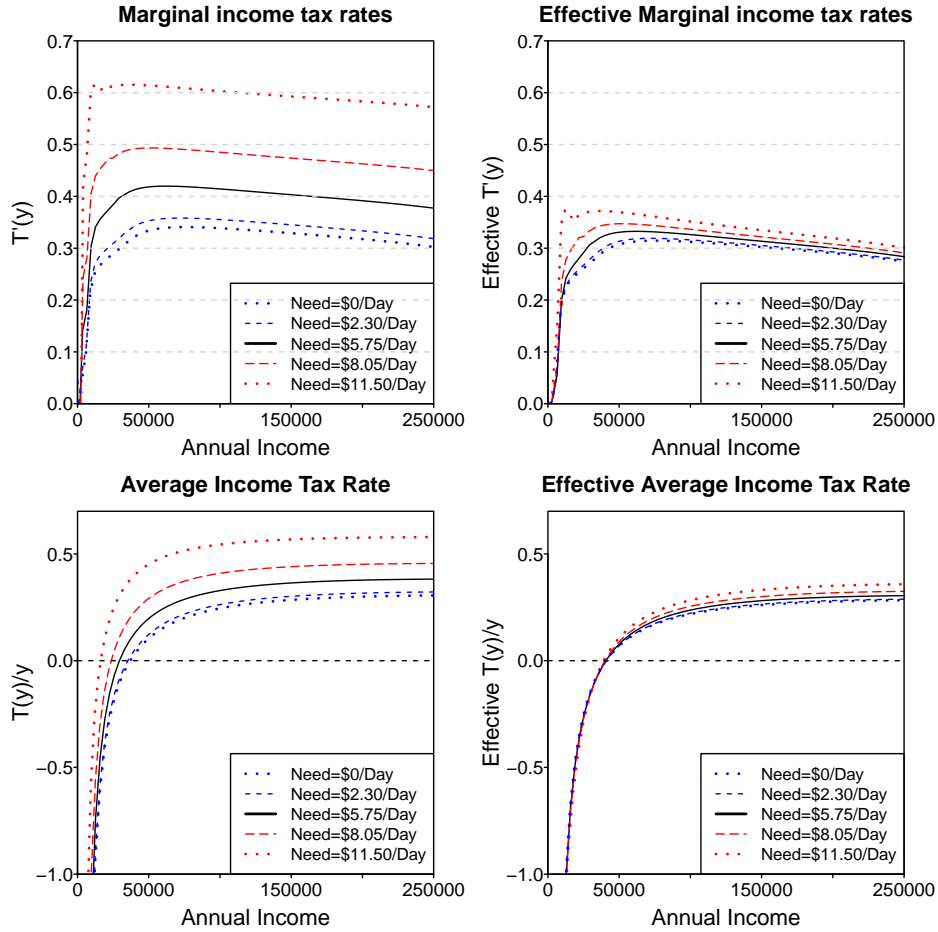


Figure 5: Optimal Labor Income Tax Rates under Different levels of Basic Need



to equalize levels of marginal utility across types, to attempt more redistribution.

4 Implications of introducing a lump-sum transfer in the first sub-period

As mentioned in the introduction, the possibility that the planner could use universal lump-sum payments to address directly the issue tackled in this paper would require those pay-

ments to be made at a similar frequency as transactions on consumption goods. If we suppose that this was a possibility, then the introduction of lump-sum payments opens the door to interesting interactions between commodity taxation and these payments.

If the lump-sum transfers could be made costlessly, e.g. whenever $\bar{r} = \underline{r}$, the planner will wish to front-load redistribution in the first period by offering a universal lump-sum payment to all individuals and then adjusting the end of period labor income tax schedule to ensure that no one will wish to borrow and be credit constrained. If this scheme is available to the planner, then the allocation would be identical to the one without credit constraints. There would be no need to use commodity subsidies and the Atkinson-Stiglitz Theorem would hold.

In the case where $r_g > \underline{r}$, the planner faces a cost to transfer money from the second sub-period to the first sub-period. In this scenario, the planner is no longer able to front-load completely redistribution in the first sub-period and adjust his labor income tax schedule to leave all workers unconstrained. This situation is similar to the one where the planner can only use commodity taxes and subsidies which offers only a partial relief to the credit constraint problem. This leaves room for the combination of both commodity taxes and lump-sum transfer to increase welfare.

When the planner has access to both lump-sum payments and linear taxes on goods, under the assumption that individual preferences give linear Engel curves, the planner will wish to create its own intra-period linear-progressive tax system. Since transferring resources across periods is costly, the planner will set high uniform commodity taxes on both goods and use the revenue to finance a large lump-sum transfer in the first sub-period. In this situation, the planner neither saves nor borrows, budgets are balanced in each sub-period. The planner decides to reduce inequality intra-period instead of trying to smooth consumption across sub-periods by decreasing the number of credit constrained workers. In fact, the optimal allocation appears to push towards more credit constrained workers. This is due to the high level of commodity taxes affecting both periods which leads the planner to adjust the labor income tax schedule to make the allocation incentive compatible. To do this, the tax burden

of workers is drastically reduced to the point of a majority of workers facing net-transfers from the labor income tax. This leads many workers to want to borrow but are unable to do so due to the credit constraint.

The uniform commodity taxes, with credit constrained workers, act as a linear tax on labor income in the first sub-period with the revenues going to finance the large lump-sum payment. We then recover the result of Deaton (1979) that demonstrates that an optimal linear progressive tax system will not use differentiated commodity taxation when Engel curves are linear. We conjecture that the uniformity result obtained will break down if Engel curves are nonlinear. Furthermore, even in the case if linear Engel curves, we conjecture that divorcing the timing of the collection of commodity tax revenues from the payment of the lump-sum transfer will also lead to differentiated commodity taxes. In fact, we should be able to recover many of the results found in this paper. Proving these conjectures is left to further research.

5 Conclusion

This paper studies an optimal tax system when transactions on goods happen more frequently than the payment of income-tested transfers. The credit constraints arise because individuals cannot fully use future transfers as collateral. Our results show that when the optimal policy is able to unconstrain all individuals, it involves proportional subsidies on goods. When the cost of doing so is too high, differentiation may happen when constrained individuals spend a higher proportion of their disposable income on a good (for instance a necessity) than the general population. Then, the government can either subsidize all goods, with a higher subsidy on commodities, or tax some goods to fund the subsidies on commodities.

References

- Aguila, E., Kapteyn, A., & Perez-Arse, F. (2017). Consumption smoothing and frequency of benefit payment of cash transfer programs. *American Economic Review Papers and Proceedings*, 107, 430–435.
- Atkinson, A. B. & Stiglitz, J. (1976). The design of tax structure: direct vs. indirect taxation. *Journal of Public Economics*, 6, 55–75.
- Baker, S. (2017). Debt and the consumption response to household income shocks. *Journal of Political Economy*, Forthcoming.
- Baker, S. & Yannelis, C. (2015). Income changes and consumption: Evidence from the 2013 federal government shutdown. *Review of Economic Dynamics*.
- Bastani, S. (2015). Using the discrete choice model to derive optimal income tax rates. *FinanzArchiv*, 71, 106–117.
- Christiansen, V. (1984). Which commodity tax should supplement the income tax? *Journal of Public Economics*, 24, 185–220.
- Deaton, A. (1991). Saving and liquidity constraints. *Econometrica*.
- Edwards, J., Keen, M., & and, M. T. (1994). Income tax, commodity taxes and public good provision: A brief guide. *Finanzarchiv*, 51, 472–497.
- Gruber, J. (1997). The consumption smoothing benefits of unemployment insurance. *American Economic Review*, 87, 192–205.
- Guesnerie, R. (1995). *A contribution to the pure theory of taxation*. Cambridge University Press.
- Hellwig, M. (2007). A contribution to the theory of optimal utilitarian income taxation. *Journal of Public Economics*, 91, 1449–1477.

- McGranahan, L. & Schanzenbach, W. (2013). The earned income tax credit and food consumption patterns. *Federal Reserve Bank of Chicago Working Papers, WP 2013-14*.
- Nichols, A. & Rothstein, J. (2016). The earned income tax credit (eitc). In R. A. Moffitt (Ed.), *Economics of Means-Tested Transfer Programs in the United States, Volume I* chapter 2. University of Chicago Press.
- Parker, J. A. (1999). The reaction of household consumption to predictable changes in social security taxes. *American Economic Review, 89*, 959–973.
- Shapiro, J. M. (2005). Is there a daily discount rate? evidence from the food stamp nutrition cycle. *Journal of Public Economics, 89*, 303325.

A First-order conditions of the general problem

The Lagrangian of the government is

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^N n_i \Phi(V^i) + \sum_{i=2}^N \gamma^i [V^i(Y^i, I^i, q_c, \alpha; w^i) - \widehat{V}^i(Y^{i-1}, I^{i-1}, q_c, \alpha; w^i)] \\ & + \lambda \left[\sum_{i=1}^N n_i \left(\left(\frac{2+r}{2} \right) \frac{Y^i}{q_c} - \frac{I^i}{q_c} \right) + \left(1 - \frac{1}{q_c} \right) \sum_{i=1}^N n_i \sum_t (1+r_g)^{2-t} c_t^i + \left(\alpha - \frac{1}{q_c} \right) \sum_{i=1}^N n_i \sum_t (1+r_g)^{2-t} d_t^i \right]. \end{aligned}$$

We present the first-order conditions in their most general form to keep the notation as compact as possible. Note that, by the definition of the problem, $\gamma^1 \equiv 0$ since the lowest type cannot mimic any lower adjacent type. Note, also, that all types that are not constrained in the optimum have $\mu^i = 0$. Those who are constrained have $\partial c_1^i / \partial I^i = \partial d_1^i / \partial I^i = 0$ and those who are not constrained have $\partial c_1^i / \partial Y^i = \partial d_1^i / \partial Y^i = 0$. The first-order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial I^i} = & n_i \Phi'(V^i) \frac{\theta^i}{q_c} + \gamma^i \frac{\theta^i}{q_c} - \gamma^{i+1} \frac{\theta^i}{q_c} - \lambda \frac{n_i}{q_c} + \lambda n_i \left(1 - \frac{1}{q_c} \right) \sum_t \frac{\partial c_t^i}{\partial I^i} (1+r_g)^{2-t} \\ & + \lambda n_i \left(\alpha - \frac{1}{q_c} \right) \sum_t \frac{\partial d_t^i}{\partial I^i} (1+r_g)^{2-t} = 0, \forall i; \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial Y^i} = & n_i \Phi'(V^i) \left(\frac{\mu^i}{2q_c} - \frac{1}{w^i} h' \left(\frac{Y^i}{w^i} \right) \right) + \gamma^i \left(\frac{\mu^i}{2q_c} - \frac{1}{w^i} h' \left(\frac{Y^i}{w^i} \right) \right) - \gamma^{i+1} \left(\frac{\mu^i}{2q_c} - \frac{1}{w^{i+1}} h' \left(\frac{Y^i}{w^{i+1}} \right) \right) \\ & + \lambda \left(\frac{2+r}{2} \right) \frac{n_i}{q_c} + \lambda n_i \left(1 - \frac{1}{q_c} \right) \sum_t \frac{\partial c_t^i}{\partial Y^i} (1+r_g)^{2-t} + \lambda n_i \left(\alpha - \frac{1}{q_c} \right) \sum_t \frac{\partial d_t^i}{\partial Y^i} (1+r_g)^{2-t} = 0, \forall i, \end{aligned} \quad (23)$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \alpha} &= \sum_i n_i \Phi'(V^i) \left(-\theta_i \sum_t (1+r_i)^{2-t} d_t^i - \mu^i d_1^i \right) + \sum_{i=1}^{N-1} (\gamma^i - \gamma^{i+1}) \left(-\theta^i \sum_t (1+r_i)^{2-t} d_t^i - \mu^i d_1^i \right). \\
&+ \lambda \left(1 - \frac{1}{q_c} \right) \sum_i n_i \sum_t \frac{\partial c_t^i}{\partial \alpha} (1+r_g)^{2-t} + \lambda \left(\alpha - \frac{1}{q_c} \right) \sum_i n_i \sum_t \frac{\partial d_t^i}{\partial \alpha} (1+r_g)^{2-t} + \lambda \sum_i n_i \sum_t d_t^i (1+r_g)^{2-t} = 0.
\end{aligned} \tag{24}$$

B Proofs

Proposition 1: When $i = 1, \dots, N$ are unconstrained and there is no credit spread, then the Atkinson-Stiglitz Theorem holds and commodity taxes are undifferentiated.

Proof: Take the first-order condition with respect to I^i in (22), multiply it by $q_c \sum_t (1+r)^{2-t} d_t^i$, the set $q_c = 1$ and obtain

$$\begin{aligned}
&n_i \Phi'(V^i) \theta^i \sum_t (1+r)^{2-t} d_t^i + (\gamma^i - \gamma^{i+1}) \theta^i \sum_t (1+r)^{2-t} d_t^i - \lambda n_i \sum_t (1+r)^{2-t} d_t^i \\
&+ \lambda n_i (\alpha - 1) q_c \sum_t \frac{\partial d_t^i}{\partial I^i} (1+r)^{2-t} \sum_t (1+r)^{2-t} d_t^i = 0, \forall i;
\end{aligned}$$

Reorganizing the last term, one gets

$$\begin{aligned}
&n_i \Phi'(V^i) \theta^i \sum_t (1+r)^{2-t} d_t^i + (\gamma^i - \gamma^{i+1}) \theta^i \sum_t (1+r)^{2-t} d_t^i - \lambda n_i \sum_t (1+r)^{2-t} d_t^i \\
&+ \lambda n_i (\alpha - 1) \sum_t \left(q_c \frac{\partial d_t^i}{\partial I^i} \sum_t (1+r)^{2-t} d_t^i \right) (1+r)^{2-t} = 0, \forall i.
\end{aligned}$$

Substituting the compensated demands into this equation and summing over all i gives

$$n_i \sum_t \Phi'(V^i) \theta^i \sum_t (1+r)^{2-t} d_t^i + \sum_i (\gamma^i - \gamma^{i+1}) \theta^i \sum_t (1+r)^{2-t} d_t^i - \lambda \sum_i n_i \sum_t (1+r)^{2-t} d_t^i \\ + \lambda \sum_i n_i (\alpha - 1) \sum_t \left(\frac{\partial \tilde{d}_t^i}{\partial \alpha} - \frac{\partial d_t^i}{\partial \alpha} \right) (1+r)^{2-t} = 0,$$

which, after substituting for (24) gives us

$$\lambda \sum_i n_i (\alpha - 1) \sum_t \frac{\partial \tilde{d}_t^i}{\partial \alpha} (1+r)^{2-t} = 0.$$

which requires that $\alpha^* = 0$. ■

Proposition 2: If in the optimum $\mu^i = 0, \forall i$, meaning that the policy can relax all credit constraints in the economy, then $t_c^* = t_d^* < 0$.

Proof: When nonlinear income taxes are optimal, the optimal commodity tax system is characterized by

$$t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1+r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1+r_g)^{2-t} - \sum_i \pi^i c_1^i = 0.$$

$$t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1+r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1+r_g)^{2-t} - \sum_i \pi^i d_1^i = 0.$$

Restating the system in matrices gives

$$\begin{pmatrix} \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1+r_g)^{2-t} & \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1+r_g)^{2-t} \\ \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1+r_g)^{2-t} & \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1+r_g)^{2-t} \end{pmatrix} \begin{pmatrix} t_c \\ t_d \end{pmatrix} = \begin{pmatrix} \sum_i \pi^i c_1^i \\ \sum_i \pi^i d_1^i \end{pmatrix}$$

where the leftmost matrix is a linear combination of per-period Slutsky matrices. With time-separable utility, this matrix is necessarily negative semi-definite. We denote it by S

and its determinant by $|S| < 0$. Then, by Cramer's rule

$$t_c = |S|^{-1} \left(- \sum_i \pi^i c_1^i \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1 + r_g)^{2-t} + \sum_i \pi^i d_1^i \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1 + r_g)^{2-t} \right)$$

and

$$t_d = |S|^{-1} \left(- \sum_i \pi^i d_1^i \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1 + r_g)^{2-t} + \sum_i \pi^i c_1^i \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1 + r_g)^{2-t} \right).$$

We can first demonstrate that if, in the optimum, $\mu^i = 0, \forall i$, then we must have two subsidies such that $q_c = q_d < 1$. Notice that if $\mu^i = 0$, $sign(\pi^i) = sign(\underline{r} - r_g)$. To get a contradiction, suppose that both taxes are positive, then the government raises revenue and $r_g = \underline{r}$, so $\pi^i = 0, \forall i$ and there are no commodity taxes.

Therefore, at least one tax has to be negative, and government's revenue in the first-period has to be negative. Then, $r_g > \underline{r}$, and $\pi^i = -n_i(r_g - \underline{r}) < 0, \forall i$. Then, we are interested in the sign of $\tau_c - \tau_d \equiv t_c/q_c - t_d/q_d$, or in the difference between $t_c q_d$ and $t_d q_c$. Using the homogeneity of the Slutsky matrix, we want to know if there is differentiation, which happens when

$$\begin{aligned} & \sum_i (r_g - \underline{r}) c_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_d} (r_g - \underline{r}) - \sum_i (r_g - \underline{r}) d_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_c} (r_g - \underline{r}) \\ & \neq \\ & \sum_i (r_g - \underline{r}) d_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_t^i}{\partial q_c} (r_g - \underline{r}) - \sum_i (r_g - \underline{r}) c_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_1^i}{\partial q_d} (r_g - \underline{r}) \end{aligned}$$

or, cancelling similar terms,

$$\sum_i c_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_d} - \sum_i d_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_c} \neq \sum_i d_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_t^i}{\partial q_c} - \sum_i c_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_1^i}{\partial q_d}.$$

Putting terms of the same sign together, the equation becomes

$$\sum_i c_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_d} + \sum_i c_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_1^i}{\partial q_d} \neq \sum_i d_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_t^i}{\partial q_c} + \sum_i d_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_c}$$

where both sides equal zero by the per-period homogeneity properties of Slutsky sub-matrices under time-separability. Therefore, taxes are undifferentiated and negative. \blacksquare

Proposition [not in paper]: When commodity taxes are not initially used, it is welfare-improving to marginally subsidize both goods.

Proof: Take (22), multiply by $q_c \sum_t (1+r_i)^{2-t} d_t^i$, normalize $q_c = 1$, substitute compensated demands, aggregate over all i , and substitute the result into (24) to get

$$(\alpha-1)\lambda \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial \alpha} (1+r_g)^{2-t} + \lambda \sum_i n_i (r_g - \underline{r}) d_1^i - \sum_i \mu_i (n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) d_1^i = 0. \quad (25)$$

Since $(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) > 0$, which can be verified in the first-order conditions, we can get two possibilities: (i) if $\underline{r} = r_g$ and at least one i is constrained, then $\alpha < 0$. (ii), if $\underline{r} < r_g$ and $\alpha^* > 0$, then the government collects revenue and r_g becomes \underline{r} , which is inconsistent. Therefore, $\alpha^* \leq 0$, and strictly so if $\mu^i > 0$ for at least one i , meaning that $q_d^* < 0$. Using a symmetric argument with $\beta \equiv q_c/q_d$ and normalizing $q_d = 1$, we obtain that $q_c^* < 0$. \blacksquare

Proposition [not in paper]: If only one commodity tax is used (here t_c), then the tax formula is

$$t_c^* = \frac{\sum_i (\mu_i/\lambda) (n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) c_1^i - \sum_i n_i (r_g - \underline{r}) c_1^i}{\sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1+r_g)^{2-t}} < 0. \quad (26)$$

(and same thing for τ_d^*). The tax satisfies $\tau_c^* \leq 0$, with strict inequality when at least one i is constrained in the optimum.

Proof: Take (22) without normalizing any price to one, multiply by $q_c \sum_t (q + r_i)^{2-t} d_t^i$, sum over i , and substitute for compensated demands. Substitute the result into, (24). We obtain

$$\begin{aligned} & \left(\alpha - \frac{1}{q_c} \right) \lambda \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial \alpha} (1 + r_g)^{2-t} + \left(1 - \frac{1}{q_c} \right) \lambda \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial \alpha} (1 + r_g)^{2-t} \\ & + \lambda \sum_i n_i (r_g - \underline{r}) d_1^i - \sum_i \mu_i (n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) d_1^i = 0. \end{aligned} \quad (27)$$

Use the fact that $q_d/q_c = \alpha$, and also that $\partial x/\partial \alpha = q_c \partial x/\partial q_d$. Obtain

$$\begin{aligned} & q_c \left(\alpha - \frac{1}{q_c} \right) \lambda \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1 + r_g)^{2-t} + q_c \left(1 - \frac{1}{q_c} \right) \lambda \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1 + r_g)^{2-t} \\ & + \lambda \sum_i n_i (r_g - \underline{r}) d_1^i - \sum_i \mu_i (n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) d_1^i = 0. \end{aligned} \quad (28)$$

and simplify to

$$\begin{aligned} & (q_d - 1) \lambda \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1 + r_g)^{2-t} + (q_c - 1) \lambda \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1 + r_g)^{2-t} \\ & + \lambda \sum_i n_i (r_g - \underline{r}) d_1^i - \sum_i \mu_i (n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) d_1^i = 0. \end{aligned} \quad (29)$$

Finally, divide all terms by λ , define $\pi^i \equiv (\mu_i/\lambda)(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) - n_i(r_g - \underline{r})$, and $t_c = q_c - 1$, $t_c = q_d = 1$, to get

$$t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1 + r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1 + r_g)^{2-t} - \sum_i \pi^i d_1^i = 0. \quad (30)$$

Setting $t_c = 0$ gives the tax formula. Analyzing (26), we see that if $\tau_c > 0$ and $\tau_d = 0$ then the government collects revenue in the first period, so $r_g = \underline{r}$. Since $(\mu_i/\lambda)(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) > 0$ and the denominator is negative, this would require $\tau_c^* < 0$, a contradiction. Therefore, $\tau_c^* \leq 0$, and with strict inequality when $\mu^i > 0$ for at least one individual i . A symmetric proof applies for τ_d^* . Symmetrically, one can take q_d as given and optimize on q_c by choosing

a ratio $\beta = q_c/q_d$ which yields

$$t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1+r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1+r_g)^{2-t} - \sum_i \pi^i c_1^i = 0. \quad (31)$$

■

Proposition 3: Optimal commodity tax system when some individuals are still credit-constrained: condition for differentiation.

Proof: Let us characterize the optimal commodity tax system when nonlinear income taxes are optimal. Let us suppose that, in the optimal tax system, $i \in \mathcal{C}$ if the individual is constrained and $i \in \mathcal{U}$ if he is unconstrained, with $\mathcal{C} \neq \emptyset$. To characterize q_d^* , take (22) and (24), and rewrite them using the equivalence that $q_d/q_c = \alpha$, and also that $\partial x/\partial \alpha = q_c \partial x/\partial q_d$. Then, multiply (22) by $q_c \sum_t (1+r_i)^{2-t} d_t^i$, normalize $q_c = 1$, substitute compensated demands, aggregate over all i , and substitute the result into (24). We obtain that q_d^* is characterized by:

$$\begin{aligned} & \sum_{i \in \mathcal{U}} n_i \left[t_c \sum_t (1+r_g)^{2-t} \frac{\partial \tilde{c}_t^i}{\partial q_d} + t_d \sum_t (1+r_g)^{2-t} \frac{\partial \tilde{d}_t^i}{\partial q_d} \right] \\ & + \sum_{i \in \mathcal{C}} n_i \left[t_c \left(\frac{\partial c_1^i}{\partial q_d} (1+r_g) + \frac{\partial \tilde{c}_2^i}{\partial q_d} + \frac{\partial c_2^i}{\partial I} d_1^i (1+r_i) \right) + t_d \left(\frac{\partial d_1^i}{\partial q_d} (1+r_g) + \frac{\partial \tilde{d}_2^i}{\partial q_d} + \frac{\partial d_2^i}{\partial I} d_1^i (1+r_i) \right) \right] \\ & - \sum_i \pi^i d_1^i = 0 \end{aligned}$$

where, again, $\pi^i \equiv (\mu_i/\lambda)(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) - n_i(r_g - \underline{r})$. Using compensated demands, we know that, for instance,

$$\frac{\partial c_1^i}{\partial q_d} (1+r_g) + \frac{\partial \tilde{c}_2^i}{\partial q_d} + \frac{\partial c_2^i}{\partial I} d_1^i (1+r_i) = \frac{\partial \tilde{c}_1^i}{\partial q_d} (1+r_g) - d_1 \frac{\partial c_1^i}{\partial \phi} (1+r_g) + \frac{\partial \tilde{c}_2^i}{\partial q_d} + \frac{\partial c_2^i}{\partial I} d_1^i (1+r_i)$$

which is

$$\sum_i (1+r_g)^{2-t} \frac{\partial \tilde{c}_t^i}{\partial q_d} + \frac{\partial c_2^i}{\partial I^i} d_1^i (1+r_i) - d_1 \frac{\partial c_1^i}{\partial \phi} (1+r_g).$$

Re-expressing again,

$$\sum_i (1+r_g)^{2-t} \frac{\partial \tilde{c}_t^i}{\partial q_d} + \left(\frac{\partial c_2^i}{\partial I^i} - \frac{\partial c_1^i}{\partial \phi} \right) d_1^i (1+r_i) - (r_g - r_i) d_1^i \frac{\partial c_1^i}{\partial \phi}.$$

The first-order condition with respect to q_d is therefore

$$\begin{aligned} & t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1+r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1+r_g)^{2-t} \\ & + t_c \sum_i n_i z_i \left[\left(\frac{\partial c_2^i}{\partial I^i} - \frac{\partial c_1^i}{\partial \phi} \right) d_1^i (1+r_i) - (r_g - r_i) d_1^i \frac{\partial c_1^i}{\partial \phi} \right] \\ & + t_d \sum_i n_i z_i \left[\left(\frac{\partial d_2^i}{\partial I^i} - \frac{\partial d_1^i}{\partial \phi} \right) d_1^i (1+r_i) - (r_g - r_i) d_1^i \frac{\partial d_1^i}{\partial \phi} \right] \\ & - \pi^i d_1^i = 0 \end{aligned} \quad (32)$$

where z_i is an indicator variable that equals one if the individual is constrained, and zero otherwise. By symmetry, we also have a symmetric condition for t_c . Since we concentrate in the case in which Engel curves are linear, optimal commodity taxes are characterized by

$$\begin{aligned} & t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1+r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1+r_g)^{2-t} \\ & - t_c (r_g - r_i) \sum_i n_i z_i c_1^i \frac{\partial c_1^i}{\partial \phi} - t_d (r_g - r_i) \sum_i n_i z_i d_1^i \frac{\partial d_1^i}{\partial \phi} - \sum_i \pi^i c_1^i = 0 \end{aligned} \quad (33)$$

and

$$\begin{aligned} & t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1+r_g)^{2-t} + t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1+r_g)^{2-t} \\ & - t_c (r_g - r_i) \sum_i n_i z_i d_1^i \frac{\partial c_1^i}{\partial \phi} - t_d (r_g - r_i) \sum_i n_i z_i d_1^i \frac{\partial d_1^i}{\partial \phi} - \sum_i \pi^i d_1^i = 0. \end{aligned} \quad (34)$$

For simplicity, rename the system:

$$t_c s_{cc} + t_d s_{dc} - t_c \Delta \sum_i n_i z_i c_1^i \frac{\partial c_1^i}{\partial \phi} - t_d \Delta \sum_i n_i z_i c_1^i \frac{\partial d_1^i}{\partial \phi} - \sum_i \pi^i c_1^i = 0. \quad (35)$$

$$t_c s_{cd} + t_d s_{dd} - t_c \Delta \sum_i n_i z_i d_1^i \frac{\partial c_1^i}{\partial \phi} - t_d \Delta \sum_i n_i z_i d_1^i \frac{\partial d_1^i}{\partial \phi} - \sum_i \pi^i d_1^i = 0 \quad (36)$$

where $\Delta \equiv r_g - \underline{r} > 0$, and s_{kj} is a sum of discounted substitution effects following a change in price j through consumption of good k , such as $s_{cc} \equiv \sum_i n_i \sum_t \frac{\partial \bar{c}_t^i}{\partial q_c} (1 + r_g)^{2-t}$, and so on. In matrices, the system is:

$$\begin{pmatrix} s_{cc} - \Delta \sum_i n_i z_i c_1^i \frac{\partial c_1^i}{\partial \phi} & s_{dc} - \Delta \sum_i n_i z_i c_1^i \frac{\partial d_1^i}{\partial \phi} \\ s_{cd} - \Delta \sum_i n_i z_i d_1^i \frac{\partial c_1^i}{\partial \phi} & s_{dd} - \Delta \sum_i n_i z_i d_1^i \frac{\partial d_1^i}{\partial \phi} \end{pmatrix} \begin{pmatrix} t_c \\ t_d \end{pmatrix} = \begin{pmatrix} \sum_i \pi^i c_1^i \\ \sum_i \pi^i d_1^i \end{pmatrix}.$$

Under the assumption that c and d are normal, the 2x2 matrix is negative definite. Call its determinant $|S'| < 0$. By Cramer's rule,

$$|S'| t_c^* = - \sum_i \pi^i c_1^i \left(s_{dd} - \Delta \sum_i n_i z_i d_1^i \frac{\partial d_1^i}{\partial \phi} \right) + \sum_i \pi^i d_1^i \left(s_{dc} - \Delta \sum_i n_i z_i c_1^i \frac{\partial d_1^i}{\partial \phi} \right)$$

Multiply by q_d and get $q_d t_c$

$$|S'| t_d^* = - \sum_i \pi^i d_1^i \left(s_{cc} - \Delta \sum_i n_i z_i c_1^i \frac{\partial c_1^i}{\partial \phi} \right) + \sum_i \pi^i c_1^i \left(s_{cd} - \Delta \sum_i n_i z_i d_1^i \frac{\partial c_1^i}{\partial \phi} \right)$$

To see if there is differentiation and what sign it takes, we compare taxes when transformed in ad-valorem tax. We know that $\tau_c < \tau_d$ if $q_d t_c < q_c t_d$ or, equivalently, if

$$\begin{aligned} & - \sum_i \pi^i c_1^i \left(q_d s_{dd} - q_d \Delta \sum_i n_i z_i d_1^i \frac{\partial d_1^i}{\partial \phi} \right) + q_d \sum_i \pi^i d_1^i \left(q_d s_{dc} - q_d \Delta \sum_i n_i z_i c_1^i \frac{\partial d_1^i}{\partial \phi} \right) \\ & > \\ & - \sum_i \pi^i d_1^i \left(q_c s_{cc} - q_c \Delta \sum_i n_i z_i c_1^i \frac{\partial c_1^i}{\partial \phi} \right) + \sum_i \pi^i c_1^i \left(q_c s_{cd} - q_d \Delta \sum_i n_i z_i d_1^i \frac{\partial c_1^i}{\partial \phi} \right) \end{aligned}$$

where the reversal of the inequality sign comes from from $|S'| < 0$. Using the homogeneity properties of the per-period Slutsky matrices (marginal rate of substitution between c_1^i and d_1^i independent of that between c_2^i and d_2^i), the condition can be expressed as

$$\sum_i \pi^i c_1^i q_d \Delta \sum_i n_i z_i d_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right) > \sum_i \pi^i d_1^i q_c \Delta \sum_i n_i z_i c_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right).$$

Assuming that at the optimum $\sum_i \pi^i c_1^i > 0$ and $\sum_i \pi^i d_1^i > 0$, meaning that both taxes are negative. The condition for differentiation (rebate on good c) is

$$\frac{\sum_i \pi^i c_1^i}{\sum_i \pi^i d_1^i} > \frac{\sum_i n_i z_i q_c c_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right)}{\sum_i n_i z_i q_d d_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right)}. \quad (37)$$

Recall that $(\mu_i/\lambda)(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) - n_i(r_g - \underline{r})$ is the benefit on social welfare, net of IC constraints and of its funding costs, of relaxing the credit constraint of type i . When some type i , is unconstrained, this is a cost. Therefore, interpret the left-hand side of (37) as the ratio of the (net benefits) of slacking constraints by increasing consumption of good c to that of good d . To interpret the right-hand side, recall that $\sum_i n_i z_i q_c c_1^i \left(\frac{\partial c_1^i}{\partial \phi} + \frac{\partial d_1^i}{\partial \phi} \right)$ is the magnitude of income effects created by reducing the tax on good c in the first period, and which will have to be financed through borrowing by the government. The same principle applies to the denominator. Therefore, the right-hand side is a cost ratio.

If one tax is positive and the other is negative, this happens when (for instance for $t_c < 0 < t_d$), $\sum_i \pi^i d_1^i < 0 < \sum_i \pi^i c_1^i$, so when the weighted first-period consumption of good c generates net social benefits through slacking credit constraints, but that through good d is negative. ■