

# Endogenous Growth, Inequality and the Composition of Government Expenditures

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## Abstract

This paper considers an endogenous growth model with public capital and heterogeneous agents. Heterogeneity is due to differences in discount factors and inherent abilities. This allows us to closely approximate the 2007 pre-Great Recession U.S. income and wealth distributions. Government expenditures, including public investment, are financed through a progressive income taxation scheme along with a flat tax on consumption. Four permanent and revenue-neutral fiscal policy reforms are considered: (a) an increase in the public consumption-to-output ratio by 1% financed by either a 1% decrease in the public investment-to-output ratio, or an increase in the degree of progressivity of the tax schedule, and (b) an increase in the public investment-to-output ratio by 1% financed by either a 1% decrease in the public consumption-to-output ratio, or an increase in the degree of progressivity of the tax schedule. It is shown that increasing investment in public capital is the only policy that simultaneously enhances growth and reduces both types of inequality (income and wealth). Furthermore, if the increase in public investment is accompanied by a reduction in public consumption, the positive impact on growth is larger relative to an increase in the progressivity of the tax schedule, while the reduction in wealth and income inequality is smaller. On the other hand, an increase in public consumption reduces both growth and inequality. If this increase is associated with a rise in the progressivity of the tax schedule, then the effect on growth and inequality is larger relative to a reduction in public investment.

Keywords: Nonlinear Income Taxation; Inequality; Endogenous Growth; Welfare; Government Expenditure; Public Capital

JEL Classification: E62; E20; H54

# 1 Introduction

The effect of government expenditures on the economy is a subject with a long and controversial history. However, although it has been widely acknowledged that fiscal policy reforms have an impact on both growth and the distributions of income and wealth, the theoretical models used to study the effects of changes in government expenditures have focused on the impact on growth ignoring the distributional aspects of the policy change.

This paper considers an endogenous growth model with heterogeneous agents who are subjected to progressive income taxes. In terms of public spending, we distinguish between government consumption which provides direct utility to households, and government investment which enhances the productive capacity of firms. Because these two types of expenditure have a distinct impact on preferences and technology, a change in the composition of government expenditures can have a significant impact on the economy. The model allows us to study the interaction between the growth and distributional effects resulting from such a shift in the composition of government expenditures. As a result, it offers a more complete assessment of the overall effects of this policy change relative to the previous literature which determined the growth effects independently of the distributional effects. From a policy perspective, this is an important issue, especially if one considers the recent findings of Picketty and Saez (2013) on the sharp increase in the share of U.S. national income accruing to the upper income groups in recent decades.<sup>1</sup>

Barro (1990) uses an endogenous growth model to show that when the government increases public consumption while reducing public investment, then growth rates fall regardless of the level of total expenditure. Turnovsky and Fisher (1995) use the neoclassical growth model to study the impact of the composition of government expenditures on growth. They show that, under plausible conditions, a unit increase in government expenditure on infrastructure has a larger impact on capital accumulation than a corresponding increase in government consumption expenditure.

It is important to note that the relationship between income inequality and growth is ambigu-

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<sup>1</sup>Lansing and Markiewicz (2013) develop a dynamic general equilibrium model of skill-biased technological change that captures the sharp increase in the share of total income for the top decile in the U.S. since 1980.

ous. This, in turn, has varying implications about the appropriate type of fiscal policy that should be implemented. Barro (2000) and Forbes (2000) argue that some inequality is necessary to provide incentives for investment and growth. In contrast, Berg, Ostry and Zettelmeyer (2012) find that inequality may be harmful for growth. Bastagli, Coady and Gupta (2012) show that income inequality has increased in most advanced and many developing economies in recent years. Furthermore, they demonstrate that the variation in income inequality across regions can be largely accounted for by differences in the progressivity of tax policies, as well as differences in spending policies.

In the present paper we consider the endogenous growth model with public capital by Cassou and Lansing (1998), and incorporate two new features: (i) heterogeneous agents and (ii) a progressive income tax schedule. Heterogeneity in our model arises from two sources. The first source is differences in discount factors between households as in Li and Sarte (2004). Krusell and Smith (1998) and Hendricks (2007) demonstrate that time preference heterogeneity is an important factor in explaining the observed wealth inequality in the United States. The second source of heterogeneity is differences in labor productivity across agents due to inherent ability. This specification is similar to the one in Suen (2014), Carroll and Young (2011) and Koyuncu (2011). It allows us to closely approximate the U.S. before-tax income and wealth distributions simultaneously.<sup>2</sup> Furthermore, it is consistent with the empirical evidence provided by Lawrance (1991) and Warner and Pleeter (2001) that impatient households tend to have lower wages and wealth.<sup>3</sup>

Apart from progressive income taxes, an additional source of revenue for financing government expenditures is a flat consumption tax.<sup>4</sup> The model is tractable enough that allows the study of the

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<sup>2</sup>Suen's (2013) model involves endogenous human capital accumulation that leads to differences in labor productivity and wages. In our case, these differences are due to exogenously determined inherent abilities. Carroll and Young (2011) consider an environment similar to ours in which heterogeneous households differ in terms of their discount factor and permanent labor productivity. In their model, a progressive income tax schedule is used to finance wasteful government expenditures. Koyuncu (2011) develops an endogenous growth model in which agents are heterogeneous with respect to their rates of time preference and labor skills. Progressive income taxes are used to finance wasteful government expenditures as well.

<sup>3</sup>There are alternative ways in which heterogeneity can be introduced in an otherwise standard growth model. For instance, García-Peñalosa and Turnovsky (2011) examine how changes in tax policies affect the wealth and income distribution in a neoclassical growth model where agents differ in terms of their initial capital endowments.

<sup>4</sup>Arnold (2008) studies the relationship between different tax structures and economic growth for a panel of 21 OECD countries. His results suggest that income taxes are generally associated with lower economic growth compared to consumption and property taxes. He also finds evidence of a negative relationship between the progressivity of

effects of various fiscal policy reforms on both the growth rate, and income and wealth distributions simultaneously. This is in contrast to previous studies that analyze the effect of fiscal policy on growth or the effect of fiscal policy on these distributions in isolation of each effect from the other.

Four revenue-neutral fiscal policy reforms are considered: (a) an increase by 1% in the public consumption-to-output ratio financed by either a 1% decrease in the public investment-to-output ratio or an increase in the degree of progressivity of the tax schedule, and (b) an increase by 1% in the public investment-to-output ratio financed by either a 1% decrease in the public consumption-to-output ratio or an increase in the degree of progressivity of the tax schedule. The model is calibrated to the postwar U.S. economy. We closely approximate the 2007 U.S. wealth distribution as described in Díaz-Giménez, Glover and Ríos-Rull (2011), and the before-tax distribution of the same year as reported in CBO (2012). It is shown that increasing investment in public capital is the only policy that simultaneously enhances growth and reduces inequality. Furthermore, if the increase in public investment is accompanied by a reduction in public consumption, the positive impact on growth is larger relative to an increase in the progressivity of the tax schedule, while the reduction in wealth and income inequality is smaller. On the other hand, an increase in public consumption reduces both growth and inequality. If this increase is associated with a rise in the progressivity of the tax schedule, then the effect on growth and inequality is larger relative to a reduction in public investment.

The study closest to ours is by Chatterjee and Turnovsky (2012). These authors consider an endogenous growth model with public capital and heterogeneous agents, where heterogeneity is due to differences in initial private capital endowments. They study the effect of different financing schemes of public investment on growth, wealth and income inequality, as well as welfare. The main result of their analysis is that public investment increases wealth inequality over time regardless of its source of financing. The main difference with our work is that these authors consider differential flat rate taxation of capital and labor, while we consider a progressive income tax schedule.

The paper is organized as follows. Section 2 presents the model with public capital and personal income taxes and growth.

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sive taxation. Section 3 discusses the calibration of the model. Section 4 presents the simulation results. The final section concludes.

## 2 The Model

Consider a closed economy populated by a large number of households uniformly distributed in the interval  $[0, 1]$ . Assume that there are  $N$  types of households. Each type is indexed by a discount factor  $\beta_j$  where  $0 < \beta_1 < \dots < \beta_N < 1$  and a level of inherent ability  $e_j > 0$  that determines the individual's labor productivity. The measure of households within each group is  $1/N$ .

Following Cassou and Lansing (1998), we assume that the private sector consists of a large but fixed number of identical firms that have a measure of one. The representative firm produces output  $Q_t$  according to the technology:

$$Q_t = AK_t^{\theta_1} (H_t L_t)^{\theta_2} K_{gt}^{\theta_3}, \quad (1)$$

where  $K_t$  denotes the stock of private capital,  $H_t$  is an index of knowledge,  $L_t$  represents the labor supply and  $K_{gt}$  denotes the stock of public capital. Regarding the parameters of production function (1), it is assumed that  $A > 0$ ,  $\theta_i > 0$  for  $i = 1, 2, 3$  and  $\theta_1 + \theta_2 + \theta_3 = 1$ .

The firm chooses  $K_t$  and  $L_t$ , but takes  $K_{gt}$  as exogenously determined by the government. Output is affected by  $H_t$ , which is also outside the firm's control. Following Arrow (1962) and Romer (1986), it is assumed that the mechanism of knowledge accumulation involves "learning-by-doing". The implication is that knowledge grows proportionally to, and is a by-product of, accumulated private investment and research activities. Hence, the following condition can be imposed after the firm chooses its optimal labor and capital input levels:

$$H_t = K_t. \quad (2)$$

Condition (2) and the assumption that  $\theta_1 + \theta_2 + \theta_3 = 1$  imply that production function (1) displays constant-returns-to-scale in the two reproducible factors  $K_t$  and  $K_{gt}$ . As a consequence, the model exhibits endogenous growth.

Each period the representative firm solves the static profit maximization problem:

$$\max_{\{K_t, L_t\}} \Pi_t = AK_t^{\theta_1} (H_t L_t)^{\theta_2} K_{gt}^{\theta_3} - r_t K_t - \delta_K K_t - W_t L_t, \quad (3)$$

where  $r_t$  and  $W_t$  denote the rental rate of private capital and wage rate, respectively. The depreciation rate of private capital is given by  $0 < \delta_K < 1$ . The first-order conditions are:

$$r_t = \theta_1 \left( \frac{Q_t}{K_t} \right) - \delta_K, \quad (4)$$

and

$$W_t = \theta_2 \left( \frac{Q_t}{L_t} \right). \quad (5)$$

Combining (3) with (4) and (5) implies that aggregate profits are equal to  $\Pi_t = \theta_3 Q_t$ .

The government is assumed to maintain a balanced budget every period. Following Li and Sarte (2004), the government chooses a tax schedule summarized by the tax rate,  $\tau(Y_j/Y)$ , where  $Y_j$  is a representative household's taxable income and  $Y$  is aggregate taxable income.<sup>5</sup> This specification implies that the tax rate that applies to a given household depends on its relative standing in the economy.<sup>6</sup> We further assume that the tax schedule is given by

$$\tau \left( \frac{Y_j}{Y} \right) = \zeta \left( \frac{Y_j}{Y} \right)^\phi, \quad \forall j = 1, \dots, N \quad (6)$$

where  $0 \leq \zeta < 1$  and  $\phi > 0$ . Parameter  $\zeta$  determines the level of the tax schedule, while parameter  $\phi$  determines its slope. When  $\phi > 0$ , tax rate  $\tau$  increases with the household's taxable income. Therefore, households with higher taxable income are subject to higher tax rates. Proportional taxation is the most common case considered in the literature. This case is obtained by setting  $\phi = 0$  in (6), which implies that  $\tau(Y_j/Y) = \zeta$ . In deciding how much to consume and invest over their lifetime, households take into account the effect of the tax schedule on their after-tax earnings. Specifying the income tax schedule using (6) allows for an explicit analysis of how changes in  $\phi$  simultaneously affect the distributions of pre-tax income and wealth, and the growth rate.

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<sup>5</sup>Note that GDP is given by (1). This is not equal to  $Y_t$  which represents household taxable income. The latter consists of the sum of capital incomes, labor incomes and profit dividends minus the private capital stock depreciation allowances. Formally,  $Y_t = Q_t - \delta_K K_t$ .

<sup>6</sup>This modeling assumption ensures that not all households eventually face the highest marginal tax rate simply as a result of economic growth. In other words, it allows us to abstract from the so-called "bracket-creep" considerations.

The distinction between marginal and average tax rates is important when studying progressive tax schedules. The total amount of taxes paid by a household with income  $Y_j$  is equal to  $\tau(Y_j/Y) Y_j$ . The marginal tax rate,  $\tau_m(Y_j/Y)$ , which is the tax rate applied to the last dollar earned, is

$$\tau_m(Y_j/Y) = \frac{\partial(\tau(Y_j/Y) Y_j)}{\partial Y_j} = (1 + \phi) \zeta \left( \frac{Y_j}{Y} \right)^\phi. \quad (7)$$

The average tax rate,  $\tau_a(Y_j/Y)$ , is simply equal to  $\tau(Y_j/Y)$ . The ratio of the marginal to the average tax rate indicates the progressivity of the tax schedule. The latter is more progressive the more the marginal tax rate exceeds the average tax rate at all income levels. Combining (6) and (7) yields  $\tau_m(Y_j/Y) / \tau_a(Y_j/Y) = 1 + \phi$ . Hence, parameter  $\phi$  captures the degree of progressivity in the tax schedule. If  $\phi = 0$ , then  $\tau_m(Y_j/Y) = \tau_a(Y_j/Y)$  and the tax schedule is “flat”.

Government expenditures,  $G_t$ , consist of public investment,  $I_{gt}$ , and public consumption,  $C_{gt}$ :

$$G_t = I_{gt} + C_{gt}. \quad (8)$$

Households are assumed to derive utility from public consumption goods  $C_{gt}$ . On the other hand, public investment leads to the accumulation of public capital:

$$I_{gt} = K_{gt+1} - (1 - \delta_G) K_{gt}, \quad (9)$$

where  $0 < \delta_G < 1$  is the depreciation rate of  $K_g$ .

In addition to income tax revenues, the government raises revenues from taxing consumption. These revenues are equal to  $\omega C_t$ , where  $C_t = \sum_{j=1}^N C_{jt} (1/N)$  denotes aggregate consumption at time  $t$  and  $C_{jt}$  represents the consumption of a type  $j$  household. Parameter  $0 \leq \omega < 1$  denotes a flat and time-invariant consumption tax. Using tax schedule (6) to obtain income tax revenues, the government’s balanced budget constraint is given by

$$G_t = I_{gt} + C_{gt} = \sum_{j=1}^N \zeta \left( \frac{Y_{jt}}{Y_t} \right)^\phi Y_{jt} \left( \frac{1}{N} \right) + \omega C_t. \quad (10)$$

Regarding preferences, we adopt the commonly used in the business cycle and growth literature specification of Greenwood, Hercowitz and Huffman (1988). Each type  $j$  household chooses paths

for consumption,  $\{C_{jt}\}_{t=0}^{\infty}$ , labor supply,  $\{L_{jt}\}_{t=0}^{\infty}$ , and private capital,  $\{K_{jt+1}\}_{t=0}^{\infty}$ , to maximize lifetime utility

$$\sum_{t=0}^{\infty} \beta_j^t \left[ \frac{1}{1-\sigma} \left( \left[ C_{jt} - H_t \frac{L_{jt}^{1+\gamma}}{1+\gamma} \right]^{1-\sigma} - 1 \right) + D \ln(C_{gt}) \right], \quad \sigma, \gamma, D > 0, j = 1, \dots, N, \quad (11)$$

subject to the flow budget constraint

$$(1 + \omega) C_{jt} + K_{jt+1} = \left[ 1 - \zeta \left( \frac{Y_{jt}}{Y_t} \right)^\phi \right] Y_{jt} + K_{jt} \quad (12)$$

where

$$Y_{jt} = r_t K_{jt} + W_t e_j L_{jt} + \Pi_{jt}, \quad (13)$$

$$Y_t = \sum_{j=1}^N Y_{jt} \left( \frac{1}{N} \right), \quad (14)$$

and  $C_{jt}, K_{jt} \geq 0, 0 \leq L_{jt} \leq 1, K_{j0} > 0$  for all  $j$  and  $t$ .

Variable  $\Pi_{jt}$  denotes the profits share of each of type  $j$  household. Parameter  $\sigma$  is the coefficient of relative risk aversion and  $1/\gamma$  is the intertemporal elasticity of substitution in labor supply. Parameter  $D$  captures the degree of substitutability between public and composite private consumption. Lansing (1998) points out that the assumption of additive separability in public consumption goods is supported by the empirical estimates in McGrattan, Rogerson and Wright (1997) based on post-war U.S. data. This specification simplifies the computations, since the term involving  $C_{gt}$  in the utility function can be ignored when the optimality conditions for the household's problem are derived.

Aggregating budget constraint (12) across all types of household and using (3), (10), (13) and (14) yields the economy-wide resource constraint:

$$C_t + G_t + K_{t+1} - (1 - \delta_K) K_t = AK_t^{\theta_1} (H_t L_t)^{\theta_2} K_{gt}^{\theta_3}. \quad (15)$$

Households take the sequence of factor payments  $\{r_t, W_t\}_{t=0}^{\infty}$ , profit dividends  $\{\Pi_{jt}\}_{t=0}^{\infty}$  and the government's fiscal policy as given when maximizing (11). Furthermore, in every period labor

supply and private capital satisfy the following aggregation conditions:

$$L_t = \sum_{j=1}^N e_j L_{jt} \left( \frac{1}{N} \right) \quad (16)$$

and

$$K_t = \sum_{j=1}^N K_{jt} \left( \frac{1}{N} \right), \quad (17)$$

respectively.

Along the balanced-growth path equilibrium (BGP), all individual and aggregate variables grow at the same constant rate  $\mu$ . Furthermore, relative income  $Y_j/Y$  remains constant for each  $j$ . Following King, Plosser and Rebelo (2002), we perform a stationary transformation in order for the transformed model to possess a steady state. Letting  $x_t \equiv X_t/H_t$  for an arbitrary variable  $X_t$ , the optimality conditions for a type  $j$  household in the transformed economy are:

$$\left( \frac{H_{t+1}}{H_t} \right)^\sigma \left[ c_{jt} - \frac{L_{jt}^{1+\gamma}}{1+\gamma} \right]^{-\sigma} = \beta_j \left[ c_{jt+1} - \frac{L_{jt+1}^{1+\gamma}}{1+\gamma} \right]^{-\sigma} \left\{ \left[ 1 - (1+\phi) \zeta \left( \frac{y_{jt+1}}{y_{t+1}} \right)^\phi \right] r_{t+1} + 1 \right\}, \quad j = 1, \dots, N, \quad (18)$$

$$L_{jt}^\gamma = \left[ 1 - (1+\phi) \zeta \left( \frac{y_{jt}}{y_t} \right)^\phi \right] \left( \frac{w_t e_j}{1+\omega} \right), \quad j = 1, \dots, N, \quad (19)$$

and

$$(1+\omega) c_{jt} + k_{jt+1} \left( \frac{H_{t+1}}{H_t} \right) = \left[ 1 - \zeta \left( \frac{y_{jt}}{y_t} \right)^\phi \right] y_{jt} + k_{jt}, \quad j = 1, \dots, N. \quad (20)$$

Expression (18) is the standard Euler equation for a type  $j$  household. Expression (19) yields the labor supply of the household at time  $t$ . Finally, expression (20) is the transformed version of the household's budget constraint (12).

Evaluating Euler equation (18) along the BGP, and using (2) and (4) yields:

$$\mu^\sigma = \beta_j \left\{ \left[ 1 - (1+\phi) \zeta \left( \frac{y_j}{y} \right)^\phi \right] (\theta_1 q - \delta_K) + 1 \right\}, \quad j = 1, \dots, N \quad (21)$$

where  $q$  is the constant output-to-private capital ratio. Using (1), (2) and (5) allows us to express the balanced growth path version of labor supply equation (19) as

$$L_j^\gamma = \left( \frac{1}{1+\omega} \right) \left[ 1 - (1+\phi) \zeta \left( \frac{y_j}{y} \right)^\phi \right] \theta_2 A^{\frac{1}{\theta_2}} k_g^{\frac{\theta_3}{\theta_2}} q^{\frac{\theta_2-1}{\theta_2}} e_j, \quad j = 1, \dots, N, \quad (22)$$

where  $k_g$  is the constant public-to-private capital ratio. Combining (1), (2) and (16), and evaluating the resulting expression along the BGP yields:

$$\left( \frac{q}{A k_g^{\theta_3}} \right)^{\frac{1}{\theta_2}} = \sum_{j=1}^N e_j L_j \left( \frac{1}{N} \right). \quad (23)$$

Furthermore, substituting resource constraint (15) into the government's budget constraint (10) implies that the ratio of government expenditures to private capital along the BGP is given by

$$g = \left( \frac{q - \delta_K}{1+\omega} \right) \left[ \omega + \sum_{j=1}^N \zeta \left( \frac{y_j}{y} \right)^{1+\phi} \left( \frac{1}{N} \right) \right] + \frac{(1-\mu)\omega}{1+\omega}. \quad (24)$$

Letting  $g_I$  denote the public investment-to-output ratio, it follows from expression (9) that:

$$g_I q = (\mu - (1 - \delta_G)) k_g. \quad (25)$$

Finally, let  $\chi$  denote the government expenditure-to-output ratio:

$$\chi = \frac{g}{q}, \quad (26)$$

and  $g_C$  denote the public consumption-to-output ratio. Expression (8) then implies that:

$$g_C + g_I = \chi. \quad (27)$$

In the long-run equilibrium, given  $\chi$ , the growth rate,  $\mu$ , the output-to-private capital ratio,  $q$ , the public-to-private capital ratio,  $k_g$ , the ratio of government expenditures to private capital,  $g$ , the shares of public investment and consumption in output,  $g_I$  and  $g_C$ , respectively, the relative income earned by households,  $y_j/y$ , and their labor supply,  $L_j$ , are simultaneously determined from a system of  $2N + 6$  equations in  $2N + 6$  unknowns. These equations are (21) – (27) and

$$\sum_{j=1}^N \left( \frac{y_j}{y} \right) \frac{1}{N} = 1. \quad (28)$$

Equation (28) is simply condition (14) evaluated along the balanced growth path.

In deriving the transitional dynamics, the transformed model is log-linearized around the steady state obtained from solving system (21) – (28) above. We then apply the techniques described in King, Plosser and Rebelo (2002) to solve for the policy rules as a function of the state variables of the model. Recall that in the transformed model the relative private capital stock of each type is given by  $k_{jt}$ . Condition (17) implies that the relative private capital stocks across quintiles sum up to 1. Hence, we can use the relative private capital stock of any type as a benchmark and exclude it from the state vector. The lowest quintile is chosen as a benchmark for the simulation results reported in Section 4. Letting  $\hat{x}_t \equiv \ln(x_t/x)$  denote the percentage deviation of variable  $x$  from its steady state value at time  $t$ , it follows that the state vector in our case is  $s_t = [\hat{k}_{gt}, \hat{k}_{2t}, \dots, \hat{k}_{5t}]'$ .<sup>7</sup>

### 3 Calibration

In order to analyze the quantitative implications of the model, we assign values to parameters based on empirically observed features of the postwar U.S. economy. These values are reported in Table 1 below. Table 2 displays the main properties of the model economy in the long run and their actual data counterparts.

Based on data obtained from the Bureau of Labor Statistics, the average long-run growth rate of real output per capita during the period 1961-2010 was approximately 2.0275%. Therefore, when calibrating key parameters of the model we set  $\mu = 1.0203$ . Regarding the production function parameters, we follow Cassou and Lansing (1998) and set  $\theta_1 = 0.30$ ,  $\theta_2 = 0.60$  and  $\theta_3 = 0.10$ . In terms of preference parameters, the coefficient of relative risk aversion  $\sigma$  is set equal to 2. In addition,  $\gamma$  is set equal to 0.6 which implies an intertemporal elasticity of substitution in labor supply of 1.7. The values of both parameters are the same as the ones used by Greenwood, Hercowitz and Huffman (1988). In addition, we follow Lansing (1998) and set  $D$  equal to 0.7870.

Regarding the depreciation rate of private capital, we follow Li and Sarte (2004) and choose the

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<sup>7</sup>For all simulation results reported in Section 4, there is always the sufficient number of stable roots that support a unique saddle path. Furthermore, these roots are real and distinct. For example, in the benchmark calibration of the model with  $1/\gamma = 1.7$ , the five eigenvalues are 0.9542, 0.9972, 0.9982, 0.9986 and 0.9990.

value of  $\delta_K$  in order to match a private investment-to-private capital ratio  $i_p = I/K$  of 0.076. As a result, we set  $\delta_K$  equal to 0.0557. Using data that covers the postwar period 1946-2006, Atolia et al. (2011) determine that the private capital-to-output ratio is roughly equal to 2.17. This implies that the GDP-to-private capital ratio is 0.4608. Furthermore, these authors calculate the ratio of public capital-to-private capital to be 0.5070. Since the private capital-to-GDP ratio is 2.17, these values imply that the public capital-to-output ratio is equal to 1.1002.

Based on data obtained from the National Income and Product Accounts, the average real government gross investment as a share of output for the period 1995-2010 is approximately 0.0318. Therefore, we set the fraction  $g_I$  of output allocated to public investment equal to this value. In addition, given the values of  $\mu$ ,  $g_I$ ,  $q$  and the public capital-to-output ratio, it follows from (25) that  $\delta_G = 0.0086$ . Note that the depreciation rate of public capital is lower than that of private capital. This captures the fact that a substantial portion of public capital consists of infrastructure which tends to depreciate at a slower pace than plant and machinery.

Following Cassou and Lansing (1998), we set labor  $L$  equal to 0.30 along the balanced growth path. Given this value, and the values of  $q$  and  $k_g$ , it follows from (1) that  $A$  should be set equal to 1.0157. Note that these values imply a private investment-to-output ratio of 0.1649 along the BGP which matches the actual value from the data. In addition, we set the consumption expenditure tax rate,  $\omega$ , equal to 0.0060 in order for the private consumption-to-output ratio in the long-run equilibrium of the model to be 0.6577 which corresponds to the average for the period 1961-2010.

The parameters governing the tax code,  $\zeta$  and  $\phi$ , are calibrated based on the supplemental data provided to CBO (2012). The objective is to choose values for these parameters in order to match the distribution of total federal tax liabilities across quintiles in 2007, given the distribution of before-tax incomes in the same year and tax schedule (6). To accomplish this, we proceed in two steps. First, given an initial choice for the value of  $\zeta$ , we choose  $\phi$  to minimize the Euclidean distance between the vector of predicted shares of total federal tax liabilities and the vector of their actual counterparts. Given  $\phi$  obtained from the previous step, we choose  $\zeta$  to match the 2007 total average federal tax rate of 19.9% reported in CBO (2012). Following this calibration scheme, we

set  $\zeta = 0.1641$  and  $\phi = 0.4056$ . These values imply that the average marginal tax rate is 27.9714% and the progressivity ratio is 1.4056.

Equation (21) is used to calibrate the values of the discount factors,  $\beta_j$ ,  $j = 1, \dots, N$ , that fit the quintile distribution of before-tax income in 2007 reported in CBO (2012). These values are listed in Table 1. Note that the Gini coefficient for the 2007 before-tax income distribution is 0.50. As shown in Table 2, the model essentially replicates the U.S. before-tax income distribution since the calculated shares of income by quintile are quite close to the ones from the data. The Gini coefficient of 0.4388 is slightly lower than the one reported by CBO (2012). The reason is that CBO uses the entire pre-tax income distribution to calculate the Gini coefficient, while we use only the income shares by quintile. Furthermore, the model underpredicts the tax liabilities of the middle and fourth quintiles, and overpredicts the tax liabilities of the remaining quintiles. However, the differences with the actual values from the data are small.

Recall that it is assumed that the government maintains a balanced budget. According to the Historical Budget Data provided in the Budget and Economic Outlook reports by the CBO, the average share of revenues in GDP for the 1971-2010 period is 0.1798. Along the BGP, the model predicts a government expenditures-to-private capital ratio of 0.0817 which combined with the output-to-private capital ratio implies that the share of government expenditures in output is 0.1774. Note that the average share of real government consumption and gross investment in GDP during the 1995-2010 period is 0.1938. The value of the public consumption-to-output ratio is 0.1620, while the model yields a slightly lower value along the BGP of 0.1456.

Finally, the indices of inherent ability,  $e_j$ ,  $j = 1, \dots, N$  are calibrated using the following scheme. First, we find the relative private capital stock holdings for each quintile that allow us to closely approximate the U.S. wealth distribution in 2007 as it is provided by Díaz-Giménez, Glover and Ríos-Rull (2011)<sup>8</sup>. As shown in Table 2, the quintile distribution of relative private

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<sup>8</sup>We use the flexible function form  $z_i = e^{f(x_i)}$  to approximate the cumulative sum of wealth shares, where  $x_i$  is an element of vector  $\mathbf{x} = [0, 0.20, 0.40, 0.60, 0.80, 1]'$  and  $f(x_i)$  is a polynomial function. The following standardization of the cumulative wealth share  $z_i$  is performed to ensure that its value lies in interval  $[0, 1]$ :

$$\tilde{z}_i = \frac{z_i - z_{\min}}{z_{\max} - z_{\min}}.$$

capital stock holdings is similar to the actual wealth distribution. The model slightly overpredicts the wealth shares of the lowest and fourth quintiles, while it underpredicts these shares for the remaining quintiles. However, the differences with the actual values are small. Díaz-Giménez et al. (2011) report a Gini coefficient of 0.816, while in our case it is 0.7161. However, as it was the case for the Gini coefficient of the before-tax income distribution, one needs to take into account that Díaz-Giménez et al. (2011) use the entire sample in their calculation, while we use only the wealth share of each quintile.

Once  $k_j$  has been determined, we find the corresponding  $e_j$  as follows. It was shown earlier that  $\Pi_t = \theta_3 Q_t$ . We assume that every period each household receives a profit dividend according to its relative private capital stock holdings. It follows that along the BGP  $\pi_j = \theta_3 q k_j$ ,  $j = 1, \dots, N$ . Next, considering the transformed version of (13) in the long-run equilibrium, the effective labor supply of type  $j$  can be defined as

$$\bar{L}_j = e_j L_j = \frac{y_j - (r + \theta_3 q) k_j}{w}, \quad j = 1, \dots, N. \quad (29)$$

Combining (29) with (19) evaluated along the balanced growth path yields:

$$e_j = \left( \frac{\bar{L}_j^\gamma (1 + \omega) L}{\left[ 1 - (1 + \phi) \zeta \left( \frac{y_j}{y} \right)^\phi \right] \theta_2 q} \right)^{\frac{1}{1+\gamma}}, \quad j = 1, \dots, N. \quad (30)$$

Using  $k_j$  and  $y_j$  for each type, expressions (29) and (30) are evaluated to obtain  $e_j$ ,  $j = 1, \dots, N$ .

The values of the indices of inherent ability are reported in the bottom panel of Table 1.<sup>9</sup>

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The coefficients of polynomial  $f(x_i)$  are chosen such that the Euclidean distance of the cumulative sum of wealth shares between the actual and the simulated wealth distribution is minimized. We find that a polynomial of the form:

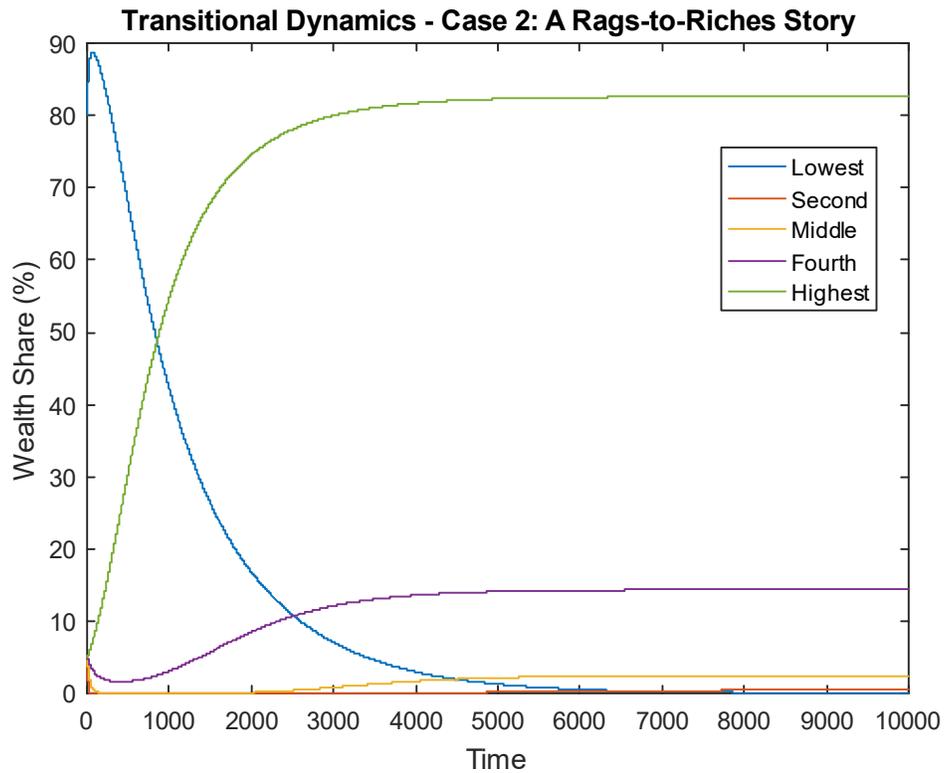
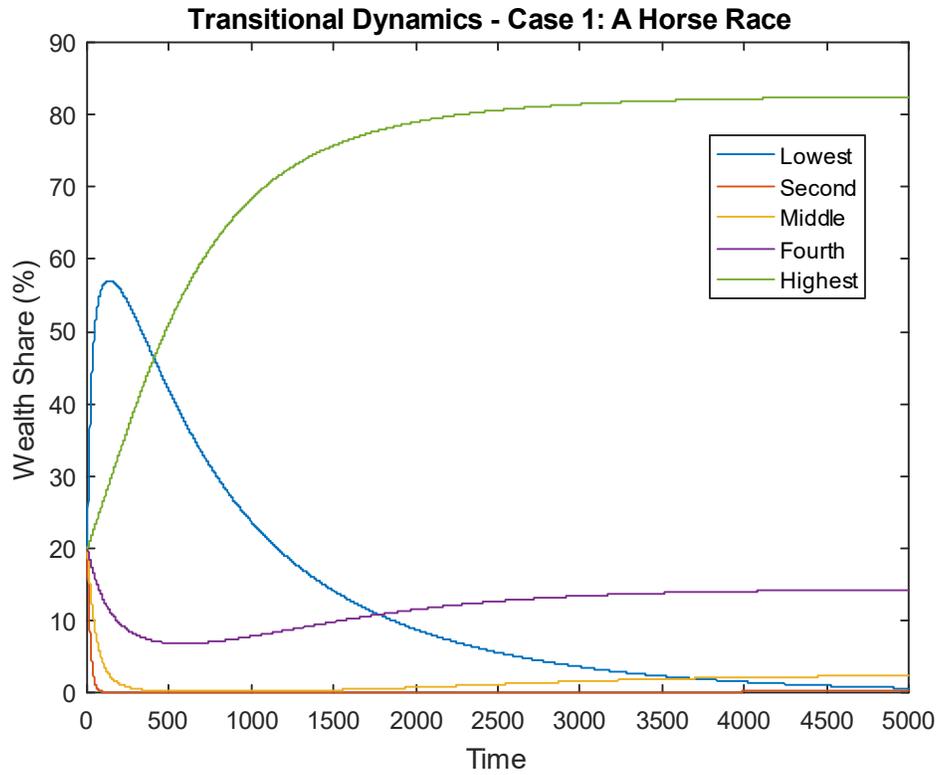
$$f(x_i) = d_1 x_i + d_2 x_i^2,$$

with  $d_1 = 8.726$  and  $d_2 = 0.010$  allows us to approximate the actual cumulative sum of wealth shares quite closely. Then, given the estimated cumulative sum, it is straightforward to obtain the wealth share of each quintile.

<sup>9</sup>Note that the labor supply of each type along the BGP is 0.2056, 0.2980, 0.3633, 0.3901 and 0.5360.

# 4 Transitional Dynamics

[IN PROGRESS]



## 5 Results

[IN PROGRESS]

## 6 Conclusions

This paper considers an endogenous growth model with public capital and heterogeneous agents. Government expenditures are financed through a progressive income taxation scheme along with a flat tax on consumption. Four revenue-neutral fiscal policy reforms are considered: (a) an increase by 1% in the public consumption-to-output ratio financed by either a 1% decrease in the public investment-to-output ratio or an increase in the degree of progressivity of the tax schedule, and (b) an increase by 1% in the public investment-to-output ratio financed by either a 1% decrease in the public consumption-to-output ratio or an increase in the degree of progressivity of the tax schedule. It is shown that increasing investment in public capital is the only policy that simultaneously enhances growth and reduces inequality. Furthermore, if the increase in public investment is accompanied by a reduction in public consumption, the positive impact on growth is larger relative to an increase in the progressivity of the tax schedule, while the reduction in wealth and income inequality is smaller. On the other hand, an increase in public consumption reduces both growth and inequality. If this increase is associated with a rise in the progressivity of the tax schedule, then the effect on growth and inequality is larger relative to a reduction in public investment.

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Table 1: Calibrated Benchmark Parameters

Parameter	Description	Value
$\theta_1$	Private capital share in output	0.3000
$\theta_2$	Labor share in output	0.6000
$\theta_3$	Output elasticity with respect to private capital	0.1000
$1/\gamma$	Intertemporal elasticity of substitution in labor supply	1.7000
$\sigma$	Coefficient of relative risk aversion	2.0000
$D$	Substitutability between private and public consumption	0.7870
$\delta_K$	Private capital depreciation rate	0.0557
$\delta_G$	Public capital depreciation rate	0.0086
$A$	Technology shift parameter	1.0157
$\zeta$	Scalar in tax schedule	0.1641
$1 + \phi$	Ratio of marginal to average tax rate	1.4056
$\omega$	Consumption tax rate	0.0060
$\beta_j$	Discount factors	0.9711, 0.9739, 0.9761, 0.9785, 0.9877
$e_j$	Inherent abilities	0.4934, 0.6418, 0.7469, 0.8101, 1.1568

Table 2: Properties of Benchmark Economy

Variables	U.S. Data	Model
Growth rate ( $\mu$ )	1.0203	1.0203
Private investment-to-private capital ratio ( $i_p$ )	0.0760	0.0760
Output-to-private capital ratio ( $q$ )	0.4608	0.4608
Public capital-to-private capital ratio ( $k_g$ )	0.5070	0.5070
Private investment-to-output ratio ( $i_p/q$ )	0.1649	0.1649
Private consumption-to-output ratio ( $c/q$ )	0.6577	0.6577
Government expenditures-to-output ratio ( $\chi$ )	0.1938	0.1774
Public investment-to-output ratio ( $g_I$ )	0.0318	0.0318
Public consumption-to-output ratio ( $g_C$ )	0.1620	0.1456
Share of total pre-tax income by quintile (%):		
Highest quintile	54.6	54.4
Fourth quintile	19.1	18.9
Middle quintile	13.3	13.1
Second quintile	9.0	8.8
Lowest quintile	4.8	4.6
Gini coefficient (pre-tax income)	0.5000	0.4388
Share of individual income tax liabilities (% by income quintile)		
Highest quintile	67.8	67.96
Fourth quintile	16.8	15.41
Middle quintile	9.4	9.22
Second quintile	4.7	5.28
Lowest quintile	1.2	2.13
Share of wealth by quintile (%):		
Highest quintile	83.4	82.62
Fourth quintile	11.2	14.37
Middle quintile	4.5	2.50
Second quintile	1.1	0.44
Lowest quintile	-0.2	0.08
Gini coefficient (wealth)	0.8160	0.7161

Table 3: Simulation Results for Fiscal Policy Reforms

Variables	Pre-Reform Economy	(a) Increase in $g_C$		(b) Increase in $g_I$	
$\phi$	0.4056	0.4056	0.4406	0.4056	0.4154
$g_I$	0.0318	0.0218	0.0318	0.0418	0.0418
$g_C$	0.1456	0.1556	0.1556	0.1356	0.1456
Growth Rate (%):	2.0299	1.9846	1.9322	2.2093	2.1486
Gini Coefficients:					
Before-Tax Income	0.4388	0.4124	0.3913	0.4071	0.3913
After-Tax Income	0.4073	0.3813	0.3580	0.3759	0.3580
Wealth	0.7161	0.6236	0.5734	0.6030	0.5734
Welfare Gains (%):					
Lowest Quintile					
Second Quintile					
Middle Quintile					
Fourth Quintile					
Highest Quintile					

Notes:  $\phi$  - slope of tax schedule,  $g_I$  - public investment-to-output ratio,  
 $g_C$  - public consumption-to-output ratio.