

Pigouvian Taxation with Costly Administration and Multiple Externalities

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Abstract

Most energy production activities are subsidized despite generating negative externalities. We explain this phenomenon by developing a model that generalizes previous work on second-best Pigouvian taxation. In this model the policymaker will optimally subsidize a harmful production activity if a constraint or cost prevents the first-best correction of an even more harmful alternative. We highlight three examples. First, it may be optimal to subsidize a harmful activity if a constraint prevents the taxation of an even more harmful substitute. Second, it may be optimal to subsidize a harmful activity if there is a large administrative cost associated with taxing an even more harmful substitute. Third, it may be optimal to subsidize a harmful production process if the activity mix at lower levels of output uses more harmful activities than the activity mix at higher levels of output.

Keywords: Administrative cost, Corrective tax, Externality, Optimal tax, Optimal tax systems, Pigouvian tax, Second-best

JEL Codes: H21, H23

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Taxes used to correct externality-generating behaviors are named for Arthur Cecil Pigou, who first described many of their features (Pigou, 1920). Pigouvian taxes improve welfare by aligning private incentives to a notion of public wellbeing. A broad class of policies that influence behavior—including carbon taxes, gasoline taxes, and toll roads—fit into a Pigouvian tax framework.

Classical Pigouvian analysis directs policymakers to tax production activities that generate external harm, setting the tax equal to the marginal harm. In practice many harmful activities are untaxed and surprisingly some are subsidized. This is particularly apparent in energy production as the following table illustrates.¹

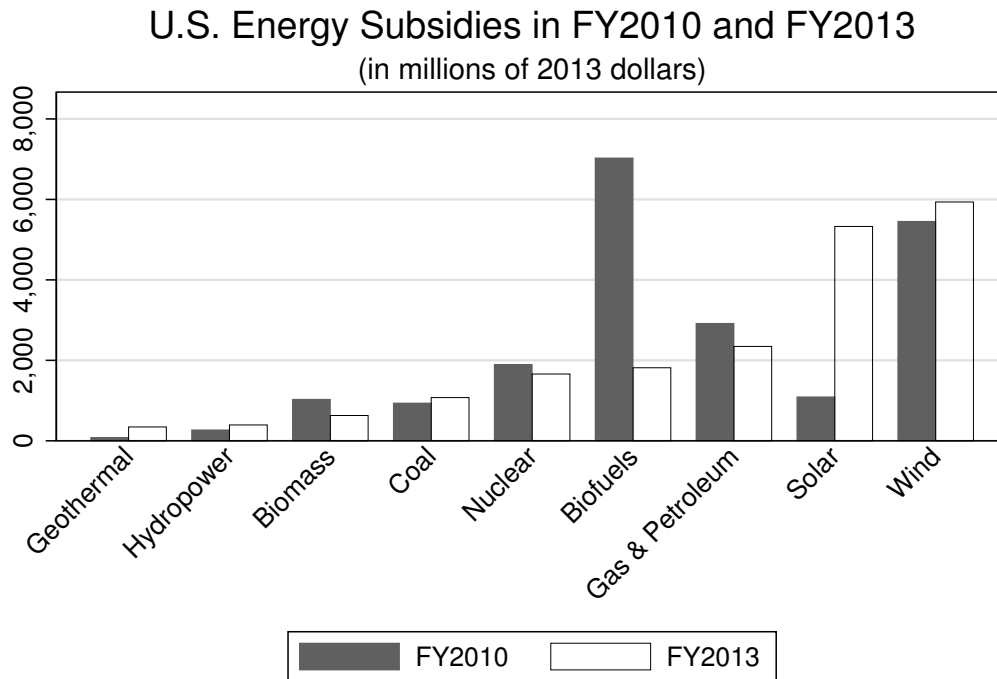
Table 1: Energy technologies, externalities, and policies

| Technology | Externalities | U.S. Policy |
|-------------|----------------------------------------------------------------------------------------------------------|-----------------|
| Coal | Greenhouse gases, Acid rain, Hazardous waste, Airborne particulates, Risk of mining accidents | Tax & Subsidize |
| Oil | Greenhouse gases, Hazardous waste, Airborne particulates, Risk of drilling accidents, Risk of oil spills | Tax & Subsidize |
| Natural Gas | Greenhouse gases, Ecosystem destruction, Airborne particulates, Water contamination | Tax & Subsidize |
| Nuclear | Hazardous waste, Risk of nuclear meltdown | Subsidize |
| Hydropower | Destruction of fisheries and environments, Risk of dam failure | Subsidize |
| Bioenergy | Greenhouse gases | Subsidize |
| Solar | Toxic production process, Ecosystem destruction | Subsidize |
| Geothermal | Toxic gases released | Subsidize |
| Wind | Toxic production (batteries), Eyesore, Harm to wildlife | Subsidize |

While this table abstracts from many of the nuances that underly these policies, it clearly demonstrates that classical (or first-best) Pigouvian analysis is insufficient to explain observed

¹This is a partial list of the external harms discussed in the World Energy Council’s 2016 World Energy Resource Report.

policy. Moreover the subsidies related to energy production exceed \$400 billion per year worldwide and substantially lower the private cost of many different energy production technologies as the following graph illustrates.



Source: U.S. Energy Information Administration

This paper resolves the apparent discrepancy between theory and practice by placing the Pigouvian taxation of multiple externalities in the second-best (Lipsey and Lancaster, 1956). In doing so, this paper becomes the first to suggest several cases in which an optimal tax system would subsidize externally harmful activities.²

Relevant literature. Most of the research on Pigouvian taxes in the second-best has explored how a policymaker should use corrective taxation in the presence of other distortionary taxes (Bovenberg and van der Ploeg, 1994; Bovenberg and Goulder, 1996; Pirttilä and Tuomala, 1997; Pirttilä, 2000; Cremer and Gahvari, 2001; Gahvari, 2014; Jacobs and de Mooij, 2015). This paper takes as a premise that a corrective tax may not be implementable in its first-best form even in the absence of distortionary taxes.

Some work has explored inherently second-best Pigouvian taxation.³ For example, Polinsky and Shavell (1982) consider Pigouvian taxes with one externality and administrative cost. Fullerton

²Bovenberg and Goulder (1996) suggest that a policymaker might subsidize a harmful activity if other taxes are set suboptimally.

³A parallel literature studies criminal sanctions that are costly to administer and enforce (Polinsky and Shavell, 1992; Kaplow, 1990*a,b*).

and West (2002) study the indirect taxation of a single externality, specifically when emissions are not taxable, but the policymaker can tax gasoline consumption and certain attributes of vehicles. Fullerton and Wolverton (2005) examine optimal policy when a two-part instrument must be used in lieu of a direct tax on a harmful activity. Jacobsen et al. (2016) use a sufficient statistics approach that enables comparison between imperfect corrective tax policies.

This paper generalizes existing work, allowing both direct and indirect taxation and accounting for a wide range of constraints and costs that would push analysis to the second-best. More importantly, this paper extends existing work by allowing multiple externalities in the second-best. The policymaker must adhere to policy constraints while minimizing lost private benefit, external harm, and the administrative cost of taxation. The policymaker must choose the optimal tax system (Slemrod, 1990; Slemrod and Yitzhaki, 2002), selecting both the optimal tax base and the optimal tax rates (Yitzhaki, 1979). Following Mayshar (1991) each implemented tax instrument is used up to the point where its marginal benefit begins to be eclipsed by its marginal harm.

A central feature of our model is that when first-best policy is unavailable, the complementarity between different activities matters (Corlett and Hague, 1953; Sandmo, 1978; Cremer and Gahvari, 1993; Cremer, Gahvari and Ladoux, 1998; Cremer and Gahvari, 2001). In plausible settings the policymaker will optimally subsidize a harmful activity because a constraint or cost prevents the first-best correction of an even more harmful alternative.

Roadmap. The next section introduces a simple and flexible model with multiple production activities each of which may have an associated externality. Unsurprisingly, the first-best Pigouvian tax on each activity is equal to the marginal external harm from each activity. The remaining sections explore Pigouvian taxation in the second-best, emphasizing novel scenarios in which the policymaker would optimally subsidize a harmful activity.

Section two describes the optimal tax when the social planner faces constraints. The social planner is first constrained to tax only a subset of production activities, perhaps because she cannot observe all activities or because a powerful lobby protects some activities. If a very harmful activity cannot be taxed, then it may be optimal to subsidize a less harmful substitute. The social planner is then constrained to tax only output, perhaps because she can collect data on market transactions but not production activities. Even if all production activities were harmful, the social planner should subsidize output if the activity mix at lower levels of output uses more harmful activities than the activity mix at higher levels of output.⁴

Section three describes how the optimal tax changes when taxes are administratively costly. Generalizing Polinsky and Shavell (1982) this paper explores several different functional forms

⁴This generalizes Plott (1966) which shows that optimal direct and indirect taxes may have opposite signs if a single harmful activity is used more at lower levels of output.

of administrative costs in a setting with multiple externalities. Administrative costs may (1) be a function of tax rates because higher rates cause more evasion, (2) be a function of activity levels because higher levels of activity require more monitoring, (3) have fixed costs, or (4) be a any combination of (1) - (3).

If administrative costs are a function of taxes rates, the social planner should tax every activity. However, the administrative cost imposes a tradeoff on the policymaker. Higher tax rates reduce the externality but increase the administrative cost. In the single activity case, administrative costs that increase with the tax rate always lower the tax relative to the first-best case, which means that the externality is not fully corrected. In the multiple activity case, administrative costs that increase with tax rates also optimally leave externalities only partially corrected. If an activity has a large externality and its tax has a large administrative cost, it may be optimal to subsidize a less harmful substitute.

If administrative costs are a function of activity levels, it may no longer be optimal to tax every activity because the reduced external harm may be smaller than the administrative cost and lost private benefit. When it is optimal to tax every activity, the social planner should set the tax equal to the externality added to the marginal cost. At that tax rate, the private market internalizes both the externality and the administrative cost. When it is not optimal to tax every activity, the social planner must optimize under incomplete taxation. The analysis of fixed costs follows a similar line of reasoning. Lastly, section three explores how Pigouvian taxation in the second-best must be modified to account for a revenue requirement.

1 First-best Pigouvian taxation

This section introduces a model of Pigouvian taxation with multiple externalities and explores optimal policy when the social planner faces neither constraint nor cost. The result is as expected: the optimal tax vector is equal to the optimal vector of externalities.

Let x be an n -dimensional vector of activities.⁵ Activities are used to generate goods and thus indirectly increase utility. Activities also have private costs and can only be performed in nonnegative quantities. Let $X \subseteq \mathbb{R}_+^n$ be the plausible set of activities.⁶

Following Ramsey (1927), a net benefit function $b : X \rightarrow \mathbb{R}$ maps activity levels to private net benefit. b is twice continuously differentiable, strictly concave, and achieves its maximum

⁵We find activity levels the most natural interpretation. However, x could alternatively be interpreted as emission levels, consumption goods, or inputs. One disadvantage of interpreting x as a consumption good is that goods may be consumed in different ways that don't all cause the same external harm (Sandmo, 1978).

⁶This set is convex, open, has finite measure, and lies entirely in the positive orthant.

somewhere in the interior of X .⁷ If x consists of all productive activities, this is a general equilibrium model. As shown in the appendix, this net benefit function generalizes the maximization of a strictly increasing, strictly concave utility function subject to a convex production possibility frontier.

Both external harm and tax burden are linear functions of activities.⁸ Let e be the n -dimensional vector of activity externalities and t be the n -dimensional vector of activity taxes. Tax revenues are assumed to be lump-sum redistributed.

Proposition 1. *In the first-best (i.e. when activity taxes are complete and tax administration is costless), the optimal tax vector is equal to the externality vector.*

Proof. The private market⁹ solves the following problem:

$$\max_x b(x) - t^\top x$$

which leads to the first order condition $b'(x) - t^\top = 0$.¹⁰ Because b is strictly concave it has an invertible Hessian. Therefore, by the implicit mapping theorem, there exists a continuously differentiable function, $x(t) : \mathbb{R}^n \rightarrow \mathbb{R}^n$, such that $b'(x(t)) - t^\top = 0$. $x(t)$ is the private market's best response function to the tax vector. Note that $b''(x(t))x'(t) = I$, the identity matrix, so $x'(t)$ is also invertible and $x'(t)^{-1} = t'(x)$. The social planner solves the following problem:

$$\max_t b(x(t)) - e^\top x(t)$$

which leads to the first order condition $b'(x(t^*))x'(t^*) - e^\top x'(t^*) = 0$. Substituting $b'(x(t^*)) = t^{*\top}$ yields $(t^* - e)^\top x'(t^*) = 0$. $t^* = e$ is clearly a solution, and the invertibility of $x'(t^*)$ ensures that it is the unique solution. \square

This is Pigou (1920)'s remarkable result generalized to arbitrary dimensions. When $t = e$, the private market fully internalizes every externality, and the policymaker does not need to know or use information other than the externality vector. Using the tax on activity A to induce changes in activity B is not welfare improving because there is no benefit to changing the activity level in a market that already internalizes the externality.

⁷One way to ensure this outcome is to assume that $\lim_{x_i \searrow \partial X} \frac{\partial b}{\partial x_i} = \infty$ and $\lim_{x_i \nearrow \partial X} \frac{\partial b}{\partial x_i} = -\infty$.

⁸Linearity makes the problem more tractable, but the main results are preserved under weak convexity

⁹We describe the private market as an agent for brevity's sake. More precisely, individuals and firms make choices in the private market that result in an aggregate quantity of activity. We represent their behavior as a maximization problem. The solution to the maximization problem is the market equilibrium quantity of activity.

¹⁰We use matrix calculus and the associated notation.

Proposition 2. *In the first-best, $t = e$ is a global maximizer.*¹¹

Proof. Consider two arbitrary activity tax vectors, v and w . Assuming no administrative cost, the change in welfare of moving from v to w is:

$$\begin{aligned}\Delta b - e^\top \Delta x &= b(x(w)) - b(x(v)) - e^\top [x(w) - x(v)] \\ &= b(x(\gamma(1))) - b(x(\gamma(0))) - e^\top [x(\gamma(1)) - x(\gamma(0))]\end{aligned}$$

where $\gamma : \mathbb{R} \rightarrow \mathbb{R}^n$, $\gamma(r) = v + r(w - v)$. By the fundamental theorem of calculus:

$$= \int_0^1 [b'(x(\gamma(r))) - e^\top] x'(\gamma(r)) \gamma'(r) dr$$

Recalling that $b'(x(t)) = t^\top$ and $\gamma'(r) = (w - v)$:

$$= \int_0^1 (v + r(w - v) - e)^\top x'(\gamma(r)) (w - v) dr$$

If $v = e$ (i.e. the social planner shifts away from the optimal activity tax), then:

$$\Delta b - e^\top \Delta x = (w - e)^\top \left[\int_0^1 r x'(\gamma(r)) dr \right] (w - e) \quad (1)$$

The integral is the sum of negative definite matrices and the expression is a quadratic form. Thus the optimal activity tax is strictly superior to any other activity tax. \square

Note that the welfare lost increases with the square of the uncorrected externality and increases with the weighted average size of $x'(t)$ over the set $[e, w]$. $x'(t)$ is weighed by r because close to the optimum the marginal benefit and marginal social harm are equal but diverge more the further w is from e —for the same reason dead weight loss approximations are a triangle.

Proposition 3. *The welfare lost from choosing an arbitrary tax w over the optimal tax can be approximated by $\frac{1}{2} \Delta t^\top \Delta x$.*

Proof. In general, the welfare change of moving from v to w is:

$$\Delta b - e^\top \Delta x = \int_0^1 (\gamma(r) - e)^\top x'(\gamma(r)) \gamma'(r) dr$$

¹¹This is true because the net benefit function is strictly concave. If activities were perfect complements or substitutes, the optimal tax vector would not be unique. In the second-best, it is possible that there exists no finite t^* or that t^* is not unique even with strictly concave b .

Integrating by parts:

$$\begin{aligned}
&= (\gamma(r) - e)^\top x(\gamma(r)) \Big|_{r=0}^1 - \int_0^1 \gamma'(r)^\top x(\gamma(r)) dr \\
&= (w - e)^\top x(w) - (v - e)^\top x(v) - (w - v)^\top \int_0^1 x(\gamma(r)) dr
\end{aligned}$$

Assuming $\int_0^1 x(\gamma(r)) dr \approx \frac{1}{2}(x(w) + x(v))$, which is analogous to Harberger (1964)'s approximation:

$$\begin{aligned}
&\approx (w - e)^\top x(w) - (v - e)^\top x(v) - \frac{1}{2}(w - v)^\top (x(w) + x(v)) \\
&= \left[(w - e) - \frac{1}{2}(w - v) \right]^\top x(w) - \left[(v - e) + \frac{1}{2}(w - v) \right]^\top x(v) \\
&= \frac{1}{2}(w + v - 2e)^\top (x(w) - x(v)) \\
&= \frac{1}{2}(w + v - 2e)^\top \Delta x
\end{aligned}$$

The optimal tax, $t = e$, is thus approximately $\frac{1}{2}\Delta t^\top \Delta x$ better than the arbitrary tax w . \square

This generalizes the *Harberger Triangle* to a setting with more than one taxed good and an initial set of taxes.¹² Assuming there are no externalities, the approximate change in welfare from moving to one tax vector to another is:

$$\Delta b \approx \frac{1}{2}(w + v)^\top (x(w) - x(v)) = \frac{1}{2}(w + v)^\top \Delta x \quad (2)$$

This equation corresponds to the sum of *Harberger Trapezoids* associated with a change from an existing set of taxes to another set of taxes. Setting $v = 0$ simplifies this equation to the *Harberger Triangle* associated with a change from no taxes to a new set of taxes, in a setting with more than one good and more than one tax:

$$\Delta b \approx \frac{1}{2}w^\top (x(w) - x(0)) = \frac{1}{2}w^\top \Delta x \quad (3)$$

¹²See generally Auerbach (1985) and Auerbach and Hines (2002).

2 Constrained Pigouvian taxation

This section modifies the model presented in the previous section by exploring two different constraints. First, the policymaker is constrained to tax only a subset of activities. We call this *incomplete taxation*. Second, the policymaker is constrained to tax only market transactions. We call this *output taxation* in contrast to *activity taxation*. A policymaker may not be able to tax some activities because she faces political constraints or because the measurement of some activities is technologically impossible, prohibitively expensive, or particularly susceptible to evasion. Even if no activities can be taxed, the policymaker may be able to tax output, which is easier to measure because of the record keeping associated with market transactions.

2.1 Incomplete taxation

Leaving even one of the relevant activities untaxed alters the analysis because the tax vector no longer sets the marginal social benefit of each activity equal to the marginal social cost of each activity.

Example 1. Let x_w be operating a wind turbine and x_c be operating a coal plant. Assume there is only one tax, t_w on x_w . The private market maximizes $b(x_w, x_c) - t_w x_w$, leading to the first order conditions $\frac{\partial b}{\partial x_w} = t_w$ and $\frac{\partial b}{\partial x_c} = 0$. The social planner's problem is:

$$\max_{t_w} b(x_w(t_w), x_c(t_w)) - e_w x_w(t_w) - e_c x_c(t_w)$$

with first order condition $\frac{\partial b}{\partial x_w} \frac{\partial x_w}{\partial t_w} + \frac{\partial b}{\partial x_c} \frac{\partial x_c}{\partial t_w} - e_w \frac{\partial x_w}{\partial t_w} - e_c \frac{\partial x_c}{\partial t_w} = 0$. Substituting the private market first order condition leads to $(t_w^* - e_w) \frac{\partial x_w}{\partial t_w} - e_c \frac{\partial x_c}{\partial t_w} = 0$. Since $\frac{\partial x_w}{\partial t_w} < 0$ the optimal tax is:

$$t_w^* = e_w + e_c \frac{\partial x_c}{\partial t_w} \frac{\partial t_w}{\partial x_w}$$

The sign of $\frac{\partial x_c}{\partial t_w}$ is ambiguous and depends on the complementarity of x_w and x_c . If the two activities are substitutes then $\frac{\partial x_c}{\partial t_w} > 0$; if they are complements then $\frac{\partial x_c}{\partial t_w} < 0$. A subsidy will be optimal if the taxed activity, operating the wind turbine, has the smaller externality and the two activities are very substitutable. In our example, if the coal plant has a much larger external harm, the wind turbine is a substitute, and the social planner cannot administer a tax on coal, then a subsidy on wind will be optimal, even though using wind to generate electricity has an external harm.

In the above example, the optimal tax depends on complementarity because the marginal pri-

vate benefit of each untaxed activity will be 0 regardless of that activity's external harm. Because of this uncorrected externality the marginal social benefit of an untaxed activity will not equal that activity's marginal social cost. Using taxes to change the levels of untaxed, externally harmful activities improves welfare. At the optimum, the marginal social benefit of each tax is equal to that tax's marginal social cost. The marginal social benefit of a tax is the reduction in external harm of the taxed activity and all untaxed activities, and the marginal social cost of the tax is the lost private benefit. The following proposition generalizes the example to arbitrary dimensions.

Proposition 4. *If tax administration is costless, the optimal tax on each taxed activity is equal to the externality generated by that activity plus the externalities of all untaxed activities weighted by the responsiveness of the untaxed activity to changes in the taxed activity.*

Proof. Let Θ be the power set of $\{1, \dots, n\}$, θ an arbitrary element of Θ , m the dimension of θ , x_θ the m -dimensional vector of taxed activities, \bar{x}_θ the $n - m$ -dimensional vector of untaxed activities, t_θ the m -dimensional vector of taxes on x_θ , e_θ the m -dimensional vector of externalities generated by x_θ , and \bar{e}_θ the $n - m$ -dimensional vector of externalities generated by \bar{x}_θ . The private market solves the following problem:

$$\max_x b(x) - t_\theta^\top x_\theta$$

with solution $\frac{\partial b}{\partial x_j} = 0$ for $j \notin \theta$ and $\frac{\partial b}{\partial x_i} = t_i$ for $i \in \theta$. As before, the concavity of b ensures that a continuously differentiable best response function, $x(t_\theta)$ exists. The social planner solves the following problem:

$$\max_{t_\theta} b(x(t_\theta)) - e_\theta^\top x_\theta(t_\theta) - \bar{e}_\theta^\top \bar{x}_\theta(t_\theta)$$

which leads to the first order condition $b'(x(t_\theta^*))x'(t_\theta^*) - e_\theta^\top x'_\theta(t_\theta^*) - \bar{e}_\theta^\top \bar{x}'_\theta(t_\theta^*) = 0$. Note that the marginal harm of the tax is equal to the marginal benefit of the tax. Substituting the private market first order condition yields $(t_\theta^* - e_\theta)^\top x'_\theta(t_\theta^*) - \bar{e}_\theta^\top \bar{x}'_\theta(t_\theta^*) = 0$. Rearranging and applying the invertibility of $x'_\theta(t_\theta^*)$ yields $t_\theta^{*\top} = e_\theta^\top + \bar{e}_\theta^\top \bar{x}'_\theta(t_\theta^*)x'_\theta(t_\theta^*)^{-1}$. For each individual tax¹³:

$$t_i^* = e_i + \sum_{k \in \theta} \sum_{j \notin \theta} e_j \frac{\partial x_j}{\partial t_k} \frac{\partial t_k}{\partial x_i} \quad (4)$$

□

Note the special case, $m = n$ in which $x'(t_\theta^*) = x'_\theta(t_\theta^*)$ gives $t_\theta^* = e$ as before. If $m < n$ each

¹³In the second-best, the optimal tax equations are often implicit equations—both sides are a function of the tax.

activity tax may be above or below the associated externality; it may even have the opposite sign as the associated externality.

2.2 Output taxation

An output tax cannot generally induce the private market to select the socially optimal combination of activities. Whereas activity taxes can induce firms to substitute one activity for another, an output tax generally cannot. Consider, for example, producing electricity from either coal or wind. Assume using coal is privately cheaper (by an arbitrarily small amount) but also has a larger externality. If coal and wind are perfect substitutes, an output tax will not discourage coal use relative to wind use, but an activity tax on coal will.

Output taxes can, however, induce the private market to substitute between activities if different combinations of activities are optimal at different scales of production. A tax on electricity would cause substitution from coal to wind if using coal exhibited better economies of scale. Similarly, a subsidy on electricity would cause substitution from coal to wind if using wind exhibited better economies of scale.

The model used in the previous section is also used in this section albeit with some modification. In the previous section, the model made no explicit reference to output. In fact, the model could implicitly include many different outputs. In this section, we restrict the model to one output and introduce the increasing and weakly concave function $q(x) : X \rightarrow \mathbb{R}$ which maps the vector of activities to the quantity of output produced.¹⁴ Let τ be the tax on output. Then the private market's problem is:

$$\max_x b(x) - \tau q(x)$$

with first order condition $b'(x(\tau)) - \tau q'(x(\tau)) = 0$, where $x(\tau) : \mathbb{R} \rightarrow \mathbb{R}^n$ is the private market's best response function to τ .¹⁵ The planner's problem is then:

$$\max_{\tau} b(x(\tau)) - e^{\top} x(\tau)$$

with first order condition $b'(x(\tau^*))x'(\tau^*) - e^{\top} x'(\tau^*) = 0$, which sets the marginal benefit of the tax equal to the marginal cost of the tax —where the benefit is reduced external harm and the cost

¹⁴The appendix contains further discussion of $q(x)$.

¹⁵The appendix contains a proof that $x(\tau)$ exists and also describes some of $x(\tau)$'s properties.

is reduced private net benefit. Substituting in the private market optimum:

$$\tau^* q'(x(\tau^*)) x'(\tau^*) = e^\top x'(\tau^*)$$

With a single activity, $x'(\tau^*)$ is a negative scalar. Dividing the planner's problem by $x'(\tau^*)$ and rearranging yields:

$$b'(x(\tau^*)) = e \quad (5)$$

Thus, since at the optimal activity tax $b'(x(\tau^*)) = e$, the activity tax and output tax can both achieve the first-best if there is only one activity. The optimal output tax, $\tau^* = e/q'(x(\tau^*))$, is equal marginal externality of output, which is the externality of the activity, divided by the marginal output of the activity at the optimal tax.

When there are multiple activities an output tax cannot generally achieve the first best because $x'(\tau^*)$ will be vector valued. In that case, the optimal tax is a ratio of the marginal external harm of the tax to the marginal product of the tax:

$$\tau^* = \frac{e^\top x'(\tau^*)}{q'(x(\tau^*)) x'(\tau^*)} = \frac{e^\top A q'(x(\tau^*))^\top}{q'(x(\tau^*)) A q'(x(\tau^*))^\top} \quad (6)$$

where $A = (b''(x(\tau)) - \tau q''(x(\tau)))^{-1}$ and is negative definite as described in the appendix. The denominator is the marginal output of an increase in the output tax and is always negative. The numerator is the marginal external harm of an increase in the output tax and may be either positive or negative. The output tax will be larger when the activities that decrease the most in response to the output tax have large externalities. The output tax will have smaller magnitude if the activities that are most responsive to the output tax have large marginal products. The output tax will be negative if very harmful activities are relatively less productive at high levels of output.

Example 2. Let x_w be operating a wind mill and x_c be operating a coal plant to produce electricity. Assume there is a tax τ levied on electricity. The private market maximizes $b(x_w, x_c) - \tau q(x_w, x_c)$ with first order conditions $\frac{\partial b}{\partial x_w} = \tau \frac{\partial q}{\partial x_w}$ and $\frac{\partial b}{\partial x_c} = \tau \frac{\partial q}{\partial x_c}$. The social planner's problem is:

$$\max_{\tau} b(x_w(\tau), x_c(\tau)) - e_w x_w(\tau) - e_c x_c(\tau)$$

with first order condition $\frac{\partial b}{\partial x_w} \frac{\partial x_w}{\partial \tau} + \frac{\partial b}{\partial x_c} \frac{\partial x_c}{\partial \tau} - e_w \frac{\partial x_w}{\partial \tau} - e_c \frac{\partial x_c}{\partial \tau} = 0$. Substituting in the private market

optimum yeilds $\tau^* \frac{\partial q}{\partial x_w} \frac{\partial x_w}{\partial \tau} + \tau^* \frac{\partial q}{\partial x_c} \frac{\partial x_c}{\partial \tau} - e_w \frac{\partial x_w}{\partial \tau} - e_c \frac{\partial x_c}{\partial \tau} = 0$. Rearranging, the optimal tax is:

$$\tau^* = \frac{e_w \frac{\partial x_w}{\partial \tau} + e_c \frac{\partial x_c}{\partial \tau}}{\frac{\partial q}{\partial x_w} \frac{\partial x_w}{\partial \tau} + \frac{\partial q}{\partial x_c} \frac{\partial x_c}{\partial \tau}}$$

By assumption both wind mills and coal plants pollute ($e_w > 0$ and $e_c > 0$) and more electricity is produced if power stations increase operations ($\frac{\partial q}{\partial x_w} > 0$ and $\frac{\partial q}{\partial x_c} > 0$). The signs of $\frac{\partial x_w}{\partial \tau}$ and $\frac{\partial x_c}{\partial \tau}$ are ambiguous, but at least one of them must be negative. A subsidy will be optimal if $e_w \frac{\partial x_w}{\partial \tau} + e_c \frac{\partial x_c}{\partial \tau} > 0$, which would happen if the use of coal increases with the tax ($\frac{\partial x_c}{\partial \tau} > 0$), coal produces a large externality relative to wind ($e_c > e_w$), and coal must be more responsive to the tax than wind ($|\frac{\partial x_w}{\partial \tau}| < |\frac{\partial x_c}{\partial \tau}|$). Note that this implies that the marginal output of wind is larger than coal, ($\frac{\partial q}{\partial x_w} > \frac{\partial q}{\partial x_c}$)—if not, increasing the output tax would increase output.

In order to make a welfare comparison between the optimal output tax and the optimal activity tax, we find a map from the output tax to activity taxes.

Proposition 5. *For any output tax the private market will respond as if there is a tax on each activity equal to the output tax times the marginal product of that activity.*

Proof. We want to find $t(\tau)$ such that $x(t(\tau)) = x(\tau)$. Recall that $b'(x(t)) = t^\top$ and $b'(x(\tau)) - \tau q'(x(\tau)) = 0$. Thus

$$t(\tau)^\top = \tau q'(x(\tau)) \tag{7}$$

□

Substituting the optimal output tax derived above:

$$t(\tau^*)^\top = \frac{e^\top x'(\tau^*)}{q'(x(\tau^*))x'(\tau^*)} q'(x(\tau^*)) \tag{8}$$

The higher the marginal product of an activity, the higher the effective tax on that activity. This relationship makes sense because with a tax on output, an increase in an activity causes an increase in the tax burden proportional to the activity's marginal product.

Applying that derivation from the previous section shows that the welfare lost if the social planner is constrained to tax only output is:

$$\Delta b - e^\top \Delta x = (t(\tau^*) - e)^\top \left[\int_0^1 r x'(\gamma(r)) dr \right] (t(\tau^*) - e) \tag{9}$$

where $\gamma(r) = e + r(t(\tau^*) - e)$. Note that the integral is the sum of negative definite matrices and that the expression is a quadratic form. Thus the optimal activity tax is always weakly better than any output tax.¹⁶ Note that the welfare lost increases with the square of the uncorrected externality and increases with the weighted average size of $x'(t)$ over the set $[e, t(\tau^*)]$. $x'(t)$ is weighed by r for the same reason dead weight loss approximations are a triangle; close to the optimum the marginal benefit and marginal social harm are equal but diverge the further $t(\tau^*)$ is from e . Applying the approximation derived in the previous section:

$$\Delta b - e^\top \Delta x \approx \frac{1}{2}(t(\tau^*) - e)^\top (x(e) - x(\tau^*)) \quad (10)$$

$$= \frac{1}{2}(\tau^* q'(x(\tau^*)) - e)^\top (x(e) - x(\tau^*)) \quad (11)$$

At the optimal activity tax there are no uncorrected externalities. At the optimal output tax the uncorrected externality is $(t(\tau^*) - e)$ or $(\tau^* q'(x(\tau^*)) - e)$. Note that the uncorrected externality is small if activities with large external harms also have high marginal products because the output tax discourages activities with high marginal products relatively more. Using the approximation from the previous section, the average uncorrected externality for each activity is half of the uncorrected externality under the output tax, $\frac{1}{2}(t(\tau^*) - e)$. Multiplying the uncorrected externality by the ‘excess’ amount of the activity that occurs under the output tax, $(x(e) - x(\tau^*))$ yields the total welfare loss.

The results presented in this section could be generalized further. The planner could be constrained to tax output and a subset of activities. The planner would achieve the first-best if she could tax $n - 1$ activities, because the output tax would provide the additional degree of freedom necessary. With fewer activity taxes, the planner would generally remain in the second-best. Another possible extension would be to allow multiple outputs, with either complete or incomplete output taxation. In that case, with some functional form assumptions on b and q , the planner would be able to achieve the first-best if the number of output taxes equalled or exceeded the number of activities.

3 Costly Pigouvian taxation

Even when policymakers are unconstrained, tax policy will be second-best if corrective taxes carry an administrative cost. This section highlights how the optimal tax is influenced by the functional

¹⁶If the optimal activity tax is equal to marginal product times the optimal output tax, then both taxes achieve the same welfare because $t(\tau^*) = e$.

form of the administrative cost.

Administrative cost includes expenditures on measurement, enforcement, collections, legislation, and litigation. Some of these costs appear in the Internal Revenue Service budget, which was 11.5 billion dollars for fiscal year 2015.¹⁷ We consider both fixed and variable administrative costs. We call the case when administrative cost increases with activity levels *measurement costs*. Measurement costs arise because it is costly to determine the level of pollution generating activities, regardless of who is making these measurements and even if all parties are behaving honestly. Specific examples of these costs include the monitoring devices and the scientists who design and operate them. We call the case when administrative cost increases with tax rates *enforcement costs*. Enforcement costs arise if evasion increases with tax rates and the government pours more resources into tax enforcement, including more auditors, more lawyers, and more evasion detection software. We call the case when administrative cost increases with tax revenue collected *bureaucracy costs*. Bureaucracy costs resemble the Flypaper effect (Hines and Thaler, 1995)—larger revenues induce larger bureaucracies—for whatever reason the money sticks.

3.1 Fixed administrative costs

If there are fixed administrative costs, it may not be optimal to tax some activities, leading to incomplete taxation. Leaving even one of the relevant activities untaxed alters the analysis (compared to complete taxation) because the tax vector no longer sets the marginal benefit of each activity equal to the marginal harm of each activity—for all untaxed activities, the marginal private benefit is equal to 0 regardless of the tax on the other variables. At the optimum each tax must not only account for the activity it is applied to but also every other untaxed activity.

If each activity tax has its own fixed cost, the social planner must optimize the social welfare function 2^n times—once for each possible combination of taxes—using incomplete taxes as described above. Let the fixed cost for each tax be f_i . Recall that the power set of $\{1, \dots, n\}$ is Θ . The social planner's problem is:

$$\max_{\theta \in \Theta} \left\{ \max_{t_\theta} b(x(t_\theta)) - e^\top x(t_\theta) - \sum_{i \in \theta} f_i \right\}$$

No closed form solution exists, but the optimal tax expression for each subproblem is:

$$t_\theta^{*\top} = e_\theta^\top + \bar{e}_\theta^\top \bar{x}'_\theta(t_\theta^*) x'_\theta(t_\theta^*)^{-1} \quad (12)$$

¹⁷See IRS (2016) for additional information on the IRS budget.

The planner should choose the θ^* , the set of taxes which if set optimally will maximize welfare.

3.2 Variable administrative costs with one activity

In this section, the social planner must construct an administrative apparatus to collect and enforce taxes. Let c map the activity level and tax rate to administrative cost. Formally c is continuously differentiable and weakly convex and $\arg \min(c) = (0, 0)$. These assumptions are made for tractability, but they are consistent with a planner who employs the most effective tax collecting and enforcement resources first. Administrative cost and marginal administrative cost (with respect to x) are both increasing in activity level. A subsidy should also be costly to administer, so administrative cost and marginal administrative cost (with respect to t) are both increasing in the tax rate above 0 and are both decreasing in the tax rate below 0.

The social planner solves the following problem:

$$\max_t b(x(t)) - ex(t) - c(x(t), t)$$

which leads to the first order condition $b'(x(t^*)) \frac{\partial x}{\partial t} - e \frac{\partial x}{\partial t} - c_1 \frac{\partial x}{\partial t} - c_2 = 0$ where c_i denotes the partial derivative of c with respect to its i^{th} argument. Note the general rule: the marginal social benefit of the tax (reduced externality and decreased administrative cost) is equal to the marginal social cost of the tax (reduced private benefit and increased administrative cost). Substituting $b'(x(t^*)) = t^*$, dividing by $\frac{\partial x}{\partial t}$, and rearranging yields:

$$t^* = e + c_1 + c_2 \frac{\partial t}{\partial x} \tag{13}$$

The following table presents the optimal tax associated with several possible functions of administrative cost. Note that for all cases with administrative cost the optimal tax equations are implicit equations.

Table 2: Single activity optimal taxes

| Case | Cost function | Optimal Pigouvian tax | Previous Reseach |
|-------------------|----------------|----------------------------------------------------------------------------------------|---------------------------|
| No cost | 0 | $t^* = e$ | Pigou (1920) |
| Enforcement costs | $c(t)$ | $t^* = e + \frac{\partial c}{\partial t} \frac{\partial t}{\partial x}$ | |
| Measurement costs | $c(x)$ | $t^* = e + \frac{\partial c}{\partial x}$ | Polinsky & Shavell (1982) |
| Bureaucracy costs | $c(xt) = c(R)$ | $t^* = e + \frac{\partial c}{\partial R} (t^* + x(t^*) \frac{\partial t}{\partial x})$ | Polinsky & Shavell (1982) |
| Arbitrary costs | $c(x, t)$ | $t^* = e + c_1 + c_2 \frac{\partial t}{\partial x}$ | |

When administrative cost takes the $c(t)$ functional form, the marginal administrative cost at $t = 0$ is 0. Thus the policymaker will always implement a tax. However, the tax is not equal to the externality and at the optimum the marginal social benefit of the activity is smaller than the marginal social cost of the activity. This is because when administrative cost is a function of tax rates, higher taxes reduce the external harm but increase administrative cost. The marginal benefit of a higher tax is lower external harm, and the marginal cost of a higher tax is lower private net benefit and higher administrative cost. Thus the tax rate is always lower than the case with no administrative cost,¹⁸ which leaves an uncorrected externality. This shows up in the model because $\frac{\partial c}{\partial t} / \frac{\partial x}{\partial t} < 0$.¹⁹

When administrative cost takes the $c(x)$ functional form, there is a discrete jump in the administrative cost if policy maker increases the tax from 0—i.e. when the policymaker decides to implement a tax. It is possible that $b(x(0)) - ex(0) > b(x(t^*)) - ex(t^*) - c(x(t^*))$ in which case the optimal tax is $t = 0$, and the externality is uncorrected. When implementing a tax is optimal, the optimal tax is not equal to the externality. If administrative cost is a function of x , the social planner should raise the tax until the marginal social benefit of the activity equals the marginal social cost of the activity. Because administrative cost increases only with x the administrative cost can be interpreted as an additional externality. Thus the optimal tax induces the private market to fully internalize the externality and the administrative cost. The tax rate is always higher than the case with no administrative cost.²⁰ This shows up in the model because $\frac{\partial c}{\partial x} > 0$.

3.3 Variable administrative costs with multiple activities

The cost function $c(x, t)$, $c : X \times \mathbb{R}^n \rightarrow \mathbb{R}$, is the multidimensional analogue of the single activity case. We assume that $c(x, t)$ is continuously differentiable, weakly convex, and has $\arg \min(c) = (0, 0)$. The private market problem is the same as in the multiple activity, complete taxation, costless administration case. The social planner solves:

$$\max_t b(x(t)) - e^\top x(t) - c(x(t), t)$$

which leads to the first order condition $b'(x(t^*))x'(t^*) - e^\top x'(t^*) - c_1(x(t^*), t^*)x'(t^*) - c_2(x(t^*), t^*) = 0$, where c_1 denotes the partial derivative of c with respect to its first vector argument and c_2 is the partial derivative of c with respect to its second vector argument. Substituting in the private market

¹⁸Similarly, if there is a positive externality the subsidy rate is always lower than the case with no administrative cost.

¹⁹For positive externalities $\frac{\partial c}{\partial t} / \frac{\partial x}{\partial t} > 0$. In either case the optimal tax will never be 0 because the marginal administrative cost is 0 at $t = 0$.

²⁰However, the optimal subsidy is always smaller than the case with no administrative cost.

first order condition and applying $x'(t^*)$'s invertibility yields:

$$t^{*\top} = e^\top + c_1 + c_2 x'(t^*)^{-1} \quad (14)$$

The table below describes some special cases. Note that for both cases with administrative cost the optimal tax expressions are implicit formulas.

Table 3: Multiple activity optimal taxes

| Case | Cost function | Optimal Pigouvian tax matrix notation | Optimal Pigouvian tax element notation |
|-------------------------------|---------------|--------------------------------------------|------------------------------------------------------------------------------------------|
| No administrative cost | $c = 0$ | $t^{*\top} = e^\top$ | $t_i^* = e_i$ |
| Enforcement costs | $c = c(t)$ | $t^{*\top} = e^\top + c'(t^*)x'(t^*)^{-1}$ | $t_i^* = e_i + \sum_j \frac{\partial c}{\partial t_j} \frac{\partial t_j}{\partial x_i}$ |
| Measuring costs ²¹ | $c = c(x)$ | $t^{*\top} = e^\top + c'(x(t^*))$ | $t_i^* = e_i + \frac{\partial c}{\partial x_i}$ |

When administrative cost is a function of only tax rates, the optimal tax on each activity is equal to the externality generated by that activity plus the sum of all the marginal administrative costs weighted by the responsiveness of the tax rate to changes in activity. The social planner should tax each activity because the marginal cost of increasing or decreasing a tax of 0 is 0.²² However, just as as in the single activity case, an increase in the tax presents a tradeoff: less externality, but higher administrative cost. Thus, optimal taxes do not induce the private market to fully internalize the externality. The social planner should raise (or lower) taxes until the marginal social benefit of the tax is equal to the tax's marginal social harm. In particular, the planner should use taxes with low marginal administrative cost to affect other activity levels. A subsidy may be optimal for some externally harmful activities.²³ c' will take negative values whenever there is a subsidy because the cost of administration will decrease the less negative the tax becomes.

Example 3. Let x_w be operating a wind mill and x_c be operating a coal plant to produce electricity. Assume that both activities are taxed and administrative cost increases with t . The private market maximizes $b(x_w, x_c) - t_w x_w - t_c x_c$, leading to the first order conditions $\frac{\partial b}{\partial x_w} = t_w$ and $\frac{\partial b}{\partial x_c} = t_c$. The concavity of b ensures that $x_w(t_w, t_c)$ and $x_c(t_w, t_c)$ exist. The social planner's problem is:

$$\max_t b(x_w(t_w, t_c), x_c(t_w, t_c)) - e_w x_w(t_w, t_c) - e_c x_c(t_w, t_c) - c(t_w, t_c)$$

²²It is possible that $t_i^* = e_i + \sum_j \frac{\partial c}{\partial t_j} \frac{\partial t_j}{\partial x_i} = 0$.

²³The matrix $t'(x(t^*)) = b''(x(t^*))$ describes the effect of a change in the activity vector on tax rates at the optimal tax rate. Because b is concave, this matrix is negative definite, so the diagonals are all negative. An increase in t_i will, thus, reduce x_i although it may increase or have no effect on x_j . This implies that t^* may have negative entries.

with first order conditions:

$$\begin{aligned} \frac{\partial b}{\partial x_w} \frac{\partial x_w}{\partial t_w} + \frac{\partial b}{\partial x_c} \frac{\partial x_c}{\partial t_w} - e_w \frac{\partial x_w}{\partial t_w} - e_c \frac{\partial x_c}{\partial t_w} - \frac{\partial c}{\partial t_w} &= 0 \text{ and} \\ \frac{\partial b}{\partial x_w} \frac{\partial x_w}{\partial t_c} + \frac{\partial b}{\partial x_c} \frac{\partial x_c}{\partial t_c} - e_1 \frac{\partial x_w}{\partial t_c} - e_c \frac{\partial x_c}{\partial t_c} - \frac{\partial c}{\partial t_c} &= 0 \end{aligned}$$

Substituting the private market first order condition and manipulating the equation leads to:

$$\begin{aligned} t_w^* &= e_w + \frac{\partial c}{\partial t_w} \frac{\frac{\partial x_c}{\partial t_c}}{\frac{\partial x_w}{\partial t_w} \frac{\partial x_c}{\partial t_c} - \frac{\partial x_w}{\partial t_c} \frac{\partial x_c}{\partial t_w}} - \frac{\partial c}{\partial t_c} \frac{\frac{\partial x_c}{\partial t_w}}{\frac{\partial x_w}{\partial t_w} \frac{\partial x_c}{\partial t_c} - \frac{\partial x_w}{\partial t_c} \frac{\partial x_c}{\partial t_w}} \text{ and} \\ t_c^* &= e_c + \frac{\partial c}{\partial t_c} \frac{\frac{\partial x_w}{\partial t_w}}{\frac{\partial x_w}{\partial t_w} \frac{\partial x_c}{\partial t_c} - \frac{\partial x_w}{\partial t_c} \frac{\partial x_c}{\partial t_w}} - \frac{\partial c}{\partial t_w} \frac{\frac{\partial x_w}{\partial t_c}}{\frac{\partial x_w}{\partial t_w} \frac{\partial x_c}{\partial t_c} - \frac{\partial x_w}{\partial t_c} \frac{\partial x_c}{\partial t_w}} \end{aligned}$$

Noting that $x'(t)^{-1} = t'(x)$, we have:

$$\begin{aligned} t_w^* &= e_w + \frac{\partial c}{\partial t_w} \frac{\partial t_w}{\partial x_w} + \frac{\partial c}{\partial t_c} \frac{\partial t_c}{\partial x_w} \text{ and} \\ t_c^* &= e_c + \frac{\partial c}{\partial t_c} \frac{\partial t_c}{\partial x_c} + \frac{\partial c}{\partial t_w} \frac{\partial t_w}{\partial x_c} \end{aligned}$$

A subsidy on wind will be optimal if the cost of administering the subsidy is small ($\frac{\partial c}{\partial t_w}$ is negative and has a small magnitude), administering the tax on coal is costly ($\frac{\partial c}{\partial t_c}$ is positive and has a large magnitude), coal is more harmful than wind ($e_c > e_w$), and coal x_c and wind x_w are very substitutable ($\frac{\partial x_c}{\partial t_w}$ is positive with large magnitude).

When administrative cost is a function of only activity levels, it may not be optimal to tax every activity. This is most apparent if the cost function is separable in each of the activities and the planner incurs the cost associated with any tax. If the planner decides to tax a particular activity, there will be a discrete increase in the administrative cost. In general, just as in the fixed cost case, the social planner must optimize 2^n different problems, each with a different variable cost function, c^θ , depending on which activities are taxed. If it is optimal to tax all of the activities, the planner should set the tax equal to the externality plus the marginal administrative cost. At this tax, the private market internalizes both the externality and the administrative tax. If it is not optimal to tax every activity, the complementarity between activities comes into play and it may be optimal to subsidize externally harmful activities.

This section could be generalized to show the optimal tax expression when there fixed and variable costs and also when there is an output tax with costly administration. The intuition highlighted above remains intact. When there is an output tax and a complete set of activity taxes, the private market's problem does not yield a one-to-one best response function, $x(t, \tau)$ —there are infinitely

many t 's and τ 's that lead to the same x . Because of this the implicit expressions that come from the first order conditions of the social planner's problem are only informative about t and τ jointly.

3.4 Revenue requirement

Pigouvian taxes generate revenue in addition to aligning incentives. In this subsection we show how the optimal taxes described above are altered when there is a revenue constraint. Assume that the revenue requirement is $R \geq t_\theta^\top x_\theta(t_\theta)$. We begin with the Lagrangian:

$$\mathcal{L} = b(x(t_\theta)) + \lambda(R - t_\theta^\top x_\theta(t_\theta))$$

with first order condition:

$$b'(x(t_\theta^*))x'(t_\theta^*) - \lambda(t_\theta^{*\top} x'_\theta(t_\theta^*) + x_\theta(t_\theta^*)^\top) = 0$$

which sets the marginal social cost of each tax equal to the marginal revenue of that tax times the Lagrange multiplier and leads to the optimal tax expression:

$$t_\theta^{*\top} = \frac{\lambda}{1 - \lambda} x_\theta(t_\theta^*)^\top x'_\theta(t_\theta^*)^{-1} \quad (15)$$

Note that the Karush Kuhn Tucker conditions require that $\lambda \leq 0$, which makes sense since increasing the required revenue should decrease private net benefit. Now we determine the optimal tax when there is a revenue constraint and Pigouvian taxation using the most general model with incomplete taxation and costly administration.

Proposition 6. *The optimal Pigouvian tax with a revenue constraint is equal to the expression for the optimal revenue constrained tax added to the expression for the optimal unconstrained Pigouvian tax times a scaling factor.*

Proof. Starting with the Lagrangian:

$$\mathcal{L} = b(x(t_\theta)) - e^\top x(t_\theta) - c(x_\theta(t_\theta), t_\theta) + \lambda(R - t_\theta^\top x_\theta(t_\theta))$$

with first order condition:

$$b'(x(t_\theta^*))x'(t_\theta^*) - e^\top x'(t_\theta^*) - c_1 x'_\theta(t_\theta^*) - c_2 - \lambda t_\theta^{*\top} x'_\theta(t_\theta^*) - \lambda x_\theta(t_\theta^*)^\top = 0$$

which sets the marginal social cost of each tax equal to the marginal revenue of that tax times the

Lagrange multiplier. Rearranging:

$$(1 - \lambda)t_\theta^{*\top} x'_\theta(t_\theta^*) = e^\top x'(t_\theta^*) + c_1 x'_\theta(t_\theta^*) + c_2 + \lambda x_\theta(t_\theta^*)^\top$$

The optimal tax expression is:

$$t_\theta^{*\top} = \frac{1}{1 - \lambda} \underbrace{\left(e^\top x'(t_\theta^*) x'_\theta(t_\theta^*)^{-1} + c_1 + c_2 x'_\theta(t_\theta^*)^{-1} \right)}_{\text{optimal Pigouvian tax}} + \underbrace{\frac{\lambda}{1 - \lambda} x_\theta(t_\theta^*)^\top x'_\theta(t_\theta^*)^{-1}}_{\text{optimal revenue constrained tax}} \quad (16)$$

□

The Karush Kuhn Tucker conditions require that $\lambda \leq 0$. If the revenue generated by the Pigouvian tax exceeds R , then $\lambda = 0$ and the optimal tax is equal to the Pigouvian tax.

This version of the Pigouvian tax problem has been analyzed by many papers. Sandmo (1975) suggests an additivity property:²⁴

[T]he optimal tax structure is characterized by what might be called an additivity property; the marginal social damage of commodity m enters the tax formula for that commodity additively, and does not enter the tax formulas for the other commodities, regardless of the pattern of complementarity and substitutability.

Our result generalizes Sandmo (1975)'s additivity property. Strictly speaking, Sandmo's additivity does not hold in our more general setting because complementarity matters in the optimal tax formula.²⁵ However, the optimal tax is the expression for the optimal Pigouvian tax multiplied by $\frac{1}{1-\lambda}$ added to the expression for the optimal revenue constrained tax.

4 Conclusion

Pigou's seminal insight demonstrated that in a first-best world taxes can fully correct externalities. This paper extends his insight into a second-best world, in which the policymaker faces constraints and costs. In this world there are several cases in which an externality cannot or should not be fully

²⁴See Kopczuk (2003), generalizing Sandmo's additivity property; Bovenberg and De Mooij (1994), describing the relationship between a tax on labor and a Pigouvian tax on a polluting consumption good; and Kaplow (2012), demonstrating that changing a commodity tax to the first best Pigouvian tax while making compensating changes in the income tax creates a Pareto improvement in a model where utility is separable in labor and other activities.

²⁵The literature reinforcing complementarity irrelevance is large. See Bovenberg and De Mooij (1994), Bovenberg and van der Ploeg (1994), Bovenberg and Goulder (1996), Fullerton (1997), Pirttilä and Tuomala (1997), Ng (1980), Kopczuk (2003), and Kaplow (2012). A notable exception is Cremer, Gahvari and Ladoux (1998), where complements and substitutes matter in a model that includes heterogeneous consumers and nonlinear external harm.

corrected and even cases in which a harmful activity should be subsidized. Intuitively, the policymaker should use the cheapest tax instruments available to her that effect the greatest reduction in external harm.

We highlight energy production examples, but there are other policy relevant examples of harm subsidies, including needle exchanges. Needle exchanges subsidize the use of heroin with clean needles in an attempt to induce substitution away from the use of heroin with dirty needles. Both activities are thought to be socially harmful, but using dirty needles is more harmful. If second-best considerations prevent the full correction of both activities, it may be optimal to subsidize the clean needles.

Our paper also suggests that optimal policy requires more than determining marginal external harm. In the second-best, the net private benefit function and administrative cost function are also relevant for policy decisions. While the examples of subsidizing harm suggest that policymakers intuitively understand how to set optimal policy in the second-best, our hope is that this paper helps formalize that intuition. Ideally, policymakers will collect the relevant market and administrative cost data to be able to set more precise policy. Future work could estimate optimal policy parameters and identify other interesting optimal harm subsidies.

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A Relationship between b and u

Let $u(x, x_{n+1}) : X \in \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ be a strictly increasing, strictly concave utility function over the choice set X of a representative agent. Let $p(x, x_{n+1}) : \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ be a strictly decreasing, weakly convex function, such that $p = 0$ defines a production possibility frontier. Under the implicit mapping theorem there exists a function f such that for all (x, x_{n+1}) such that $p(x, x_{n+1}) = 0$, $x_{n+1} = f(x)$. $p(x, f(x)) = 0$ and $f_i = -p_i/p_{n+1}$.

Proposition 7. Define $b(x) = u(x, f(x))$. Then

- (i) x^* maximizes b if and only if $(x^*, f(x^*))$ maximizes u subject to $p = 0$; and
- (ii) $\max_x \{b(x)\} = \max_{x, x_{n+1}} \{u(x, x_{n+1}) \text{ s.t. } p(x, x_{n+1}) = 0\}$.

Proof. (i) The first order conditions of the utility maximization problem are $\forall_{i \in \{1, \dots, n\}} : \frac{u_i}{u_{n+1}} = \frac{p_i}{p_{n+1}}$. The first order conditions of the private net benefit function are $b_i = 0 \implies u_i + u_{n+1} f_i = u_i + u_{n+1}(-p_i/p_{n+1}) = 0 \implies \frac{u_i}{u_{n+1}} = \frac{p_i}{p_{n+1}}$. (ii) follows. \square

Proposition 8. $b(x) = u(x, f(x))$ is strictly concave.

Proof. First, note that weakly convex p implies weakly concave f . By the concavity of p :

$$p(y, y_{n+1}) - p(x, x_{n+1}) \leq \sum_{i=1}^{n+1} p_i(x, x_{n+1}) \cdot (y_i - x_i)$$

Then $\forall (x, x_{n+1}), (y, y_{n+1})$ such that $p(x, x_{n+1}) = p(y, y_{n+1}) = 0$

$$\begin{aligned}
0 &\leq \sum_{i=1}^{n+1} p_i(x, x_{n+1}) \cdot (y_i - x_i) \implies \\
-p_{n+1}(x, x_{n+1}) \cdot (y_{n+1} - x_{n+1}) &\leq \sum_{i=1}^n p_i(x, x_{n+1}) \cdot (y_i - x_i) \implies \\
(y_{n+1} - x_{n+1}) &\leq \sum_{i=1}^n (-p_i(x, x_{n+1})/p_{n+1}(x, x_{n+1})) \cdot (y_i - x_i) \implies
\end{aligned}$$

$$\begin{aligned}
p(y, y_{n+1}) - p(x, x_{n+1}) &\leq \sum_{i=1}^{n+1} p_i(y_i - x_i) \implies \\
0 &\leq \sum_{i=1}^{n+1} p_i(y_i - x_i) \implies \\
-p_{n+1}(y_{n+1} - x_{n+1}) &\leq \sum_{i=1}^n p_i(y_i - x_i) \implies \\
y_{n+1} - x_{n+1} &\geq \sum_{i=1}^n (-p_i/p_{n+1})(y_i - x_i) \implies \\
f(y) - f(x) &\geq \sum_{i=1}^n (-p_i/p_{n+1})(y_i - x_i) \implies \\
f(y) - f(x) &\geq \sum_{i=1}^n f_i(y_i - x_i)
\end{aligned}$$

$$\forall \theta \in [0, 1]$$

$$\begin{aligned}
b(\theta x + [1 - \theta]y) &= u(\theta x + [1 - \theta]y, f(\theta x + [1 - \theta]y)) \\
&\geq u(\theta x + [1 - \theta]y, \theta f(x) + [1 - \theta]f(y)) \\
&= u(\theta x + [1 - \theta]y, \theta x_{n+1} + [1 - \theta]y_{n+1}) \\
&> \theta u(x, x_{n+1}) + [1 - \theta]u(y, y_{n+1}) \\
&= \theta b(x) + [1 - \theta]b(y)
\end{aligned}$$

□

This shows that the private net benefit problem generalizes the constrained utility problem. There are two points worth noting. First, there is no tax on x_{n+1} when the private net benefit function is used. Second, using the b function removes prices from the problem, which means that the tax must be in the same units as the objective function.

B The details of the output tax problem

Strictly increasing, weakly concave q maps activities to output. Strictly increasing, strictly concave v maps output to private benefit. Strictly increasing, weakly convex g maps the activities to private cost. $b(x) = v(q(x)) - g(x)$

Proposition 9. *The best response function $x(\tau)$ exists and is continuously differentiable.*

Proof. The private market's problem is:

$$\max_x v(q(x)) - g(x) - \tau q(x)$$

The first order condition is $v'(q(x))q'(x) - g'(x) - \tau q'(x) = 0$. Taking the derivative of first order condition with respect to x

$$\begin{aligned} & v'(q(x))q''(x) + (q'(x))^\top v''(q(x))q'(x) - g''(x) - \tau q''(x) \\ &= \underbrace{(v'(q(x)) - \tau)}_{\text{FOC} \implies +} \underbrace{q''(x)}_{\text{neg. def.}} + \underbrace{v''(q(x))}_{-} \underbrace{(q'(x))^\top q'(x)}_{\text{pos. sem. def.}} - \underbrace{g''(x)}_{\text{pos. sem. def.}} \end{aligned}$$

Since a positive semidefinite matrix multiplied by a negative scalar is a negative semidefinite matrix and since the sum of a negative definite matrix and to semidefinite matrices is a negative definite matrix, the implicit mapping theorem implies that $x(\tau)$ exists and is continuously differentiable. As an aside, $b = v(q) - g$ is concave.

□

Proposition 10. $q'(x(\tau))x'(\tau) < 0$

Proof. Take the derivative of first order condition with respect to τ . Note that the first order condition must still hold, so the derivative with respect to τ will equal 0.

